Arithmetick:

Jn° His Book

VULGAR,
DECIMAL,
INSTRUMENTAL,
ALGEBRAICAL,

1723

In four Parts.

Containing

I. VULGAR ARITHMETICK, In *whole Numbers* and *Fractions*, in a plain and easie method.

II. DECIMAL ARITHMETICK, The Ground and Reason thereof, and its Use illustrated by divers *Examples*.

III. INSTRUMENTAL ARITHMETICK, Performing, by *Decimal Scales*, all kind of *Reductions* (with more expedition than by *Decimal Tables*) Also new *Scales*, whereby the *Square* and *Cube Roots* may be extracted by Inspection only: Both of them new *Artifices*; nothing of the like kind having been before published in any Language. With the Description of *Nepair's Bones* (according to their best Contrivance) and the use of them in *Multiplication*, *Division*, and *Extracting* of *Roots*.

IV. ALGEBRAICAL ARITHMETICK, Containing an Abridgment of the Precepts of that Art, and its Use, illustrated by Questions of divers kinds:

Whereunto is added, the Construction and Use of several Tables of *Interest* and *Annuities*, *VVeights* and *Measures*, both of our own and other Countries.

The Fifth Edition, corrected and enlarged.

By WILLIAM LEYBOURN.

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MAYNOOTH.

TO THE
READER.

HERE is presented unto thee a short *Treatise* of *Arithmetick*, I confess there are enough (if not too many) already extant, notwithstanding I have *adventured* to publish this, more out of *variety* then for any *necessity*, knowing that in the perusal thereof thou wilt find something worthy thy *Labour*, and what in other *Books* of this kind is wanting.

The whole *Treatise* is divided into four Parts. The first contains *Vulgar ARITHMETICK* in *whole Numbers* and *Fractions*: And in every *Rule* there are Examples for Practice added, and *Questions* also wrought by those single *Rules*: In *Multiplication* I have added divers *Compendiums*, or brief ways of *Multiplying*, whereby Sums (having 2 or 3 Figures in the Multiplier) may be performed, without any burthen or charge to the Memory, more than in ordinary *Multiplication*, and yet no other (or at most very few) Figures set down but the Product it self. And in *Division* (which is the most difficult of the four *Species*)

there are Five varieties so that every man, may make use of that which he best understands or fancies: And in the working of the *Golden Rule*, &c. I have (to express variety) made use sometimes of one kind of *Division*, and sometimes of another.

The Second Part contains *DECIMAL ARITHMETICK*, with the Ground and Reason thereof. Also *Tables* of the *Moneys*, *Weights* and *Measures*, used in *England*, with Directions for the making of those *Tables*, and of any other. And lastly, there are *Examples* wrought in *Decimal Numbers*, in all the most usual *Rules* of *Arithmetick*, and those *Examples* are incumbered with as many *Fractions* as can possibly happen in any *Question* concerning *Buying* or *Selling*.

Unto this Second Part there is added an *Appendix* containing certain *Rules* of *Exchanges* with *Tables* of the *Weights* and *Measures* of *Forreign Countries*, compared with the *weights* and *Measures* used in *London*, with an Example to illustrate the use of each *Table*: Also, There are several Tables calculated at 6 *per Cent. Compound Interest*, by which the true valuation of any *Lease* or *Annuity*, or *Money forborn* or *rebated*, may be easily known, with an Example shewing the use of each Table.

The third Part is of *INSTRUMENTAL ARITHMETICK*, which performeth by a new Artifice, by me contrived, any *Question*

Arithmetical

Arithmetical in a *Decimal* way without the help of *Decimal Tables*, by which the whole work of *Reduction* is avoided, there being certain *Scales* of *English Money*, *Weights*, and *Measures* divided, and ſo diſpoſed, that by them (by inſpection only) the *Decimal Fraction* of either *Mony*, *Weight*, or *Measure*, may be ſet down as exactly, and in leſs time than they could have been taken out of the *Decimal Tables* in the Second Part of this *Treatiſe*. And on the contrary, any *Decimal Fraction*, may be reduced into its proper parts of the integer with the ſame facility, ſpeed and exactneſs. I have alſo in this Third part (the better to illuſtrate the uſe, and commodiouſneſs of theſe *Scales*, a Figure whereof is inſerted at the beginning of the Third part) gone through all the moſt uſual *Rules* of *Arithmetick*, giving *Examples* in each *Rule*, by which the *Reader* may plainy perceive what labour there is ſaved by uſing the *Scales*, the whole work of *Reduction* being taken away, and the *Fraction* immediately ſet down at firſt view, without *Addition*; or being already ſet down, *reduced* to the known part of the *Integer*, without *Subſtraction*.

Unto theſe *Decimal Scales*, I have now added Two other *Scales*, for the *Extraction* of *Roots*; By the one you may find the *Root* of any *Number* the *Square* thereof being given; Or, if the *Root* be given, you may find the *Square Number* anſwering thereunto, and

that by inſpection only, without the help of either *Pen* or *Compaſs*—And as this Line doth for extracting the *Square Root*, the other doth the like for the *Cube Root.*

And to make this Third part of *Inſtrumental Arithmetick*, yet the more compleat, I have more largely (than in the former *Editions*) inſiſted upon the *Deſcription* and *Uſe* of *Nepairs*, *Bones*; largely treating of their *Uſe* in *Multiplication*, *Diviſion*, and *Extraction* of *Square* and *Cube Roots*. And laſtly,

The Fourth Part containeth an *ABRIDGMENT of the PRECEPTS of ALGEBRA*, firſt written in French by *James de Billy*, a Tranſlation whereof came to my hands ſome years ſince: In the peruſal whereof, finding the *Precepts* very plain and eaſie, and conſidering that We have but very little of this kind of *Arithmetick* in our Engliſh Tongue; I have adventured to inſert it here as a Fourth Part, thereby to make this *Work* the more compleat; unto which Tranſlation there is further added divers *Queſtions* of good conſequence, which were not in the *Original*, as by comparing them together may appear.

This Treatiſe thus finiſhed, I freely offer unto thee, deſiring thy friendly acceptance, and pardon for ſuch faults as may poſſibly have eſcaped the Preſs, or my ſelf, and in ſo doing thou wilt encourage him, who is

A Friend to all that are
Mathematically affected,

William Leybourn.

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ADVERTISEMENT.

IF any Gentleman, or other Person, desire to be instructed in any of the Sciences Mathematical as *Arithmetick*, *Geometry*, *Astronomy*, the use of the the *Globes*, *Trignometry*, *Navigation*, *Surveying of Land*, *Dialling*, or the like; Either at their own Houses, his Habitation, or such other convenient Place as the Party shall direct, the *Author* hereof will be ready to attend them at times appointed.

Also, if any Persons would have their Land, or any Ground for Building *Surveyed*, , or any Edifice or Building *Measured*, either for the *Carpenters*, *Bricklayers*, *Plaisterers*, *Glasiers*, *Joyners*, or *Masons* Work, he is ready to perform the same either for *Master-Builder* or *Workman*.

Likewise, If any desire to have about his House or Garden, any kind of Sun-Dial, or Dials, of what kind soever, either fixed or movable, he will prepare or make for them such as they shall desire.

You may hear of him at the House of Mr. *Walter Hayes*, at the Cross-Daggers in *More-fields*, next door to the *Popes-Head* Tavern, who makes and selleth all sorts of Mathematical Instruments; Also, At Mr. *Robert Mordens* at the sign of the *Atlas* in *Corn-hill*, near the *Royal Exchange London*, where you may have the best and newest Globes of all Sizes, Maps, Sea Plates, and other Mathematical Instruments.

FINIS.

VULGAR Arithmetick.

The Firſt Part

NUMERATION.

NUMERATION, is accounted the Firſt Part of *Arithmetick*, and it is to know how to read a Sum of figures expreſſed in writing; or to write down any Sum to be expreſſed.

To the doing of which there are four things neceſſary.

Firſt, To know their *Number*, which is *Nine*.

Secondly, Their *ſhapes*, which are 1. 2. 3. 4. 5. 6. 7. 8. 9. Of which the firſt toward the left hand ever ſignifieth *One*, the ſecond *Two*, &c.

Thirdly, To know the *value* of their *places*.

Lastly, How their proper *signification* is altered thereby

The *value* of their places is thus, When two, three, or more figures stand in one Sum, that is without any *Point*, *Line* or *Comma* betwixt them, as 321, that place next the right hand where the figure 1 standeth, is called the place of *Unity*, or *Unites*, and the figure 1 standeth in that place only for *one*, and the figure 2 when it is found in that first place stands only for *two*; and the like of the rest.

But in the Sum 321, above expressed, the figure 2 is in the second place, and every place contains the value of that place before towards the right hand ten times, and therefore the figure 2 doth not signifie *Two*, but (in this second place) ten times two, that is *Twenty*. And so the figure 3, if it had been in that place had signified ten times *Three*, that is *Thirty*; but being here in the third place, it signifies ten times *Thirty*, that is *Three hundred*. And so the whole Sum 321, is to be read: *Three hundred twenty and one*.

It is hereby seen, how their proper significations, which were *Three*, *two*, and *One*, are altered by being thus placed, and the Sum, which otherwise had been but *Six*, is *Three hundred twenty one*, as before.

In like sort if there had been more places, as Seven, the value is quite through increased ten times, by being a place bound towards the left hand; as in the Sum 1 1 1 1 1 1 1 The figure 1 in the second place stands for ten times one (that is *Ten*,) in the third, for ten times ten (which is one *Hundred*,) in the fourth for *Ten hundred*, (which is called One Thousand,) in the fifth, for ten Thousand, in the sixth, for ten times *Ten thousand*, (which is *One hundred thousand*) in the last.

laſt (here the ſeventh) place, for *Ten hundred thouſand*, which is called a *Million*: and ſo on, if there were more places, obſerving the ſame order.

Now to read this readily, make a prick over the place of *Unity*, another the third from it, and over every third ſtill towards the left hand, for ſo thoſe points will be over the places of *Unites*, *Thouſands* and *Millions*; and ſo beginning at the laſt, that is, at the left hand; read *One Million*, and becauſe the three following towards the right, ſignifie properly *One hundred* and *eleven*, but the prick belonging to them is in the place of thouſands, call them *One hundred and eleven thouſand*, and the three remaining being under the point over *Unity*, ſignifie only *One hundred* and *eleven*; and all three points read together in one ſum is *One million, one hundred* and *eleven Thouſand, one hundred and eleven.*

In like manner, if this number 73598624, were given to be read (according to former directions) make a prick over every third figure, beginning with the firſt figure towards the right hand, (which is the place of *Unity*) and then will your number ſtand thus,

$$\dot{7}\,3\,5\,\dot{9}\,8\,6\,\dot{2}\,4$$

Then for the ready reading thereof (becauſe the third prick ſignifieth *Millions*) call all the figures towards the left hand, ſtanding from that prick, Millions, which in this example are 7 and 3, ſo then this number contains 73 *Millions*, 598 *Thouſand*, 624 *Six hundred twenty four*, Which in words at length we read, *Seventy three Millions, five hundred ninety eight thouſand, ſix hundred twenty four.*

Let thus much ſuffice concerning the placing of large numbers, for the ready reading of them, only take theſe four *Tables* following, for illuſtration of

what hath been hitherto delivered in words, the very ſight whereof is better than a whole Chapter of information.

The firſt Table is thus to be read] *One* in the firſt place, ſignifies *One*. *One* in the ſecond place, ſignifies Ten. *One* in the third place, ſignifies a *hundred*, &c. as in the Table.

The ſecond Table is thus to be read] In this Table you ſhall find the laſt number thereof to conſiſt of theſe figures, 357.846.903 with a point or a comma betwixt every third figure, for diſtinction ſake, and alſo every three figures in their order are connected together with this ⏟ brace, which denominates the places of *Millions*, *Thouſands*, *Hundreds*, ſo that the laſt number of this Table will evidently appear to be 357 *Millions*, 846 *Thouſands*, 903 *Nine hundred and three*.

The third Table] is only certain rows of figures, ſet together, and orderly diſpoſed, having the ſignification or reading of the ſame Numbers in words at length to them annexed, and is only inſerted for the better ſatisfaction of ſuch as ſhall doubt whether they perfectly underſtand what hath been before taught.

The fourth Table] Is much like the ſecond, only it conſiſteth but of one number and extends three places farther then the greateſt number in the ſecond Table doth: *viz.* to twelve places; which figures are thus to be read, 736 *Millions of Millions*, 842 *Millions*, 708 *Thouſand*, 645 *Six hundred forty five.*

(I. *Table*.)

(I. *Table.*)

One in the	first	place signifies	1 one
	second		10 ten
	third		100 a hundred
	fourth		1000 a thousand
	fifth		10000 ten thousand
	sixth		100000 a hundred thousand
	seventh		1000000 a Millions
	eight		10000000 ten Millions
	ninth		100000000 a hundred Millions

(II. TABLE.)

								8
							5	4
						7	6	2
					3:	4	8	3
				9	7.	6	2	1
			2	4	3.	7	9	4
		8.	7	4	9.	8	0	7
	5	7.	3	1	6.	2	4	8
3	5	7.	8	4	6.	9	0	3
Millions.			Thousands:			Hundreds		

(III. Table.)

Number of places.		
1	8	(eight.
2	54	(fifty four.
3	762	(ſeven hundred ſixty two,
4	3483	(three thouſand four hundred eighty three,
5	97621	(ninety ſeven thouſand, ſix hundred twenty one,
6	243794	(two hundred 43 thouſand 7 hundred 94,
7	8749807	(eight millions, 749 thouſand 8 hundred and 7.
8	57316248	(fifty ſeven mil. 3 hund. and 16 thouſ. 248.
9	357846983	(three hundred fifty ſeven millions, eight (hundred 46 thouſ. 9 hund. 83

(IIII. Table.)

7	hundred	millions of millions
3	ten	
6	one	
8	hundred	millions
4	ten	
2	one	
7	hundred	thouſands
0	ten	
8	one	
6	hundred	hundreds
4	ten	
5	one	

Addition.

Addition.

ADDITION, is the collecting or gathering together of two or more sums, either of one or of divers denominations; into one sum, which is called the [*Aggregate*] [*Total*] or [*Gross*[*sum*]

In Addition of Numbers of one denomination, the order is, to set the numbers to be added one directly under the other; that is to say, *Unites* under *Unites*, *Tens* under *Tens*, *Hundreds* under *Hundreds*, &c.

The Rule.

Having placed your numbers to be added in due order, one under another; draw a line under them, and begin at the lowermost figure toward your right hand, and add that to the next figure above, and the sum of them to the next figure above that; proceeding in this order, till you have added the whole line together; which when you have done, consider how many tens are contained in that line; and for every ten, keep one Unite *in your mind, to be added to the next row; but if there be any odd* Digits, *you must set them beneath the stroak just under the line you added together; Having thus finished the addition of one line, proceed to the next; and from thence to the third; and so forward, be there never so many.* The examples following will make this plain.

Example 1. Let the numbers given to be added together be 7832, 5609, 376, 8547, having thus pla-

ced them in order one under another, as in the Margine is done; draw a line under them, then begin your Addition, at the lowermost figure towards your right hand; Saying 7 and 6 is 13, and 9 is 22, and 2 is 24: Now (because in 24 there is two tens, and 4 remaining) I place the 4 under the line, and carry the two tens to the next row of *Tens*; Saying, 2 which I carried and 4 makes 6, and 7 makes 13, and 3 makes 16; in which row there is but one ten contained, and 6 remaining, which 6 I set under the line, and carry the ten to the next row of ***Hundreds***, saying, 1 that I carried and 5 makes 6 and 3 makes 9, and 6 makes 15, and 8 makes 23, in which 23, ten is contained two times, and 3 remaining; the 3 I place under the line, and carry the two tens to the next row of *Thousands*, saying, 2 which I carried and 8 makes 10, and 5 makes 15, and 7 makes 22, in which, ten is contained two times, and 2 remaining; which 2 I set under the line, and because there is never another row to be added (to which I should carry the two tens) I therefore set 2 down also under the line towards the left hand, as you see done in the margine: So the *total* or *gross sum* of these numbers, being added together, is 22364.

Thousands	Hundreds	Tens	Unites
7	8	3	2
5	6	0	9
	3	7	6
8	5	4	7
22	3	6	4

Example 2. *A Man hath in his Orchard* 136 *Apple-Trees,* 76 *Pear-trees,* 107 *Cherry-Trees, and* 36 *Plum-Trees, and he desires readily to know how many Trees he hath in all.*

Place your numbers one under another, as in the margine, and then begin to add them together, at

your

your right hand, Saying, 6 and 7 make 13, aud 6 make 19, and 6 make 25; place 5 under the line, and carry 2 to the next row; Saying, 2 and 3 is 5, and 7 is 12, and 3 is 15, place 5 under the line, and carry 1 to the next row; Saying, 1 and 1 is 2, and 1 is 3; which 3 I set under the line, and (because there was no tens contained in that line, therefore) the total is 355, and so many *Trees* are in the *Orchard*.

Apple-trees	136
Pear-trees	76
Cherry-trees	107
Plum-trees	36
Trees in all	355

Other Examples for Practice.

95432	321	9161
76100	1986	235
2570	23	72
832	1107	9
Total 174934	Total 3437	Total 9477

Addition of Numbers of divers Denominations.

I. Addition *of English money*.

The most usual Coyns used in *England* are *Pounds Shillings*, *Pence*, and *Farthings*, of which

4 Farthings	make	1 Penny	thus Charactred.	*d.*
12 Pence		1 Shilling		*s.*
20 Shillings		1 Pound		*li.*

For a Farthing we use *q*.

THE RULE.

In the Addition of Numbers of divers denominations this order is to be observed, *viz. Place all numbers of the same denomination one directly under another, as* Pounds *under* Pounds, Shillings *under* Shillings, Pence *under* Pence, *and* Farthings *under* Farthings. *Then draw a line under them, and begin your Addition with the least denomination first; Observing how many times the next greater denomination is contained in that least: and for every time carry one unite to the next denomination, as before you did the tens, setting down the remainder, if any be; Then adding the next denomination together, take notice how many times the next greater denomination is contained in the lesser; carrying for every time one to the next greater denomination.* Thus proceeding till you have gone over all the *denominations*, be they never so many.

Example 1. Let the numbers to be added together be 37 *l.* 16 *s.* 9 *d.* 3 *q.* 21 *l.* 9 *s.* 8 *d* 1 *q.* 13 *l.* 12 *s.* 9 *d.* 2 *q.*

li.	*s.*	*d.*	*q.*
37	16	9	3.
21	09.	8.	1
13	12	9	2
79	19	3	2

Place the numbers as in the margine, draw a line under them, and begin with the least denomination (which in this example is *farthings*,) first, Saying, 2 *q.* and 1 *q.* is 3 *q.* and 3 *q.* is 6 *q.* which is ane peny and 2 *q.* remaining; which 2 *q.* I place under the line, and carry the one peny to the next row, which is the place of pence; Saying, one peny and 9 *d.* is 10 *d.* and 8 *d.* is 18 *d.* which is 1 *s.* and 6. *d.* (*Now against the* 8, *make a prick with my pen, for my better rememberance; to signifie, that there is one shilling to be carried to the place of shillings*,) then go on and say 6. *d.* and 9. *d.* is 15 *d.* which is 1 *s.* and 3 *d.* therefore

fore against 9, I make a prick with my pen, and (because that is the last number) I set down the odd 3 *d.* under the place of pence, and (being I find two pricks in the line of pence, therefore) I carry 2 *s.* to the place of shillings, saying, 2 *s.* which I carried, and 12 *s.* is 14 *s.* and 9 *s.* is 23 *s.* which is one pound and 3 *s.* remaining, I make a prick against 9, and going on, say, 3 *s.* and 16 *s.* is 19 *s.* which (being there is no more numbers to be added, and being also less than 20 *s.*) I set under the line, and finding one prick in the line of shillings, I therefore carry one to the place of pounds, saying, one which I carried and 3 is 4, and 1 is 5, and 7 is 12. Set down the 2 under the line (as in addition of numbers of one denomination) and carry one to the next row; Saying, one that I carried and 1 is 2, and 2 is 4, and 3 is 7, which being the last I set down, and so the total or gross sum is 72 *l.* 19 *s.* 3 *d.* 2 *q.*

Example 2. Let the numbers to be added be 29 *l.* 16 *s.* 8 *d.* 32 *l.* 17 *s.* 9 *d.* 81 *l.* 13 *s.* 11 *d.* and let it be required to find the total or gross sum. Here in this Example the least denomination is *pence*, therefore I begin with them, and say, 11 *d.* and 9 *d.* is 20 *d.* which is 1 *s.* and 8 *d.* make a prick against the 9, and say 8 *d.* and 8 *d.* is 16 *d.* that is 1 *s.* and 4 *d.* make a prick against the 8, and set down the odd 4 *d.* Then (because there are two pricks in the line of pence) you must carry 2 *s.* to the place of shillings, Saying, 2 *s.* which I carry and 13 *s.* is 15 *s.* and 17 *s.* is 32 *s.* which is 1 *li.* 12 *s.* make a prick against 17, and say 12 *s.* and 16 *s.* is 28 *s.* make a prick against 16, and (because there is no more numbers to be added) set down the odd 8 *s.* under shillings, and (being there is two pricks in the line of shillings)

li.	*s.*	*d.*
29	16	8.
32	17	9.
81	13	11
144	08	4

shillings) carry 2 to the place of pounds, saying, 2 and 1 is 3, and 2 is 5, and 9 is 14, set down 4 and carry 1 to the next line; and say 1 and 8 is 9, and 3 is 12, and 2 is 14, which (because it is the last) you set down, so is the total or gross sum, 144 *li.* 8 *s.* 4 *d.*

Other Examples for Practice.

li.	*s.*	*d.*	*q.*	*li.*	*s.*	*d.*
29	18	7	3	36	2	8
63	11	2	1	29	0	2
229	4	0	2	31	16	9
3	7	10	1	6	2	5
326	1	8	3	103	2	0

II. *Addition of* Troy-weight.

Troy-weight is a weight used in *England*, by the which is weighed, *Bread*, *Gold*, *Silver*, *Pearl*, &c. the most usual denominations of which weight are *Pounds*, *Ounces*, *Peny weights*, and *Grains*; of which

24 Grains	make	1 Peny-weight	thus charact.	*pw.*
20 Peny-weight	make	1 Ounce	thus charact.	*ou.*
12 Ounces	make	1 Pound	thus charact.	*lib.*

for a grain we write *gr.*

The Addition of *Troy-weight* (and consequently of any other weight or measure whatsoever, either Domestique, or Forreign) differeth nothing at all from the Addition of English Coin last taught, if the affinity of one denomination to another be first known; for

for whereas in Money, becauſe 12 *d.* make 1*s.* you therefore obſerve how many twelves there are in the addition of your pence, and for every 12 you add one ſhilling to the place of ſhillings, ſo in the addition of *Troy-weight*, knowing that 24 *gr.* make one *peny-weight*, you muſt therefore in the addition of *Grains* of *Troy-weight*, obſerve how many times 24 you find in your line of *Grains*, and for every 24, carry one to the place of *peny-weights*; likewiſe, in the addition of *peny-weights*, you muſt conſider how many times 20 is contained in your line, and for every 20 carry one to the place of *ounces*, (becauſe 20 peny-weights make an ounce.) Alſo in the addition of *Ounces* Troy, you muſt obſerve how many times 12 you find in your line of *ounces*, and for every 12 carry one to the place of Pounds; Then laſtly, Add your Pounds together, as numbers of one denomination.

Example. Let the numbers to be added together be 7 *lb.* 11 *ou.* 13 *pw.* 19 *gr.* 6 *lb.* 7 *ou.* 16 *pw.* 19 *gr.* 3 *lb.* 7 *ou.* 9 *pw.* 6 *gr.* Place your numbers as in Addition of Money, each under other, according to their reſpective denominations, as in the margine: then draw a line under them; and begin your Addition with the leaſt denomination firſt, *viz.* Grains, Saying 6 *gr*: and 19 *gr.* is 25 *gr.* which is one peny-weight and one grain, make a prick againſt 19, and carry the odd grain to the number above, ſaying, 1 *gr.* and 19 *gr.* is 20 *gr.* which (becauſe it is leſs than one peny weight) I ſet under the line, then finding one prick in the line of grains, (I therefore) carry one to the place of peny-weights, ſaying 1 and 9 is 10, and 16 is 26, which is one ounce, and 6 *pw.* make

lb.	*ou.*	*pw.*	*gr*
7	11	3.	19.
6	07	16	19
3	07	09	06
18	02	9	20

a prick againſt 16, and ſay 6 and 03 is 09, which (being leſs than an ounce) ſet under the line, then for the one prick, carry 1 to the place of ounces, ſaying 1 and 7 is 8, and 7 is 15, which is one pound and 3 *ounces*, make a prick at 7, and ſay 3 and 11 is 14, which is one pound and 2 ounces, make a prick againſt 11, and ſet down the 2 ounces, and for the 2 pricks carry 2 pounds to the place of pounds, ſaying 2 and 3 is 5, and 6 is 11, and 7 is 18, which ſet under the place of pounds, ſo is your Addition ended, and the ſum is 18 *lb.* 2 *ou*, 09 *pw.* 20 *gr.*

Other Examples for Practice

lb.	*ou.*	*pw.*	*gr.*	*lb.*	*ou.*	*pw.*	*gr.*
32	9.	12	16	0	10	17.	11
17	11.	6	9.	0	6.	0	5
34	8.	15.	10	0	0	19.	8.
8	10	4	7	0	5	2	19
94	3	18	18	1	10	16	19

III. *Addition of* Avoridupois little weight.

There is another kind of weight moſt commonly uſed in *England*, called *Avoridupois little weight*, by which is weighed all ſorts of Wares or Merchandize, Garblable, as *Sugar*, *Pepper*, *Cloves*, &c. This weight is commonly divided into theſe denominations, *Pounds*, *Ounces and Drams*, of which

16 Drams	make	1 Ounce	thus charact.	*ou.*
16 Ounces		1 Pound		*lb.*

For a Dram we write *dr.*

In the Addition of *Avoirdupois-weight*, you muſt obſerve the very ſame method and order as in *Money* and *Troy weight*, having due reſpect to the quantity of the denominations; as in the Addition of *drams* to make a prick at every 16, ſetting down the remainder, and for every prick carrying an unite to the next place. The preceding Rules being ſo copious in this particular, I ſhall forbear to make any verbal illuſtration; but only give you ſome *Examples* ready wrought, together with the moſt uſual parts into which the *Weights* and *Meaſures* now uſed in *England* are divided; which to the ingenious will be ſufficient.

Examples *of Addition of* Avoirdupois little weight.

lb.	*ou.*	*dr.*	*lb.*	*ou.*	*dr.*
12	11.	09	06	13.	07.
76	05	12.	05	09.	12
32	10.	00	06	03	09.
91	07.	13.	10	00	00
32	13.	07	05	07	09
246	00	09	34	02	05

IV. *Addition of* Avoirdupois great Weight.

There is a weight commonly uſed in *England*, by which is weighed all commodities that are ſold by the hundred, as *Corants*, *Wooll*, *Fleſh*, *Butter*, *Cheeſe*, and the like, the which hundred weight containeth 112 pounds, and the hundred weight is divided into *Quarters*, *Pounds* and *Ounces*, ſo that

16 Ounces	make	1 Pound	thus charact.	*lb.*
28 Pounds	make	1 quarter of a C.	thus charact.	*qr.*
4 Quarters	make	1 Hund. weig.	thus charact.	*C.*

For an Ounce we write *ou.*

Exam-

Examples *of Addition of* Avoirdupois great Weight.

C.	qr.	li.	ou.	C.	qr.	li.	ou.
37	03.	21	12	05	01	07	07
09	01	06	03	03.	02	18.	06
33.	02	20.	00	00	01	06	08
10	00	00	00	11	03.	04	00
12	03.	07	03	06	01	10	05
103	02	27.	02	17	01	11	10

I might farther proceed to give you Examples of Addition of common *English Measures*, *viz.* of *Long measures*, *Liquid measures*, and *Dry measures*, as also of *Time*, *motion*, &c. but the preceding Examples being of sufficient extent, I shall forbear to trouble either my self or the Reader with that which I conceive superfluous: Only, before I leave *Addition*, I will give you a brief view of the most usual *Measures* used in *England*, which take as followeth. And

V. *Of* Liquid Measures

Liquid Measures are those by which all sorts of Liquid substances are measured, of which, (according to the Statute of 12 *Hen.* 7 *chap.* 5.) a *Pint* is the least, from which the greater *Liquid measures* are deduced, according as is expressed in the Table following.

2 Pints	make	1 Quart
2 Quarts	make	1 Pottle
2 Pottles	make	1 Gallon
8 Gallons	make	1 Firkin of Ale, Sope, or
9 Gallons	make	1 Firkin of Beer (Herrings.
10 ½ Gallons	make	1 Firkin of Salmon or Eels
2 Firkins	make	1 Kilderkin
2 Kilderkins	make	1 Barrel
42 Gallons	make	1 Tierce of Wine
63 Gallons	make	1 Hogshead
2 Hogsheads	make	1 Pipe or Butt
2 Pipes or Butts	make	1 Tun of Wine

VI. *Of*

VI Of Dry Measures.

Dry Measures are these by which all kind of dry substances are measured, as *Corn*, *Salt*, *Coles*, *Sand*, *&c.* of which a pint is the least.

2 Pints	make	1 Quart
2 Quarts		1 Pottle
2 Pottles		1 Gallon
2 Gallons		1 Peck
4 Pecks		1 Bushel Land Measure
5 Pecks		1 Bushel Water measure.
8 Bushels		1 Quarter
4 Quarters		1 Chaldron
5 Quarters		1 Wey.

VII. Of Long Measures.

Long Measure is that by which is measured *Cloth*, *Land*, *Bord*, *Glass*, *Pavement*, *Tapestry*, &c. of which measures (according to the Statute of 33 *Ed.* 1. and 25 *El.*) a *Barly Corn* is the least. So that.

3 Barly Corns	make	1 Inch
12 Inches		1 Foot
3 Foot		1 Yards
3 Foot 9 Inches		1 Ell
6 Foot		1 Fadome
5 ½ Yard, or 16 ½ Foot		1 Pole or Perch
40 Perches		1 Furlong
8 Furlongs		1 English mile.

VIII. Of Time.

Time consisteth of *Years*, *Moneths*, *Weeks*, *Days*, *Hours* and *Minutes*. So that.

60 Minutes	make	1 Hour
24 Hours		1 Day Natural
7 Days		1 Week
4 Weeks		1 Moneth of 28 days
13 Moneths, one day 6 hours.		1 Year

IX *Of* Apothecaries Weight.

The Weights used by *Apothecaries* are *Grains*, *Scruples*, *Drams*, and *Ounces*, or which.

20 Grains	make	1 Scruple	thus charact.	℈
3 Scruples		1 Dram		ʒ
8 Drams		1 Ounces		℥
12 Ounce		1 Pound		℔

By help of these Tables, and the rules and cautions before expressed, any Man may make Addition of any of the above-said measures one with another, and therefore I shall forbear to illustrate them by Examples, but leave them to every Mans own practice, and thus I conclude *Addition*.

The Proof of Addition.

Having placed you numbers in order, and added them together, and set the Total under the line, *cut off the upper number by drawing a line with your Pen betwixt that and the others, then add all the numbers together except the uppermost, and set the Total of them under the Total before found; then add this last Total, and the first number which you cut off with your Pen together, and if the sum of those two numbers be equal with your Total sum first found, then is your work right, otherwise not.*

Example. In the first Example of whole numbers the sums to be added were 7803, 5609, 376, and 8547, these numbers placed in due order, and added together, the total or gross sum of them is 22364, now to prove whether this Total be true or not

not, I cut off the uppermost number, (to wit 7832) with a dash of the Pen, and I add the other three numbers together, namely, 5609, 376, and 8547, and the Total of them is 14532; which number being added to 7832 (the number cut off) the sum of them is 22364, exactly agreeing with the Total first found; clearly evidencing that the Addition was truly performed: but if they had disagreed, then the work had been erroneous. The like course must be taken for the proof of those sums which have different denominations, as in *Money* and *Weight*, as by the examples following will appear.

	7832
	5609
	376
	8547
first Total	22364
last Total	14532
Proof	22364

Other Examples proved.

1 *Example of* Money.

	li.	s.	d.	q.
	37	16	9	3
	21	9	8	1
	13	12	9	2
1 Total	72	19	3	2
2 Total	35	2	5	3
Proof	72	19	3	2

2 *Example of* Troy-Weight.

	lb.	ou.	pw.	gr.
	32	9	12	16
	17	11	6	9
	34	8	15	10
	8	10	4	7
	94	3	18	18
	61	6	6	2
	94	3	18	18

There are other ways to prove *Addition*, by casting away all the *nines* in the number of one denomination, and of all the *twelves*, *twenties*, and *nines*, in *pounds*, *shillings* and *pence*, &c. but this as the most certain and easie I imbrace; and thus much for Addition, and the proof thereof.

Substraction.

SUBSTRACTION is the taking of one or more small sums out of a greater, as 7 *s.* out of 12 *s.* or 37 *li.* out of 100 *li.* or 137 foot out of 983 foot, and the like.

As in *Addition*, the sums to be added may be either of one, or of divers denominations; so likewise they may be in *Substraction*, and the manner of placing them is the same; for you must set *Unities* under *Unities*, *Tens* under *Tens*, *Hundreds* under *Hundreds*, &c.

Example 1. Of *Substraction* of numbers of one denomination. *Let it be required to Substract* 234 *out of* 986. Place the numbers one under the other as you see done in the Margine, draw a line under them, and begin with the first figure towards your right hand, which is 4; saying take 4 from 6, and there remains 2, place 2 under the the line, and go to the next figure which is 3; saying, take 3 from 8, and there remains 5, place 5 under the line, and go to the next figure which is 2; saying, take 2 out of 9 and there remains 7, place 7 under the line, and your Substraction is ended; and it is evident by the work, that if you take 234 out of 986, there will remain 752, which you may thus prove. For if you add the 234, to 725, you shall find the sum of that addition to be 986, which is equal to the whole sum from which 234 was substracted.

Number given	986
Number to be substracted	234
	——
Remainder.	752

Example

Example 2. *Let it be required to substract* 2976 *out of* 96527. Place the numbers one under another, as in the Margine you see done; then draw a line under them, and beginning with the first figure towards your left hand; say, take 6 out of 7, and there remains 1, place 1 under the line, and proceed to the next figure; saying, 7 out of 2 I cannot (wherefore you must always add [10] to the number above, which in this Example is 2, and it makes it 12, (therefore take 7 out of 12, and there remains 5, place 5 under the line, and (because you added 10 to the 2 to make it 12, you must) carry a Unite to the next figure; saying, one which I carried and 9 is 10, take 10 out of 5, which I cannot; therefore I must add 10 to 5, and it makes 15, and say 10 out of 15, and there remains 5; place 5 under the line, and (because you added 10 to 5 to make it 15, you must therefore) carry a unite to the next figure; saying, one which I carried, and 2 is 3, take 3 out of 6, and there remains 3, place 3 under the line, and because there is no more figures to be substracted from the number above, you must say, nothing from 9, and there remains 9, set the 9 under the line, and your Substraction is ended.

$$\begin{array}{r} 96527 \\ 2976 \\ \hline 93551 \end{array}$$

Other Examples for Practice.

	li.	*Reams of Paper*			*Sheep*
Lent	5762	Bought	9765	From	1000
Paid	378	Sold	6529	Take	394
Rests to pay	5384	Unsold	3236	Remains	606

Subſtraction of Numbers of divers denominations.

Of Engliſh Money.

In Subſtraction *of Numbers of divers denominations, you muſt obſerve the ſame order as in* Addition, *namely, to place every number in due order, with reſpect to the denomination, as* pounds *under* pounds, ſhillings *under* ſhillings, *&c. the greater number always uppermoſt; and drawing line a under them, begin with the leaſt denomination firſt, ſubſtracting it from the line above, and ſetting the* remainder *under the line, as in whole numbers; but if the* pence *or* ſhillings *in the upper row, be ſmaller than thoſe in the neather row you muſt add* 12 d. *or* 20 s. *to the ſmaller number, that ſo Subſtraction may be made, as by the* Examples *following will appear.*

Example 1. *Let it be required to ſubſtract* 38 *li.* 12 *s.* 8 *d. out of* 269 *li.* 18 *s.* 10 *d.* Place your numbers as in the Margine; then beginning with the leaſt denomination firſt, (which in this Example is pence) ſay, 8 *d.* from 10 *d.* and there remains 2 *d.* ſet the 2 *d.* under the line, and proceed to the next denomination, which is ſhillings; ſaying, take 12 *s.* out of 18 *s.* and there remains 6 *s.* place 6 *s.* under the line, and go to the pounds; ſaying, 8 out of 9, and there remains 1. place 1 under the line and ſay 3 out of 6, and there remains 3; then (becauſe there is no more figures to be ſubſtracted) ſay, nothing out of 2, and there remains 2, which ſet under the line, ſo is your Subſtraction ended and the remainder is 231 *li.* 6 *s.* 2 *d.*

	li.	*s.*	*d.*
Lent	269	18	10
Paid	38	12	8
Reſts	231	6	2

Example

Example 2. *Let it be required to ſubſtract* 2628 *li.* 16 *s.* 10 *d. out of* 9320 *li.* 10 *s.* 7 *d.* Place the numbers in order, and beginning with the pence, ſay, 10 *d.* out of 7 *d.* I cannot (therefore I muſt add 12 *d.* (which is one ſhilling) to 7 *d.* and it makes 19 *d.*) but 10 *d.* out of 19 *d.* and there remains 9 *d.* ſet the 9 *d.* under the line, and (becauſe I added 12 *d.* to 7 *d.*) I muſt therefore carry one to the place of ſhillings, ſaying, 1 *s.* which I carried, and 16 *s.* is 7 *s.* then 17 *s*, from 10 *s.* I cannot take, therefore I muſt add 20 *s.* (which is one pound) to 10 *s.* and it makes 30 *s.* then 17 *s.* out of 30 *s.* and there remains 13, ſet 13 under the line, and carry one to the place of pounds; ſaying, one which I carried and 8 is 9, take 9 out of 0 I cannot, but 9 out of 10, and there remains 1; ſet 1 under the line, and carry an unite to the next place, ſaying, 1 which I carried and 2 is 3, take 3 out of 2 I cannot, but three out of 12, and there remains 9; place, 9 under the ſame line, and carry 1 to the next place, ſaying, 1 which I carry and 6 is 7, take 7 out of 3 I cannot, but 7 out of 13, and there remains 6. place 6 under the line, and carry one to the next row; ſaying, 1 and 2 is 3, take 3 from 9, and there remains 6, place 6 under the line, ſo is your ſubſtraction ended, and the remainder is 6691 *li.* 13 *s.* 9 *d.*

	li.	*s.*	*p.*
Lent	9320	13	07
Paid	2628	16	01
Reſts	6691	13	9

Example 3. *Suppoſe a Man had let to another Man* 1000 *pound, and that the borrower had paid thereof at one time* 127 li. *at another time* 490 l. 10 s. *and at third payment* 50 li. *and the Creditor would know how much he hath received, and how much is owing of his Debtor.*

Place the numbers as here you ſee, firſt the ſum of money lent, and draw a line under it; then ſet the ſums paid at ſeveral times one under another, and draw a line under them: Then add all the ſums which have been paid at ſeveral times together, which make 607 *li.* 10 *s.* which is the ſum which the Debtor hath paid in all; then ſubſtract this 607 *li.* 10 *s.* from 1000 *li.* and there will remain 392 *li.* 10 *s.* and ſo much is ſtill owing to the Creditor.

	li.	*s.*	*d.*
Money lent	1000	00	00
	127	00	00
Paid at ſeveral times	430	10	00
	50	00	00
Paid in all	607	10	00
Reſts to pay	392	10	00

Other Examples for Practice.

	li.	*s.*	*d.*
Lent	2601	13	6
Paid	98	7	9
Reſts	2530	5	9

	li.	*s.*	*d.*
Owing in all	100	00	00
Paid in all	36	10	06
Reſts to pay	63	09	06

	li.	*s.*	*d.*	*q.*
Lent	3625	16	08	03
Paid at ſeveral times	100	00	00	00
	336	10	06	02
	039	12	09	02
	100	00	00	00
Paid in all	576	03	04	00
Reſts to pay	3049	03	04	03

The Proof *of* Subſtraction

The *Proof* of *Subſtraction* is performed by *Addition*, for adding the number to be ſubſtracted to the remainder, the ſum of them muſt be equal to the number given, if you have truly wrought: As in the firſt example of numbers of one denomination,

The

The number given is	986
The number to be Substracted is	234
The remainder is	752
Proof	986

Add the number to substracted 234, to the remainder 752, the sum of them is 986, equal to the number given.

Examples for Practice proved.

	li.	*s.*	*d.*		*li.*	*s.*	*d.*
Lent	62	18	09	Borrowed	100	00	00
Paid	37	19	06	Received	36	13	04
Rests	24	19	03	Due	63	06	08
Proof	62	18	09	Proof	100	00	00

Other Example in Weight *and* Measure.

1 Example in *Troy-weight.*

	li.	*ou.*	*pw.*	*gr.*
Bought Of Silver	07	11	13	19
Sold	05	07	03	05
Unsold	02	04	10	14
Proof	07	11	13	19

Example in *Avoirdupois great weight*

	C.	*q*	*li.*	*ou.*
Bought	37	03	22	11
Sold	13	01	23	06
Rests	24	02	27	05
Proof	37	03	22	11

3 Exmple in *Avirdupois little weight.*

	li.	*ou.*	*dr.*
Bought	84	12	13
Sold	26	08	11
Rests	58	04	02
Proof	45	12	13

4 Example in Time.

	days	*ho.*	*m.*
From	364	23	50
Take	76	09	22
Rests	288	14	28
Proof	364	23	50

Questions

Questions performed by Addition *and* Substraction.

Question 1. *What number is that which being added to* 376 *shall make* 1000? Substract 376 from 1000, the remainder is 624, the number sought.

Question 2. *What number of Pounds, Shillings and Pence must be added to* 36 *li.* 17 *s.* 3 *d. to make that sum up* 100 *l.* Substract 36 *li.* 17 *s.* 3. *d.* from 100 *li.* the remainder is 63 *li.* 2 *s.* 9 *d.* which added to 36 *li.* 17 *s.* makes 100 *li.*

Quest. 3. *In the year of our Lord* 1440, *the famous Art or Mystery of Printing was invented, I would know how long it is since that time to this year of our Lord,* 1655. From 1655, substract 1440, the remainder is 215, and so many years are expired since *Printing* was invented.

Question 4. *An Army consisting of* 13721 *Horse, and* 26850 *Foot, in an Ingagement there were slain* 3760 *Horse, and* 7523 *Foot; the Question is, how many were slain in all, and how many Horse, and how many Foot escaped* From the 13721 Horse that went out, substract the 3760 that were slain, there remains 9961 and so many Horse escaped: Also from the 26850 Foot which went out, substract the 7523 which were slain, and there remains 19327. the number of Foot which escaped; and by adding the 3760 Horse which were slain, to the 7523 Foot that were slain, their Total is 11283, and so many were slain in all.

Mult pli-

MULTIPLICATION.

MULTIPLICATION is that part of *Arithmetick* which teacheth how to increase one number by another, so that the number produced by their *Multiplication*, shall contain one of the numbers multiplied, so many times as there are *Unites* contained in the other. *Multiplication* may fitly be termed a Compendium of *Addition*, for that it performeth at one operation the same, which to effect by Addition, would require many. For instance, if it were required to know how much 7 times 5 is to perform this by Addition, I must set seven fives, or five sevens, one under another, and adding them together, I shall find that either of their Totals shall contain 35; but this by *Multiplication* is performed with far more brevity, as by *Examples* hereafter shall appear.

Before you enter upon the Practice of *Multiplication*, it is necessary to remember the Product produced by the multiplication of any one of the nine Digits, by any other of the same; as readily to know, that 4 times 5 is 20, 6 times 7 is 42, 2 times 9 is 18, 7 times 9 is 63, 8 times 9 is 72, *&c.* which this Table following will plainly declare, and must be perfectly learned by heart, before you attempt to multiply great numbers.

Multiplication Table.

2 times	2 3 4 5 6 7 8 9	makes	4 6 8 10 12 14 16 18
3 times	3 4 5 6 7 8 9	makes	9 12 15 18 21 24 27
4 times	4 5 6 7 8 9	makes	16 20 24 28 32 36
5 times	5 6 7 8 9	makes	25 30 35 40 45
6 times	6 7 8 9	makes	36 42 48 54
7 times	7 8 9	makes	49 56 63
8 times	8 9	makes	64 72
9 times	9	makes	81.

The use of the TABLE *of* MULTIPLICATION, *and the manner how it is to be read.*

This Table ſheweth what the ſum of any two Digits multiplied one by another doth amount unto, and is thus to be read, 2 times 2 makes 4, 2 times 3 makes 6, 2 times 4 makes 8: Alſo 6 times 4 makes 24 7 times 8 makes 56, 8 times 8 makes (or is) 64, 9 times 9 is 81, *&c.*

Aeother Table of Multiplication.

1	9	8	7	6	5	4	3	2	1
2	18	16	14	12	10	8	6	4	
3	27	24	21	18	15	12	9		
4	36	32	28	24	20	16			
5	45	40	35	30	25				
6	54	48	42	36					
7	63	56	49						
8	72	64							
9	81								

THis Table is thus to be read; In the first Row, or Column, towards the left hand, and also at the top of the Table, you have the nine *Digits* in bigger Figures than the rest; the Figures in the first Column beginning which 1, and so proceeding by 2, 3, 4, *&c.* to 9. Those at the top of the Table beginning with 9 towards the left hand, and so backwards by 8, 7, 6, *&c.* to 1 at the right hand.

Now if by this Table you would know how much 8 times 7 is, find 8 among the great Figures at the head of the Table, and look down that *Row* or *Column*, till you come against 7 of the great Figures in the first *Column*, against which you shall find 56, and so much is 8 times 7, or eight multiplied by 7.

In the same manner may you find that 7 times 9 is 63, 5 times 6 is 30, 3 times 4 is 12, and so of any two of the nine Digits.

In Multiplication there are three terms commonly used, that is to say;

The

The *Multiplicand*,
The *Multiplier*, and
The *Product*.

The *Multiplicand* is the number to be multiplied.
The *Multiplier*, is the number by which the Multiplicand is Multiplied: and
The *product*, is the number which is produced by the multiplication of the Multiplicand and the Multiplier together.
Thus, if it were required to multiply 8 by 7, here 8 is the *Multiplicand*, 7 the *Multiplier*, and 56 is the *Product*; for 8 times 7, or 7 times 8 is 56.

In *Multiplication* it mattereth not which of the two numbers is made, the *Multiplicand*, or which the *Multiplier*, for the *Product* produced by either, will be the same; but the usual way is to make the *greater* number the *Multiplicand*, and the *lesser* number the *Multiplier*.

THE RULE.

The numbers to be multiplied must be set one under another, viz. the Multiplicand *(or great number) above, and the* Multiplier *(or lesser number) below, the last figure of the* Multiplier *under the last figure of the* Multiplicand, *then draw a line under them, and (having learned the preceding Tables perfectly by heart) multiply every Digit of the* Multiplier, *into every Digit of the* Multiplicand, *setting the several Products under the line; then having finished your* Multiplication, *draw a line, and add all the Products together, and the sum of those Products is the general* Product *of the whole multiplication, as by the following* Examples *will appear.*

Example 1. *Let it be required to multiply 736 by 7.*
First,

Firſt, I write down 736 the Multiplicand, and under it 7 the Multiplier, and under them I draw a line; then I multiply 7 into every Digit of the Multiplicand; ſaying, 7 times 6 is 42, place 2 under the line, under 7, and for the four tens keep 4 in mind; then ſay again, 7 times 3 is 21, and 4 which I kept in mind is 25; place 5 under the line and keep the two tens in mind; then ſay again, 7 times 7 is 49, and 2 which I kept in mind is 51; place 1 under the line, and the 5 tens kept in mind (becauſe there is no more figures to be multiplied) I ſet down under the line alſo, ſo is the work ended, and the Product of this multiplication is 5152.

736	*Multiplicand*
7	*Multiplier*
5152	

Example 2. *Let it be required to multiply* 3417 *by* 5. Place the numbers one under another, and draw a line under them, as in the Margine; then begin your Multiplication, ſaying, 5 times 7 is 35, place 5 under the line, and keep the three tens in mind; then ſay again, 5 times 1 is 5, and 3 which I kept in mind is 8, place 8 under the line, and (becauſe it is leſs than 10, I keep nothing in mind) then ſay again, 5 times 4 is 20, place a cypher under the line, and keep the two tens in mind: Laſtly, ſay 5 times 3 is 15, and 2 which I kept in mind is 17, which 17 (being the laſt number) I place under the line, and ſo is my Multiplication ended, and the *Product* is 17085.

3417
5
17085

¶ You may be ſatisfied of the truth of this work, If you will take the pains to ſet down the Multiplicand 3417, five times one under another, and

and add them together, as ſo many ſeveral ſums, ſo ſhall you find the *Total* of that Addition, to be 17085, exactly the ſame with the Product of this Multiplication:

Example 3. In the two fore going examples, the *Multiplier* conſiſted but of one *Digit*, we are now to ſhew how *Multiplication* is performed, when the Multiplier conſiſts of more than one figure, therefore in this Example, *Let it be required to multiply* 5704 *by* 37. Place your numbers, and draw a line under them as you ſee in the Margine:

5704	Multiplicand
37	Multiplier
39928	
17112	
211048	Product

Then begin you Multiplication in in this manner: Saying, 7 times 4 is 28, ſet 8 under the line; and keep the two tens in mind, then ſay 7 times nothing is nothing, but the two tens in mind is 2, ſet 2 under the line, then ſay 7 times 7 is 49, ſet 9 under the line, and keep 4 in mind, then laſtly, ſay 7 times 5 is 35, and 4 in mind is 39, which being the laſt number to be multiplied, I ſet down under the line; ſo is the multiplication of one of the Digits (namely 7) finiſhed.

Then begin to multiply the ſecond digit, ſaying 3 times 4 is 12, place 2 in the ſecond line, one place towards the left hand, and keep 1 in mind, then ſay 3 times nothing is nothing, but 1 in mind is 1, ſet down 1 by the 2 in the ſecond line; thirdly, ſay 3 times 7 is 21, place 1 in in the ſecond line, and keep the two tens in mind: Laſtly, ſay 3 times 5 is 15, and 2 is 17, which 17 (becauſe there is no more figures to be multiplied) I place in the ſecond line alſo.

Having thus done, I draw a line under them, and add theſe two lines together, as in common Addition of

of numbers of one denomination; Saying 8 is 8, place 8 under the line; then say 2 and 2 is 4, place 4 under the line; then say 1 and 9 is 10, place a cipher under the line; and carry one to the next place: saying 1 and 1 is 2, and 9 is a 11, place 1 under the line, and carry 1 to the next row, saying 1 and 7 is 8, and 3 is 11, place 1 under the line, and cary 1 to the next place; saying, 1 which I carry 1 and is 2, place 2 under the line, and so is your Multiplication ended and the product is 211048.

Example 4. Let it be required to multiply 57325 *by* 4032. Place the multiplicand and the multiplier one under another, and draw a line as before, then proceed to the multiplication as formerly; saying First, 2 times 5 is 10, set down a Cypher, and keep 1 in mind; then 2 times 2 is 4, and 1 in mind is 5, place 5 under the line: then 2 times 3 is 6, set 6 under the line; then 2 times 7 is 14, set down 4 and keep one in mind; then 2 times 5 is 10, and 1 in mind is 11, which 11 (being the last) I set down.

57325	Multiplicand
4032	Multiplier
114650	
171975	
229300	
231134400	Product

The Multiplication of one of the digits being finished, proceed to the Multiplication of the next; Saying, 3 times 5 is 15, set down 5 in the second line, a place more towards the left hand, and keep 1; then 3 times 2 is 6, and 1 kept is 7, set down 7 then 3 times 3 is 9, set down 9; then 3 times 7 is 21, set down 1 and keep 2 in mind; then 3 times 5 is 15, and 2 in mind is 17; which being the last set down also.

Two of the figures of the multiplier being finished,

proceed to the third, which (in this example) being a cypher, you may wholly neglect, and proceed to the multiplication of the fourth figure; only remember to remove the Product of the fourth figure one place more to the left hand, as in the example you may ſee, for the cypher though it be not written down, yet it muſt keep its place, and the figures following muſt be removed a place farther.

Then for the Multiplication of the fourth and laſt digit, ſay 4 times 5 20, ſet down a cypher, (under 9) and keep 2 in mind: then 4 times 2 is 8, and 2 in mind is 10, ſet down a cypher; and keep 1 in mind: then 4 times 3 is 12, and 1 is 13, ſet down 3 and keep 1: then 4 times 7 is 28, and 1 kept is 29, ſet down 9, and keep 2: then 4 times 5 is 20, and 2 is 22; which 22 (becauſe the multiplication is ended) ſet down alſo.

Having thus multiplied all the digits ſeverally, draw a line under their Products, and add them altogether, as in the former Example, ſo ſhall you find their general Product to be 231134400.

Other Examples for Practice.

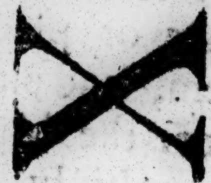
```
        73260              50762
        45003               4567
   ----------        -----------
       219780             355334
     366300              304572
   293040               253810
   ----------          203048
   3296919780        -----------
                       233830054
```

The Proof of Multiplication.

The moſt certain proof of *Multiplication* is by *Diviſion*,

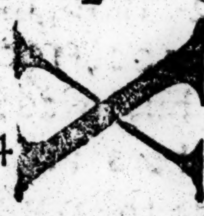
viſion, but becauſe *Diviſion* is not yet known, I will here ſhew a near way by which *Multiplication* may be proved. Which is this.

Make a Croſs *as in the Margine, then any ſums being multiplied, you may prove the truth of your work in this manner,* (1) *Caſt away all the nines which you can find in the* Multiplicand, *what remaineth ſet on the right ſide of* the Croſs. (2) *Caſt away alſo the nines in the* Multiplier, *and what remains ſet on the left ſide of the* Croſs. (3) *Multiply the figure on the right ſide of the* Croſs, *and out of that* Product *caſt away the nines alſo, ſetting the figure remaining over the* Croſs, *then* (4) *Caſt away all the nines in the* Product, *and if the figure remaining be the ſame with that which ſtandeth over the* Croſs, *then is your Multiplication truly performed, otherwiſe not.*



Example 1. Let it be required to prove the Sum in the margine

4324
23
———
12972
8648
———
99452



1. Caſt away all the nines in the *Multiplicand*, Saying, 4 and 3 is 7, and 2 is 9, which being rejected, there remains 4, which I ſet on the right ſide of the Croſs, Then,

2. Caſt away all the nines in the *Multiplier*: Saying, 2 and 3 is 5, (which being leſs then 9) I ſet on the left ſide of the Croſs. Then.

3. Multiply 4 by 5, ſaying 4 times 5 is 20, from which caſt all the nines which are two, and there remains 2, place 2 over the Croſs. And,

4. Caſt away all the nines in the *Product*; Saying, 2 and 5 is 7, and 4 is 11, caſt away 9 and there remains 2: which exactly agrees with the figure over the *Croſs*, and demonſtrates that the Multiplication is truly performed.

Compendiums in Multiplication.

1. If the *Multiplier* conſiſt of Cyphers in the laſt place or places, you may omit the Multiplication of them, and place the former figures of the Multiplier under the Multiplicand: Thus, if it were required to multiply 3257 by 2600; place the numbers as you ſee in the margent, them multiplying 3257 by 26, the Product will be 84682, to which if you add two Cyphers, (becauſe there were two Cyphers in the Multiplier) it will be 8468200, which is the true product of the multiplication.

```
  3257
  2600
-------
 19542
 6514
-------
8468200
```

2. If it be required to multiply any number by 10, 100, 1000, 10000, &c. You have no more to do, but to add ſo many Cyphers to the *Multiplicand*, as there are Cyphers in the *Multiplier*: Thus if you were to multiply 365 by 10, the product will be 3650, or by 100, it would be 36500, or by 1000, it would be 365000, or by 10000, it would be 3650000.

3. If any number given were to be multiplied by 5, you may abbreviate your work thus. Add a Cypher to the *Multiplicand*, take half that number, and it ſhall be the product required. Thus if it were required to multiply 8727 by 5, add a Cypher to the *Multiplicand*, then it is 87270 the half whereof is 43635, which is the product required.

To multiply by any of the nine Digits without charging the memory.

To multiply any number by 2, Either double the number in your mind, or add it, by ſetting it down twice, ſo 57325 produceth 114650.

To

To multiply any number by 3, To the number given add the double thereof, the ſum is the product, ſo 57325 produceth 114650.

To Multiply any number by 4, Double the duplica tion in your mind, ſo 57325 produceth 229300.

To multiply any number by 5, Conceive a cyphe added to the given number, and in your mind hal thereof is the product, thus a cypher added t 57325, maketh it 573250, the half whereof i 286625.

To multiply any number by 6. Take half adding a cypher, and add to the half, the figure ſtanding next before: Thus 573250 produceth 343950.

To multiply any number by 7. Take half and add it to the double of the former figure, ſuppoſing a cypher ad= ded as before; So 57325 thus ordered, produceth 401275.

To Multyply any number by 8. Double each former fi- gure and ſubſtract it from the following, ſo 57325 produceth 458600.

To multiply any number by 9. Suppoſe the number multiplyed by 10, then ſubſtract each former figure from the following, beginning with that next before the cypher, the remainder is the product, ſo 57325 produceth 515925.

Other Brief Rules of Multiplication:

Having ſhewed you ſome *Compendious* wayes of *Multiplying* by *Article Numbers*, as by 10, 100, 1000, *&c.* and alſo by the *nine Digits*, without any trouble or charge to the *memory*: I will now ſhew how you may expeditiouſly and certainly, *multiply* any ſum by divers other Numbers conſiſting of *two* or *three* places, without ſetting down many figures but the Product it ſelf.

To multiply any number by 11.

THE RULE.

Set the Multiplicand down twice, removing it one place to the left hand, add them together, the ſum is the Product of the Number multiplyed by 11.

Example. *Let it be required to multiply* 97 *by* 11.

Set down 97 twice, removing one of them a place more to the right hand as you ſee here in the margine, then add them together, the ſum is 1067, which is equal to the Product of 97 multiplied by 11

97
97
1067

To multiply any Number by 12, 13, 14, 15, 16, 17, 18, or 19.

THE RULE

To effect this, You have no more to do, but to *multiply the given number by* 1, 2, 3, 4, 5, 6, 7, 8, 9, *and in your Multiplication, continually to add that figure of the Multiplicand which ſtandeth on the right hand of the figure you are multiplying by, ſetting down the ſum for the figure of the Product.*

Example. *Let it be required to multiply* 3624 *by* 17.

Multiply in this manner by 7, Saying, 7 times 4 is 28, ſet down 8 and carry 2 —— then 7 times 2 is 14, and 2 which I carryed is 16, and 4 (*the figure of the Multiplicand which ſtands on the right hand of* 2) 20: Set down 0, and carry 2 —— then 7 times 6 is 42, and 2 which I carried is 44, and 2 (*which ſtands on the right hand of* 6) is 46, ſet down 6 and carry 4 —— then 7 times 3 is 21, and 4 which I carried is 25, and 6 (*which ſtands on the right hand of* 3) is 31, ſet down 1 and carry 3, which 3 add to 3 (*the left hand figure of your Multiplicand,*)

it is 6, which set down, so the Product of 3624 multiplied by 17 is 61608.

In the same manner, may you multiply by 12, 13, 14, &c. as in these Examples.

3764	625	4793
107	16	19
48932		
	10000	91067

To multiply any Number by 102, 103, 104, 105, 106, 107, 108, *or* 109.

THE RULE.

Multiply the Number given by 2, 3, 4, 5, 6, 7, 8, *or* 9, *setting the Product two places towards the right hand of the Multiplicand, the Product and the Multiplicand added together in the same order that they stand, shall be the Product of the whole Multiplication,*

Example. *Let it be required to multiply* 3924 *by* 106.

Set them down as in the Margine, then multiply 3624 by 6, it produceth 21744 which added to 3624, in the same order as they there stand, the sum of that Addition will be 384144, which is equal to the Product of 3624 multiplied by 106.

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3624
 106
------
  21744
------
384144
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Other Examples.

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 765        6374
 103         107
-----       ------
  2295       44618
-----       ------
 78795      682018
```

To multiply any number by 112, 113, 114, 115, 116, 117, 118, *or* 119.

MULTIPLICATION

THE RULE,

Multiply the given Sum by 12, 13, 14, 15, 16, 17, 18, *or* 19, *as hath been shewen already, setting the Product two places to the right hand of the Multiplicand, then add this Product and the Multiplicand together in the same order as they stand, so shall the sum of that Addition be equal to the Product of the Multiplication.* As by the Examples following is evident.

Multiply	4065
By	113
The Product multiply by 13	52845
Real Product	459345

Multiply	7632
By	119
The Product Mul. by 19	145008
Real Product	908208

Questions performed by Multiplication only.

Question 1. *If a piece of of Land be* 236 *Perches long, and* 182 *Perches broad, how many square Perches are contained therein?* Multiply 236 the length, by 182 the breath, the product is 42952, and so many square Perches are contained in such a square piece of Land.

Question 2. *In a year there are* 365 *days natural, and in every day* 24 *hours, how many hours be there in a year?* Multiply 365 the number of days by 24, the number of hours, the product is 8760, and so many hours be there in a year.

Question 3. *From* London *to* Coventry *it is accounted* 76 *mils; how many yards therefore is it from* London *to* Conventry? Multiply 1760 (which are the number of yards contained in one mile) by 79, the product is 139760, and so many yards are between *London* and *Coventry*.

DIVISION.

Division.

DIVISION is the just contrary to *Multiplication*, for *that* turns *Small* denominations to *Greater*, as *Multiplication* turns *Greater* to *Smaller*: Or (in whole Numbers, of which only we yet speak) *Division* is the asking, *how many times one sum is contained in another*? and the number which answereth to that question is called the *Quotient*.

And the Number containing, which is to be divided, is called the *Dividend*.

And the Number contained, or by which the Dividend is to be divided, is called the *Divisor*.

And as often as the *Dividend* contains the *Divisor*, so often doth the *Quotient* contain *Unity*.

The ways of performing Division are divers. I will begin with that which is most used and taught, which is as followeth.

THE RULE

Place the Divisor *under the* Dividend*, so that the figures next to the left hand stand directly one under the other, if the rest of the* Divisor *be not the greater; or if all the* Divisor *be greater than that above it, then the said* Divisor *must be devolved one place further toward the right hand; having so placed them, try how many times the lower figures are contained in the upper figures,*

and

and write that figure which answereth that question within a crooked line in the margine of the work, which is called the Quotient, *and by it multiply the first figures of the* Divisor, *and take the Product out of the figures directly over it, beginning the* Substraction *towards the left hand; then cancel that figure of the* Divisor, *and also that of the* Dividend *which hath been already used, with a light dash of a Pen, and write the* remain *(when the Product of the first figure multiplied by the* Quotient *is substracted, as before) just over the figure used and cancelled; Then proceed to do the like with the second, third, and fourth figure of the* Divisor, *if there be so many; till having cancelled it all, and set the remain orderly above the* Dividend, *you have finished one work.*

Now if the Dividend, *have still some figures untouched towards the right hand, then remove the* Divisor *still towards the right hand, but one place at a time, and then again ask or try how many times the lower may be had in the upper and write the answer in the* Quotient, *whether it be* 1 *or more, (only it cannot be above* 9*) or nothing, then put* 0 *in the* Quotient, *and multiply the* Divisor *by this new figure, and substract the Product, setting the remainder orderly above, as before; this work must be repeated by removing the* Divisor *still one place towards the right hand, until* Unity *in the* Divisor *stand under* Unity *in the* Dividend, *and then the work is done.*

Example 1.

Let it be required to divide 4096 *by* 3

Place them thus 4096(1
3

Ask how many times 3 in 4, the Answer is 1, which is put within a crooked line by it self.

Then in your mind multiply the Divisor 3 by the Quotient

Quotient 1. And having ſaid theſe words, *Once* 3 *is* 3, preſently cancel the 3. And having added theſe words, *Out of* 4, cancel the 4. And after theſe words, *and there remains* 1, write 1 juſt over the 4, as you ſee here done.

1
~~4~~096(1
~~3~~

Then remove the Diviſor one place toward the right hand, ſaying, *how many times* 3 *in* 10, the anſwer is 3, which write in the quotient; and in your mind multiply the Diviſor 3 by the quotient 3: the product is 9, wherefore ſay, three times three is nine, out of ten, and there remains one, then (having cancelled the 10 and the 3,) write over them 1.

~~1~~1
~~4~~~~0~~96(13
~~3~~~~3~~

Again, remove the Diviſor 3 one place more, asking *how many times* 3 *in* 19? the anſwer being 6, write 6 in the quotient, and ſay 6 times 3 is 18 out of 19, and there remains 1, wherefore having cancelled the 3, and the 19, write 1 over 3, and remove the Diviſor once more, and ask *how many times* 3 *in* 16? anſwer is 5, which write in the quotient, then in mind ſay 5 times 3 is 15 out of 16 and there remains 1, and cancelling the 16 and the 3, write over 3, 1: Now becauſe the *Diviſor* 3, is advanced ſo far till it is come to ſtand under 6 in the *Dividend*, which 6 is the place of *Unity* there, the ſaid *Diviſor* cannot be removed any more, and therefore the Diviſion is ended, and the *Quotient* being 1365, ſhews that the *Diviſor* 3 is contained in the *Dividend* 4096, 1365 times, and 1 remaining, which being

~~1~~~~1~~1
~~4~~~~0~~~~9~~6(136
~~3~~~~3~~~~3~~

~~1~~~~1~~~~1~~1
~~4~~~~0~~~~9~~~~6~~(1365
~~3~~~~3~~~~3~~

ing less then the Divisor 3, doth not contain it once, but one third part of once, which (after the Reader hath skill in broken numbers) must be joyned to the quotient thus 1365 $\frac{1}{3}$

The best proof of this *Division* is by *Multiplying* the quotient into the Divisor, and to the Product add the remain; then if the work be well done, the sum shall be equal to the Dividend.

So 1365 multiplied by 3 produceth 4095, to which adding the remain 1, the sum 4096 is equal to the Dividend.

Here we divided only by one figure 3, because the first example being easie and clear, should be a fair Introduction to the second.

Note, that if the Divisor had been greater than 4, as 5, the work must have begun thus 4096 (

5

So the quotient would have been 819 $\frac{1}{3}$ which is one place less.

Example 2.

Let it be required to divide 1 3 1 0 7 2 0 by 4096

Place them thus 1 3 1 0 7 2 0 (

4 0 9 6

Now the question to be asked is, how many times 4096 is there in 13107? to find and answer to this question, the Reader which hath but an indifferent) faculty of judging, may do it, (for the most part by considering the first figure in the Divisor, as here 4, for presently he knows that 4 times 4 is 16, which cannot be had in 13, therefore the first figure in the quotient must be less than 4.

Again, it cannot be much less, because the second place in the Divisor is 0.

He

He may therefore venture on 3, And having put 3 in the quotient, ſay 3 times 4 is 12, which ſubſtract out of 13 over it thus, ſay 2 out of 3, and there remains 1, and 1 out of 1 and there remains 0, cancelling the 0 and the 13, and ſet the remain 1 over the 4, as you ſee.

```
     1
   1839
  13110120(3
   4096
```

Then go on, ſaying 3 times 0 is 0, out of 11, and remains ſtill 11, again 3 times 9 is 27, take 7 out of 10 there remains 3, to be ſet over the 0, which is over 9, and 2 with 1 borrowed (to make the 0, 10, from the which the 7 was taken) is 3, and 3 out of 11, remains 8, which write over the place of 0 in the Dviſor, cancelling the 9 in it, and alſo thoſe figures of the Dividend 110, out of which you have taken any thing: Then laſtly, ſay 3 times 6 is 18, take 8 out of 7 I cannot, therefore (borrowing 10,) ſay 8 out of 17 remains 9, which write over the 7, then in the next place, the 10 borrowed is 1, and the 10 in the 18 is 1: Say therefore 1 and 1 is 2, out of 3, and the remain is 1, which write over the 3, having ſtill cancelled the figures which you have uſed orderly as you go.

Now the whole Diviſor being cancelled, it muſt be removed on one place further, and placed as here.

Now ask again, how many times 4096 in 8192? it will be found (becauſe of the 0 after the 4) as often as 4 in 8, that is two times, put therefore in the quotient 2, and work as before, ſaying, 2 times 4 is 8, out of 8 remains 0, then 2 times 0 is 0 out of 1 remains 1, then

```
       00
      011
    018390
   13110120,(320
    4096666
     40999
      40
```

then 2 times 9 is 18, take 8 out of 9 remains 1, (which put over 9,) and 1 out of 1 in the next place remains 0, then laſtly, 2 times 6 is 12, that is, 2 out of 2 there remains 0, and 1 out of 1, there alſo remains 0, cancel and put the remainder over as formerly.

Now again, the Diviſor being all cancelled, ſhould be removed; and ask how many times 4096 in 000, the anſwer is nothing 0, which being put in the quotient, the work is all done.

And the quotient 320, ſhews that the *Diviſor* 4096 is contained in the *Dividend*, 1310720, three hundred and twenty times.

And whether the Diviſor have 2, 3, 5, 7, or more places, the working is ſtill like this, not differing from it at all.

Compendiums in *Diviſion*. For brevity in ſome caſes, where the *Diviſor* towards the right hand hath one *Cypher* or more; thoſe Cyphers may be placed orderly under the *Dividend* at firſt, and remain there till the work be done, with the reſt of the *Diviſor*, which muſt needs ſhorten the *Diviſion*.

As if 2587645 *were to be divided by* 15000, *place them thus*:

```
2 5 8 7 6 4 5
1 5     0 0 0
```

And ſay how many times 15 in 25? anſwer is 1, which write in the quotient, and multiply in mind the Diviſor by the quotient, ſaying once one is one, out of 2 there remains 1, then cancel the 2, and the 1 under it, and write the remaining 1 over it, as here is done, then ſay once 5 is 5 out of 5, and there remains 0, therefore cancel the two 5, and over the uppermoſt write 0.

```
0 0
3 1
1 0 3 (7
2 5 8 7 (6 4 5 (1 7 2
1 5 5 5 0 0 0
  1 1
```

Now

Now remove the Diviſor one place, and ask how many times 15, in 108? anſwer is 17, which write in the quotient, and multiply, ſaying 7 times 1 is 7, out of 10 remains 3, which write over the 0, then ſay 7 times 5 is 35, take 5 out of 8 remains 3, which write over 8, and 3 out of 3 remains 0.

Now again, remove the Diviſor, and ask how many 15 in 37? anſwer is 2, which write in the quotient, and ſay two times 15 is 30, out of 37 remains 7, and the Diviſion is ended, the remain being 7645.

Proof of this.

Multiply the quotient	172
by the Diviſor	15
	860
	172
The Product is	2580
Before which put the three Cyphers, And then it is	
	2580000
To which add the remain	7645
The Total is	2587645

Which is equal to the Dividend, and therefore the work is right.

So if one would divide any ſum by 10, 100, 1000, 10000, &c. he need but cut off the firſt, two firſt, three firſt, or four firſt figures towards the right hand, the other figures ſhall be the quotient, and thoſe cut off, the remain.

As if 2587645 be divided by 1000, the quotient is 2587, and the remain 645.

If you would know how many pounds Sterling are in 95670 ſhillings: having placed them thus, that is, 2 under 9, and 0 under 0, then divide by 2, which is very eaſie. So the quotient 4783, ſhews that in the number of ſhillings given, is contained 4783 pounds; and 10 ſhillings remaining.

$\cancel{1}\ \cancel{1}\quad (1$
$\cancel{9}\ \cancel{5}\ \cancel{6}\ \cancel{7}\ (0\,(4783$
$\cancel{2}\ \cancel{2}\ \cancel{2}\ \cancel{2}$

The reaſon why the Diviſor was 2, is becauſe there are 20 Shillings in one Pound, and therefore any number of Shillings is turned into Pounds, by dividing by 20, that is by 2, putting the Cypher under Unity, onely to fill a place at laſt. And this way of turning any number of Shillings into Pounds, may be eaſily effected by memory, if you ſuppoſe the laſt figure of your given number to be cut off with a line or comma, and taking the half of the other figures. Thus, let the given number of Shillings be 5739; Imagine the laſt figure 9 to be ſeparated from the reſt by a line thus 573|9; now by memory take take the half o 573, by ſaying in your mind the half of 5 is 2, (and one remaining which makes the 7 following 17) the of 17 is 8, (and one remaining, which 1 is 10 *s.* to be added to the 9. and the whole is 286 *li.* 19 *s.*

In this way of Diviſion (as in all others) if the *Remain* at laſt be greater than the *Diviſor*, the *Quotient* is not juſt, but too little, which may be remedied (without beginning the work again) by dividing the remainder onely by the ſame Diviſor, for thereof will ariſe a new quotient, which added to the former quotient the ſum will be the juſt quotient.

So if 7290 be divided by 27, the numbers being placed thus.

Becauſe 27 can be had in 72 but twice, put 2 in the

the quotient, ſaying 2 times 27 is 54, out of 72, and there remains 18, which write over 72, cancelling as before is ſhewed; then rmoving the Diviſor, ſay how many times 27, in 189? anſwer is 7, but if one ſhould miſtake, and write 5 in the quotient, and ſay 5 times 27 is 137, out of 189, remains 54, and write it over as before; and remove the Diviſor, and ſay, how many times 27 in 540? anſwer is 20, but ſhould not be above 9; ſay therefore, 9 times 27 is 243, out of 540, and the remain is 297, which being the the laſt remain, and greater than the Diviſor, ſhews the quotient 259 is too little.

```
    2
   19
  1847
  7290 (259
  277
   22
```

Wherefore divide the laſt remain 297, by the Diviſor 27, ſaying twenty ſeven is in 29 once, and write 1 in the new quotient, and ſay, once 27 is 27, out of 29 remains 2, which write over 9, and remove the Diviſor, and ſay, 27 in 27, juſtly once, ſo write 1 in the quotient, ſo the quotient is 11; which added to the former quotient 259, gives 270, which is the true and whole quotient.

```
  2
  297 (11
  277
  72
```

A Second way of Diviſion.

Although (as I have ſaid) the former way is more uſed, yet this may ſeem plainer and more natural to ſome, I will therefore give on example of it.

Example.

6477734(19494	
3137734	*Rem.*
3006	*Subst.*
0131734	*R.m.*
01002	*Subst.*
0031534	*Rem.*
003006	*Subst.*
0001474	*Rem.*
0001336	
....138	*Rem.*

Let it be required to divide 6477734 *by* 334, where the first figure in the quotient is easily seen 1; substract once 334 out of 647, and write the remain under a line, then see how many times 334 in 3137, answer 9, by which multiply the Divisor 334 in another paper, the Product is 3006, which substract out of the remain, then the new remain is 131734.

Again, how many times 334 in 1317, answer is 4, for the third figure in the quotient; by which multiply the Divisor, the product is 1002, which take out of the later remain, as in the Margine, then will remain 31534.

Now try how many times 334 in 3153, answer is 9; for the fourth figure in the quotient, by which multiply the divisor (in a by-paper) the Product is 3006 (as might have been seen above in the second substraction) which substracted out of the later remain, there remains now 1474.

Lastly, ask how many times 334 in 1474, answer is 4, for the last figure in the quotient, by which multipy the Divisor 334, the Product is 1336, which substracted from 1474, there finally remains 138; which being less than the Divisor, shews the division is done.

Proof

Proof of this.

	334....
	3006...
The ſeveral ſubſtractions, and the final remain, added together	.1002..
	..3006.
	...1336
	138
The Total equal to the Dividend.	6477734

If the former (as I have ſaid more uſual) way ſeem difficult to Beginners, becauſe the Products of the Diviſor into the ſeveral figures in the quotient are not ſet down, but mentally made; and alſo becauſe the ſubſtraction of them begins towards the left hand: and laſtly, becauſe the remain is ſtill ſet above; yet this later way which agrees altogether with plain Subſtraction before taught, I hope is ſo plain, that any diligent Reader may acquire it without a Tutor. And yet for the better ſatisfaction and help of the young Learner, I will add another way or two more of Diviſion.

A Third way of Diviſion.

There is another kind of Diviſion which is very much uſed, and is in great requeſt with thoſe who have moſt occaſion to divide great numbers, the manner of working is not much unlike the way before taught, one or two *Examples* will make it plain.

Example 1. *Let it be required to divide* 162483 *by* 1321. Set down your numbers as you ſee them placed in the Margine, *viz.* Firſt, ſet down 162483

the Dividend, then on the left hand thereof set the Divisor 1321 with a crooked line between them, then on the right hand thereof make another crooked line, which must serve to set the figures of the quotient in, so are your numbers placed in due order; then draw a line under the Dividend, and make a prick under the figure 4, (because so far the figures of the Divisor would extend if they had been placed underneath the Dividend, accordin as in the other examples) this prick serves only to show how far you have proceeded in your work, and must at every division be removed a place further, till at length you come to the last figure of the Dividend: your numbers being thus placed with a line under them, you are ready for the work, which must be performed according to the directions of the following Rule.

```
1321)162483(
        .
   ─────────
```

THE RULE

Demand how often the Divisor *may be had in the* Dividend, *and place that number in the* quotient, *then multiply the* divisor *by the* quotient, *and place the* product *under the line: then substract this* product *from the* dividend, *and set the* remainder *under the* product; *then make a prick under the next figure of the* dividend, *and bring that figure down to the* remainder, *and then proceed as before.*

Example, Your numbers being placed, as is before directed, you may begin your work in this manner: First, say how many times 1321 can I have in 1624? say once, place 1 in the quotient, which 1 multiply the Divisor 1321, beginning at the left hand, saying, once one is 1, place 1 under the line, then once 2 it

is 2, set 2 under the line, then once 3 is 3, place 3 under the line: lastly, once 1 is 1, place 1 under the line, them substract this 1321 from 1624, and there will remain 303. To this 303 bring down the next figure in the dividend, namely 8; (first, making a prick under the 8) so will that number be 3038, under which draw a line, and repeat the same work again, saying, how many times 1321 can I have in 3038, which may be had two times, place 2 in the quotient, by which 2 multiply the divisor 1321; saying, 2 times 1 is 2, place 2 under the line: then 2 times 2 is 4, place 4 under the line: then 2 times 3 is 6, place 6 under the line: lastly, 2 times 1 is 2, place 2 under the line, and substract this 2642 from 3038, and there will remain 396, to this 396 bring down the next figure of the dividend, which is 3, so is this number made 3963, under which draw a line, and repeat the work once again; saying, how many times 1321 can I have in 3963, which may be had 3 times, by which 3 multiply the divisor 1321; saying, 3 times 1 is 3; then 3 times 2 is 6; then 3 times 3 is 9; and lastly, 3 times 1 is 3, which place under the line, and substract it from the line above, which in the example is the same number; therefore there remains nothing, and the work is ended; but if any remainder had been, that should have been set under the line, as by the examples following will appear.

```
1321) 162483 (123
         ...
      ------
      1321
       3038
       2642
       ----
       3963
       ----
       3963
       0000
```

Other Examples for Practice.

```
5624)793058(141
        . . .
     ---------
     5624
     23065
     ---------
     22496
       5698
     ---------
       5624
         74 remainder
```

In this Example where 793058 is divided by 5624, you may perceive that the quotient is 141, and 74 remaining, so that the real quotient is 141 $\frac{74}{5624}$.

The Proof of this Division.

This kind of Division is proved by Addition, for *if you add the several* products *arising from the multiplication of the several* quotients *into the divisor, and also add them to the* remainder *(if any be) the total of this Addition shall be equal to the dividend, if there be no error in the work.*

So in the example following, if you add 4325 the first product, and 30275 the second product, and 2796 the remainder together, in the same order as they now stand in the example, you shall find the total of this Addition to be 76321, equal to the dividend, which demonstrates the Work to be true.

```
            4325)76321(17
                   . .
                 --------
1 Product        4325
                 33071
                 --------
2 Product        30275
                  2796 remaind.
                 --------
                 76321 Proof
```

A Fourth way of Division.

There is a fourth way of Division used by some, not inferior to any of the preceding, for that it is no burthen to the memory, and it is also proved by Addition.

The manner of placing the figures is the same with the third kind of Division last taught; And for the performance of this Work, this is

THE RULE.

First write down the dividend, *and on the left hand thereof the* divisor, *with a crooked line betwixt them, and on the right hand of the* dividend *make another crooked line wherein to place the figures of the* quotient, *then draw a line under the* dividend, *and also make a prick under that figure of the* dividend *under which the last figure of the* divisor *would fall, were it to be placed as in the first kind of* division. *This done, demand how often the* divisor *may be found in those figures of the* dividend, *and place that digit in the* quotient, *then by this digit multiply the* divisor, *and set the* product *of this Multiplication directly under the* dividend, *beginning at the place where you made the prick, then substract this* product *from the figures of the* dividend, *and place the* remainder *over the* dividend, *cancelling the figure of the* dividend *as you proceed, so is the first figures of the* quotient *finished, then make a prick under the next figure of the* dividend, *and demand how often the* divisor *may be found in the last* remainder, *and the other figure being added thereto, which place in the* quotient, *and proceed in all respects as before, till you have pointed all the figures of the* dividend.

Example, *Let it be required to divide* 763258 *by*

2345) 763258 (

 2345

2345. Place your numbers as you see in the margine, and beca se there are 4 figures in the divisor, therefore make a prick under the fourth figure of the dividend, which is 2, and draw a line, then begin your Division in this manner: saying

First, how many times 2245 can I have in 7632 (or how many times 2 can I have in 7) say 3 times, place 3 in the quotient, by which 3 multiply the divisor, saying 3 times 5 is 15, place 5 under the prick, then 3 times 4 is 12, and 1 is 13, place 3 under the line; then 3 times 3 is 9, and 1 is 10, place a cypher under the line; then 3 times 2 is 6, and 1 is 7, place 7 under the line; then substract 7035 from 7632; saying, 5 from 12, and there remains 7, place 7 over 2, (cancelling the 5 and the 2) and bear one in mind, then 1 and 3 is 4, out of 13, there remains 9, place 9 over 3, and cancel 3 and 3, then 1 which I carried from 6, and there remains 5, place 5 over 6, and cancel 0 and 6; lastly, 7 from 7 there remains 0, which you need not set down, but cancel the two sevens, then will the work stand as above, aed the remainder will be 597.

     597
2345) ~~7632~~58 (3
     ———————
     ~~7035~~

Secondly, make a prick under the next figure of the dividend, namely 5, and say how many times 2245 can I have in 5975, answer two times, place 2 in the quotient, by which multiply the divisor, saying 2 times 5 is 10, place 0 under 5, and carry 1, then 2 times 4 is 8, and 1 is 9, place 9 under 7; then 2 times 3 is 6, place 6 under 9; lastly, 2 times 2 is 4, place 4 under 5; so is the product of this multiplication 4690, which you must substract from 5975, saying 0 from 5 and there remains 5, place 5 over 5, and cancel 0 and 5; then 9 out of 17, there remains

mains 8, place 8 over 7, and cancel 9 and 7, then 1 carried and 6 is 7, from 9 there remains 2, place 2 over 9, and cancel 6 and 9; lastly, 4 from 5 rests 1, place 1 over 5, and cancel 4 and 5, so have you finished your second figure, and your work will stand thus, and your remainder will be 128.

$$\begin{array}{r}
1281 \\
\cancel{1}\cancel{9}\cancel{7}5 \\
2345)\cancel{7}\cancel{6}\cancel{3}\cancel{2}\cancel{5}8(32 \\
.. \\
\hline
\cancel{7}\cancel{0}\cancel{3}\cancel{5}\cancel{0} \\
\cancel{4}\cancel{9}\cancel{6}
\end{array}$$

Thirdly, make a prick under the next figure of your dividend (namely under 8) and ask how many times 2345 can I have in 12858 (or how many times 2 can I have in 12) say 5 times, place 5 in the quotient, by which multiply the divisor, saying 5 times 5 is 25, place 5 under 8, and carry 2; then 5 times 4 is 20, and 2 is 22, place 2 under 0, and carry 2; then 5 times 3 is 15, and 2 is 17, place 7 under 9, and carry 1; then 5 times 2 is 10, and 1 is 11, place 11 under 4 and 6; so is the product of this multiplication 11725 to be substracted from 12858, saying 5 from 8 rests 3, place 3 over 8, and cancel 5 and 8; then 2 from 5 rests 3, place 3 over 5, and cancel 2 and 5; then 7 from 8 rests 1, place 1 over 8, and cancel 7 and 8; then 1 from 2 rests 1, place 1 over 2, and cancel 1 and 2; lastly, 1 from 1 rests nothing, so is your work ended, which you shall find to stand as in the Margine, the remainder being 1133.

$$\begin{array}{r}
11 \\
\cancel{1}\cancel{2}\cancel{8}3 \\
\cancel{5}\cancel{9}\cancel{7}\cancel{5}3 \\
2325)\cancel{7}\cancel{6}\cancel{3}\cancel{2}\cancel{5}\cancel{8}(325 \\
.. \\
\hline
\cancel{7}\cancel{0}\cancel{3}\cancel{5}\cancel{0}\cancel{5} \\
\cancel{4}\cancel{6}\cancel{9}\cancel{2} \\
\cancel{1}\cancel{1}\cancel{7}
\end{array}$$

I had an intent here to have put an end to *Division*, but there came into my mind

A Rule, by which you may certainly know what figure to ſet in your Quotient, and never to take one too great, or too little, but that which will juſtly ſerve: And alſo to perform (with eaſe and certainty) the hardeſt, and moſt difficult Sum that can be propoſed in *Diviſion*, without the aſſiſtance of *Multiplication*, only by *Addition* and *Subſtraction*, not burthening the memory at all.

In the practice of *Diviſion*, there is nothing more difficult then in large Sums (eſpecially if the firſt figures of the Diviſor be either 1, 2, 3, or cyphers, and the laſt figures 7, 8 or 9) to know certainly what figure to put in the Quotient, when you demand how often the diviſor may be had in the dividend; for the certain finding whereof (a little pains being taken before you begin your work) do thus.

Suppoſe you were to divide any Sum, as 1979909, by 309: Firſt, ſet down the nine Digits 1, 2, 3, *&c.* one under another, and againſt the figure 1, ſet 309 your diviſor, which doubled is 618, which ſet againſt 2, theſe added together make 927, which ſtands againſts 3: Add the diviſor 309 to 927, it makes 1236, which is againſt 4, to this add the diviſor, and it makes 1545, which ſtands againſt 5. And thus to every laſt number, ſtill add the *Diviſor* till you have gone through all the nine Digits, then will they be as in the Margine.

1	309
2	618
3	927
4	1236
5	1545
6	1854
7	2163
8	2472
9	2781

Having prepared this little Table, ſet your dividend and diviſor down, as in the third way of Diviſion pricking the dividend, and drawing a line under it, as is there directed, and as you ſee here done.

309)

309) 1097909 (3553

927
1709

1545
959

927
32

Then laying your litle Table before you, look in it for 1097, the four first figures of the dividend, which you cannot exactly find there, but the nearest number less (which you must always take when you cannot find the just number you look for) is 927, against which stands 3, set 3 in the Quotient, and substract 927. out of 1097, and there will remain 170, to which bring down 9, the next figure of your dividend, and it is 1709: Look this number in your Table, which you cannot find, but the next less is 1545, against which stands 5, set 5 in the Quotient, and substract 1545 from 1709, there will remain 164, to which bring down the next figure of your dividend (which here is a cypher) making it 1640. Look this 1640 in the Table, which you cannot find, but the next less is 1545, against which stands 5, set 5 in the Quotient, and substract 1545 out of 1640 and there will remain 95; to which bring down the last figure of your dividend 9, making it 959. Look this number in the Table, or the next less, which is 927 against which stands 3, set 3 in the Quotient, and substract 927 from 959, the remainder is 32. So is your division ended, and the Quotient is 3553 $\frac{032}{309}$ And with what ease and certainty this is effected; no Multiplication being used, I leave to the Reader to judge.

The

The Proof this Division.

This kind of division is also proved by Adition; for, *If you draw a line under the work, and add all the figures between the two lines together, (in order as they then stand) taking the remainder (if any be) the Total of this addition will be equal to the Dividend, if the work be true.*

Other Examples for Practice proved.

(4
221(73
542)76353(140
. . .

54280
216

75880
To which add 473 *the remainder*

The sum is 76353 *equal to the dividend*

6
1166
17794
5678)2345678(413
. . .

2271214
5673
110

2345014
To which add 664 *the remainder*

The Total is 2345678 *equal to the dividend.*

Questions

Questions performed by Division only.

Question 1. *If a piece of Land lying in a long square or Parallelogram, contain* 42952 *square perches, and one of the sides thereof be* 236 *perches long, how long must the other side be?* Divide 42952 by 236, *the* quotient will be 182, and so many perches long must the other side be.

Question 2. *In a year there are* 8760 *hours, and in every natural day there are* 24 *hours, I demand how many dayes be there in a year?* Divide 8760 by 24, the Quotient will be 395, and so many dayes be there in a year.

Question 3. *The distance from* London *to* Coventry *is* 133760 *yards, and in one mile there is contained* 176 *yards, now I would know how many miles it is from* London *to* Coventry; Divide 133760 by 1760 the quoteint will be 76, and so many miles it is from *London* to *Coventry*.

These Questions persomed by *Division* only, are the converse of those that were performed by *Multiplication*, which I the rather make choice of, that the Reader might see how *Multiplication* and *Division* prove each other.

There are one or to more kinds of *Division*, something like these last, but I shall forbear exemplifying them; for much variety helps to make a beok rather great than useful.

¶ Here is to be noted that in the following Rules. where there is continual use of *Division*, I sometimes use one kind of *Division*, and sometimes another, for variety sake, but the Practitioner may use which he is best skill'd in, for they all produce the same effect.

Reduction.

IS Two-fold, First, That which turns *Great* denomination into *Smaller* as Pounds into Shillings or Pence, this is done by *Multiplication*: as followeth.

Example 1.

Let it be asked how many pence are contained in 719 *l.* 11 *s.* 7 *d.* ?

First, a Shilling is contained in a pound 20 times, therefore multiply 719 by 20, or (which is the same, but shorter) by 2, and put 0 to the product, as in the Margine, this shews, that in 729 *l.* there are 14580 Shillings. To which add 11 *s.* it makes 14591 Shillings.

Again, because one peny is contained in one shilling 12 times, multiply 14591 by 12, it produceth 175092, to which add the 7 pence, so the sum will be 175099, and o many pence are contained in 729 *li.* 11 *s*, 7 *d.*

li.	
729	
20	*multiply*
14580	
11	*add*
14591	
12	*multiply*
29182	
14591	
175092	
7	*add*
175099	

Example 2.

Let it be asked how many pints are contained in 4 *Tuns,* 3 *Hogsheads, and* 27 *Gallons?*

First

First, 1. Tun is equal to 4 Hogsheads, therefore
4 Tun is equal to 16 Hogsheads, to which add the
3 Hogsheads, so there is 19 intire Hogsheads.

Again, because one Hogshead contains 63 Gallons, multiply 19 by 63, it produceth 1197 Gallons, to which add 27, it gives 1224 Gallons.

Lastly, because every Gallon contains 8 pints, multiply 1224 by 8, it produceth 9792, and so many pints are contained in 4 Tuns, 3 Hogsheads, and 27 Gallons

63	
19	*multiply*
567	
63	
1197	
27	*add*
1224	
8	*multiply*
9792	

After the same sort might dry Measures be reduced, as Quarters to Bushels, Pecks, or Gallons, and likewise all weights and Outlandish Coins, of which the proportion of the greater to the lesser is (before) known or given.

Secondly, It is often requisite to turn *Smaller* denominations to *Greater*: this is done by *Division*, as followeth.

Example 1.

Let it be asked how many pounds are contained in 80976 shillings?

Divide 80976 by 20, the quotient is 4048 *l.* and 16 *s.* remaining, which is the true answer.

~~1~~(1
8 ~~0~~ ~~9~~ ~~7~~(~~6~~(4048
~~2~~ ~~2~~ ~~2~~ ~~2~~ ~~0~~

Example 2.

Let it be asked how many pounds are in 109754 *d.*?

Because a pound contains a shilling 20 times, and a shilling contains a peny 12 times, therefore if 109754 be divided first by 12, the quotient shall be

9146 shillings and 2 pence over; then if 9146 be divided by 20, the quotient is 457 pounds and six shillings remaining; so that 109754 pence is equal to 457 *l.* 6 *s.* 2 *d.*

Or if 109754 had been at first divide by 12 times 20, that is 240, (which is the number of pence contained in a pound) the quotient had been 457 pounds and 74 pence remaining, which is all one with the former; so 74 pence is equal to 6 shillings 2 pence. More instances shall not need herein, because the thing of it self is very clear.

Progression.

IS also of two sorts, the first is of certain numbers in *Arithmetical Proportion* from 1; that is such as differ equally, as 1, 2, 3, 4, 5, 6, where the common difference is 1, (as is easily seen or 1, 3, 5, 7, 9, 11, where the common difference is 2, or any other, as 1, 8, 15, 22, 29, 36, where the common difference is 7, this is called *Arithmetical Progression.*

2. Secondly, of certain numbers in *Geometrical* Proportion from 1, that is such as increase by a common Multiplication, as 1, 3, 4, 8, 16, 32, where the common Multiplier is 2, that is the first by 2, produceth the second, and the second multiplied by 2, produceth the third, and so on.

Or as 3, 9, 27, 81, 243, where the common Multiplier is 3, this is called *Geometrical Progression.*

Both the common difference (in the first) and the common Multiplication (in the latter) shall for shortness hereafter be called *the common excess.*

First,

First, now of the first sort, or *Arithmetical Progression*, the principal use of this is,

1. *If the number of place, and common excess be given, to find the last number.*

2. *When the number of places, and the last number is given, to find the aggregate, or total sum of all the numbers.*

3. *When the last number, and the total sum is given, to find the number of places.*

4. *The number of places, and total sum being given, to find the last number.*

5. *The last number, and number of places given, to find the common excess.*

6. *The last number and common excess being given, to find the number of places.*

I will instance in no more, few of these ever hapning to be used.

For the first of these, let there be given the number of places. 100
The common excess 1
To find the last number also; 100

THE RULE.

Multiply the number of places less by 1 by the common excess, and to the product add the first number: the sum is equal to the last number.

So here, multiply 99 by 1, the Product is 99, (for 1 neither multiplies nor divides) to this add the first number 1, it gives 100 for the last number.

Or let the numbers be 1, 7, 13, 19, 25, 31, where the common excess is 6, and the number of places also 6.

Now, if the number of places less by 1, that is 5, be multiplied by the common excess, which is 6, the product is 30, to which adding the first num-

ber which is 1, the last number 31, is thereby composed. This is so easie that it is needs no proof.

2. For the second, which is, The last number, and the number of places given, to find the total sum of all the numbers.

THE RULE.

Add the first and last numbers together, and multiply the sum by half the number of places, the product is equal to the aggregate or sum of all the numbers added together.

So if to the first number 1 be added the last number 100, it gives 101, which multiplied by 50 (which is half the number of places) produceth 5050, which is equal to all the hundred numbers added together.

And hereby may that vulgar question be answered, which is,

If a man take up 100 stones placed a yard one from another, all in a right line by one at once, and bring them back one by one to his first standing, how many yards doth he go backwards and forward?

It is shewed before that he goes forward 5050 yards and he must needs come back just as much, that is, in all 10100 yards, which is 5 miles and 3 quarters; wanting 20 yards.

Or secondly, suppose the numbers were 1, 9, 17, 25, 33, 41. Whereof the common excess is 8, the first and last added gives 42, which multiplied by 3, (half the number of places) the Product is 126, which is the sum of them all.

3. For the third thing, that is, by the last number and the total, to find the number of places.

THE

THE RULE.

Add the first and last numbers, and by the sum of them divide the total, the quotient will be equal to half the number of places.

~~1~~ ~~2~~ 6 (3
~~4~~ 2

This is so plain, it needs no clearing.

4. For the fourth, if the total, and number of places, be given, to find the last number.

THE RULE.

Divide the total by half the number of places, the quotient is a number, from which if 1 *be taken, the rest is the last number.*

As let the numbers be 1, 3, 5, 7, 9, 11, 13, 15, or any other (in Arithmetical proportion) whatsoever. The sum of these is to be 64, and the number of places is 8, the half of it 4. Now if 64, be divided by 4, the quotient is 16, from which if 1 be taken, there remains 15 for the last number.

~~2~~ 0.
~~6~~ ~~4~~ (16
~~4~~ ~~4~~

5. Now for the fifth variety, if the last numbers and number of places be given, to find the common excess.

THE RULE.

From the last number take 1, *and the remain shall be the Dividend; then from the number of places also take* 1, *and make this later remain the Divisor; then the quotient of this Division shall be the common excess.*

Example. Let the numbers be 1, 4, 7, 10, 13, 16, from 16 take 1, remains 15, for the dividend, then from 6 (which is the number of places) take also 1 remains 5 for the Divisor.

Now when 15 is divided by 5, the quotient is 3.

And 3 is alſo the common exceſs, or difference beween 1 and 4, or 4 and 7, *&c.*

6. Laſtly, let the laſt number, and the common exceſs be given, to find the number of places.

THE RULE.

From the laſt number take 1, *and divide the remain by the common exceſs; then to the quotient add* 1, *the ſum is the number of places.*

As, let the numbers be 1, 5, 9, 13, 17, 21, 25, 29, from 29 take 1, remains 28, which divided by 4 (which is the exceſs) the quotient is 7, to which add 1, the ſum is 8, which is the number of places, as the Reader may eaſily count.

Geometrical Progreſſion.

I ſhall not be ſo large in this as in the former, becauſe theſe things are of little uſe to the Arithmetician, except where a number is to be many times doubled, tripled, or the like, which cannot be ſo eaſily abridged here, as in the other, becauſe there the laſt number ariſing of many *Additions* of the exceſs to 1, was eaſily found by one multiplication: but here the laſt number being made by many *Multiplications* of the exceſs, is therefore many times harder than the other.

The varieties here ſhall be but two.

1. *The common exceſs, and number of places being given, to find the laſt number.*

2. *The exceſs, and laſt number being given, to find the total ſum.*

The firſt of theſe may thus be found. Let the numbers be 1, 2, 4, 8, 16, 32, 64, 128, 256, 512, the exceſs is 2, the places 10, find, out the fifth number

(which

(which is easily done, for any one may reckon so far by heart, that is here 16, and multiply 16 by 16, it produceth 256, which is the ninth number: lastly, multiply 256 by the excess 2, thence ariseth 512, the number desired.

So if the places had been more, as 72, having found the 9 number 256, multiply it by 256, thence comes 65536 for the 17 th number, which multiplied by the excess 2, gives 131072 for the eighteenth place, which multiplied by 131072, gives 17179869184, for the 35 place; and that multiplied again by the excess 2, gives 34359738368, for the 36 place, that multiplied by 34359738368, the product will be 1180591620717411327424, for the 71 place, which lastly, multiplied by the excess, gives 2361183241434822654848, for the 72 place, which is the last number of the Progression required to be found.

Perhaps this may seem somewhat tedious, but where things cannot be performed without labour, the Reader must content himself with such Rules as make it less; for it is certain, that this way is much shorter than to have multiplied still by the excess 71 times, which else he must have done.

All this notwithstanding, he is not bound to use the same numbers, much less in other questions where the number of places is not the same: but whereas I began from the place 9, he may begin at 8, 10 or 12, or where he pleases, so as he remembers still where he is; for this is general, *If the number belonging to any place whatsoever, be multiplied by it self, the product shall be the number belonging to twice so many places want one place.*

Now for the second thing, which is to find the sum of all the numbers.

THE RULE.

From the last number take the first, and divide the remain by the excess want 1, then multiply the quotient by the excess, and to the product add the first number, the sum of them is equal to the sum of all the numbers.

So if from the last numbers, or 72 place, be taken 1, remain is 2361183241434822654847 which should be divided by 1, (that is the excess want 1, for the excess is but 2) but because 1 neither multiplies nor divides, that labor is saved: Now multiply this remain by the excess, the Product is 4722366482869645309696, to which adding the first number 1, by making the figure 6 next the right hand to be 7, you have the total sum of all the 72 numbers,

A Question resolved by Geometrical Progression.

A Londoner *sojourning in a Country Market Town in Winter made himself a new Frez Suit and Coat, on which were set 6 dozen of Buttons of Silk and Silver; a Baker being in his company liked it so well, he would buy it of him; the Citizen consented to let him have it, paying for the first Button a single Barly-corn, for the second 2, for the third 4, and so on doubling to the last.*

The Bargain was liked on both parts for the present, but shortly after revoked, for it could not be performed, and no man can be holden to an impossibility.

But why this could not be performed, may be judged: First, by inquiring the worth of so much Barly in Mony: And secondly, the weight of it; and how it should be removed.

1. For the first, allowing 10000 corns to a pint (which is more than enough) then 5120000 Corns make a quarter; and yet (for shortning the Division)

on) we will allow 10000000 Corns to a quarter; by which dividing the whole number of Corns (which is done by cutting off the first 7 figures towards the right hand) the quotient will be 47223664828696, and so many whole quarters there are, omitting the remain, as in this case inconsiderable.

Now allowing Barley were to be sold at 15 *d.* the Bushel (which is cheap) it is so many Angels; and therefore dividing by 2, it is 23611832414348 pounds sterling; which is in words, *Two hundred thirty six millions of millions, one hundred and eighteen thousand, three hundred twenty four millions, one hundred forty three thousand, four hundred eighty two pounds*, which I take to be too much for any Trades-man to get or keep.

And reckoning Land for ever at twenty years purchase, if this sum of pounds be divided by 20, the quotient is the yearly rent of 1180591620717 pounds.

And this divided again by 365 (the number of days in a year) the quotient is 3234497591, that is above *thirty two thousands of millions a day for ever.* So great a vanity may be concluded one for want of a little premeditation.

2. Now secondly, for the weight of it, if we put 8 Bushells to weigh 2 hundred pounds weight, (for sure it doth weigh more) then the whole number of quarters multiplied by 2, gives the weight of all to be 94447329657392 hundred weight, and if this be divided by 20, (which is but cutting off one figure towards the right hand, and dividing the rest by 2) or which is all one, cut off one figure from the number of quarters, the quotient 4722366482869 is so many Tuns. And therefore it will require 4722366482 ships of 1000 Tuns a piece to carry it: And consequently, if every Nation in the World

 had

had above 10000 ſuch ſhips, yet there muſt be above *four millions* of ſuch Nations; which I ſuppoſe are not to be found in this World.

And here I will leave this, having uſed this long example, (which though it require more labour as all great examples do, yet the ſame skill will do it, as if the places had been fewer) that the Reader being throughly exerciſed thereby, may the eaſier leap over others which are ſhorter.

The GOLDEN RULE, or, Rule of Three Direct.

THis is the moſt *uſeful* and moſt *eaſie Rule* in *Arithmetique*, and deſerve a *Golden Name*. It is when there are three numbers given, or known, to find a fourth in proportion with them.

But 4 numbers, are in proportion, and called *Proportional*, when as the firſt is to the third, ſo is the ſecond to the fourth.

As if there were given 3, 4, and 6, to find a fourth, which may be to 4, as 6 to 3, that is double, and that fourth number is 8; and this is called *Proportion Direct*; and the Rule whereby it is done, *The Direct Rule*.

There is alſo another proportion called *Reciprocal*; which is when as the firſt is to the third, ſo is the fourth to the ſecond: As 3, 4, 6, and 2, this is called *The Reverſe Rule*.

In *Direct proportion*, the product of the two middle numbers multiplied together, is ever equal to the

product

product of the first and last multiplied together, which serves not only for a *Proof*, but a ground of the *Rule*, which *Rule* shall here follow: the *Reverse Rule* being deferred till we have done with this.

The Rule Direct.

Multiply the second term (or number) by the third, and divide the Product *by the first; the* Quotient *shall be the fourth number desired.*

Example. Let the three numbers given be 2, 6, 3, multiply 6 by 3, the Product is 18; then divide 18 by 2, the Quotient is 9, which is the fourth number in proportion with 2, 6, and 3.

For as 2 to 3, so 3 times 2, which is 6, is to 3 times 3, which is 9

And so the Product 18 divided by 2, and the Quotient 9, causeth that the Product of 2 into 9 shall be also 18, and consequently if 2 be the first of the 4 proportional numbers, and 6 and 3 the two middlemost, then 9 is the last.

Otherwise.

Divide the second by the first, and multiply the third by the quotient, the product shall be the fourth.

So if one divide 6 by 2, the quotient is 3, by which multiply 3, the product is 9, for the fourth number, as before. Other ways this Rule might be expressed, but where the first way is so short and clear, there many other ways would rather trouble than help the person that should use them.

In the first way, which here we mean to use, and no other) if the *first* number be 1; then the product of the *second* and *third* gives the *fourth*, without any *Division*: Or, if the *second* or *third* number be 1, then there needs no *Multiplication*, but dividing the greater of them by the first, the quotient (in whole numbers, for yet we speak

whole numbers, for yet we speak of them) is the fourth number which was sought.

Note 1

To know when to use the Direct, or the Reverse Rule. Consider, if *more*, require *more*; or if *less*, require still *less*, then use the *Direct Rule.* But if *more* require *less*, or *less more*, then use the *Reverse Rule*, this will be easily understood when we come to Example.

Note 2.

To know how to place the three numbers when they are confusedly given. Remember that 2 of them are always of one denomination, as both *pounds*, or both *sheep*, or both *yards*, or *acres*; and the other number hath another denomination: now know, that this single number is ever the second number in order.

And one of the other two, namely, that which hath some relation to this *second*, is the *first*; and the other is the *third* number, whose relation is sought for in the *fourth*, whence its plain that the *second* and *fourth* are also of the same denomination.

And having premised these things, let us now exemplifie the Rule in some questions.

Question 1.

If three yards of Cloth cost 4 li. *what shall* 21 *yards cost?*

Set the numbers in order, as in the example. If 3 yards cost 4 *li.* what 21 yards? Here you see that the first number, and the third number, are both of one denomination, *viz.* both *yards*, and the second number is of another denomination, namely *pounds*, wherefore the *fourth* number which is sought for, must be also *pounds*; therefore multiplying (according

to the Rule before given) the *second* number by the *third*, and dividing the Product by the *first*, the quotient shall answer the question.

First, 21 multiplied by 4, (which is the third number multiplied by the second) produceth 84, which divided by 3 the first number, the quotient is 28 *li.* and so much shall 21 yards cost: for 28 is to 4, as 21 to 3, seeing each contains either 7 times.

And the work will stand thus.

Yards		*Pounds*		*Yards*
If 3	cost	4	what	21
				4
				——
$\cancel{8}\cancel{4}$		(28 *li.*		84
$\cancel{3}\cancel{3}$				

Question 2.

If 4 men eat 2 Pecks of Corn in a Week, how many Pecks shall serve 100 men?

Place your numbers as here you see, then multiply 100 by 2, (that is the third number by the second, and the Product is 200, which divided by 4, the quotien is 50, for the number of Pecks required.

Men		*Pecks*		*Men*
If 4	eat	2	what	100
				2
				——
				200
$\cancel{2}\cancel{0}\cancel{0}$		(50 *Pecks*		
$\cancel{4}\cancel{4}$				

Question 3.

If 20 sheep cost 13 pound 13 shillings 4 pence, what is that for every sheep?

Turn

Turn the Shillings and Pounds into Pence; thus:

s.	*li.*
13	13
12	240
26	520
13	26
156	3120

```
3120
 156
   4
----
3280
```

Multiply 12 *s.* by 12, the Product is	156
And 13 *li.* by 240 (because 240 pence make one pound) the Product is	3120
To which add the 4 *d.*	4
It makes in all	3280

Then the question will be, If 20 sheep cost 3280 pence, what shall one sheep cost?

sheep *pence*
If 20 cost 3280 what 1?

```
                       1  d.
1          d.          4 (8 s.
3 2 8 0(1 6 4          1 6 4 (1 3
2 2 2 0                1 2 2
                         1
```

By the Rule before delivered, I should multiply, the second number by the third, but in this example,

the

the third number being 1, it doth not multiply; I therefore divide 3280 the second number by 20 the first number, and the quotient 164, is the price of one sheep in pence, which divided by 12, the quotient is 13 *s.* and 8 *d.* remaining, the price of every sheep therefore is 13 *s.* 8 *d.*

Question 4.

How many 10 *inch Tiles will pave a Floor that contains* 16 *square yards?*

First remember there are 36 inches in one yard in length; which multiplied into 36, gives 1296, for the square inches in one square yard; multiply 1296 therefore by 16, thence comes 20736, the sum of all the 16 yards in inches.

Secondly, seeing every Tile is 10 inches in length, and 10 in bredth, multiply 10 by 10, it produceth 100 for the square inches in one tile, See the manner of work.

```
   36      10
   36      10
  ----    ----
  126     100
 108
 -----
 1296
   16
 -----
 7776
1296
-----
20736
```

Then by The Golden Rule.

If 100 *inches require* 1 *tile*; *what shall* 20736 *inches require?*

inches *tile* *inches*
If 100 require 1 what 20736.

o
~~2~~ ~~0~~ ~~7~~ | 36(207
~~1~~ ~~1~~ ~~1~~ | oo

Here because 1 doth neither multiply nor divide (as hath been several times intimated) I therefore divide the third 29736, by the first 100, the quotient is 207, and 36 remaining.

So it appears, that 207 is too little, and 208 too much to do the Work: the just number being $207 \frac{12}{100}$, we shall not trouble the Reader with this till he know something of Fractions.

Question 5.

If 100 *li. give* 6 *li. interest for a year, how much shall* 750 *li. give?*

Multiply 750 by 6, the product is 4500, which divided by 100, the quotient is 45 *l.* for the thing required.

li. *li.* *li.*
If 100 give 6, what 750?
6

4500

li.
~~4~~ ~~5~~ oo(45
~~1~~ ~~1~~ oo

Question 6.

If 750 *l. gives* 45 *l. interest for a year, what shall* 100 *l. give?*

Mul-

Multiply 45 by 100, the product is 4500. which divided by 750, the quotient is 6 *l.* for the intereſt of 100*l.* for a year.

	li.		*li.*		*li.*
If	750	give	45,	what	100
			100		
			4500		

3
$\cancel{4}\cancel{5}\cancel{0}\cancel{0}$ (6
$\cancel{7}\cancel{5}\cancel{0}$

Many other queſtions might be added, but the Rule is ſo plain, that it needs them not; and ſo general, that he which can reſolve one, may as well reſolve any other: And for that reaſon, and becauſe in all the Rules which follow, this Rule will be conſtantly made uſe of, I will ſay no more of it here.

The Golden Rule *Reverſe.*

IF 12 *Workmen do any piece of work in* 8 *months, how many Workmen ſhall do the ſame in* 2 *months?*

THE RULE.

Multiply the firſt *term by the ſecond; and divide the Product by the* third, *the quotient is the number deſired.*

Here 12 is not the firſt number, though it be firſt named; but the three numbers placed in order, ſtand thus, 8, 12, 2, for the middle term muſt always be of the ſame denomination with that which is required.

Now multiply 12, by 8, the product is 96, which divided by 2, the quotient is 48, which anſwers the queſtion, As in this example,

months

months	*men*	*months*
8	12	2
	8	
	2) 9 6 (48	
	8	
	16	
	16	

For if 8 months require 12 men; then (a fourth part of 8) 2 months, shall require four times 12, that is 48 men.

For here *less* requires *more*; that is, *less time*, *more hands*; and therefore it is wrought by the *Reverse Rule.*

Question 2.

How many Ells of Tapestry will serve to hang a Room 3 *yards high*, 6 *yards long*; *and* 5 *yards broad?* not regarding Doors, Windows or Chimney, but as if there were no such.

First, multiply 6 by 3, the product is 18, which doubled (because there are 2 sides called *lengths*) is 36 yards for all the length.

Secondly (for the same reason) multiply 3 by twice 5, that is by 10, the product is 30 yards, for all the bredth; which added to 36, gives 66 yards, equal to all the length and bredth in yards.

But now because Ells, that is, *Flemish* Ells (for such measure are Hangings sold by) is equal to 3 quarters of a yard, that is, their Ell is to our Yard as 3 to 4. Say therefore, if 4 give 66, what 3? multiply 66 by 4, it produceth 264; then divide 264 by 3, the quotient is 88. Again, multiply 88 by 4, and divide the product (which is 352) by 3, the quotient is 117, and

and 1 remaining. to which the divisor 3 being applyed; the number justly answering the question is 117 Ells, and one third part of an Ell.

Note 1.

Because here we had to deal with things which had equal length and breadth, that is square yards, and square ells; therefore one multiplication and division was not sufficient to proportion this: but if instead of working by 4 and 3, we had done it by their squares which is 16 anc 9, it might have been performed at once; thus multiply 66 by 16, the product is 1056, which divide by 9, the quotient is 117 $\frac{1}{3}$, as before, but I began not with this way, for I supposed my Reader ignorant of squares.

Note 2.

It might also have been done, by reducing all the terms into quarters of a yard at the first, and after the number is found, reducing them again to ells, but because it is more proper to work thus, till fractions have been taught: I leave that, and proceed to another question.

Questions 3.

If 1 *Close would graze* 21 *Horses for* 6 *weeks; then (supposing no waste to be made) how many Horses would it feed for* 7 *weeks?*

Multiply 21 by 6, it produceth 126, which divided by 7, the quotient is 18. At that rate therefore it would keep 18 Horses for 7 weeks.

Question 4.

If 1 *Close will feed* 18 *Horses for* 7 *weeks, how long shall it feed* 63 *Horses?*

Multiply (according to the rule) 18 by 7, the

 product

product is 126, which divide by 63, the quotient is 2, therefore 2 weeks it ſhall keep them.

The like way ſerves for Hay, Oats, or any other proviſion for Man or Beaſt; which may be of uſe in *Garriſons*, and ſuch like caſes where ſcarcity may be feared, to proportion either the *mouths* to the *meat*, or *meat* to the *mouths*.

Before I leave this *Rule*, (becauſe it comes not ſo much in uſe and Practice as the *direct Rule* doth, and therefore may be more apt to be forgotten) I will, to exerciſe the Reader therein, propoſe the following *Queſtions*, giving the Anſwers of them, and leave the Practice to the Reader to find out of himſelf, the better to fix it, the Rule in his memory.

Queſtion 1.

If 12 *men would raiſe a Frame in* 10 *days; in how many days would* 8 *men raiſe the ſame?*

Here, becauſe the fewer men would require the longer time, though the number be 12, 10, 8, yet you ſhall (by obſerving what hath been already delivered in this Rule) find the fourth porportional (which is the number anſwering the Queſtion) to be 15, and ſo many men will do the work in 8 days.

Queſtion 2.

If 60 *yards of Hangings of three quarters broad would hang a Room; How many yards of half a yard in breadth would ſerve to hang the ſame Room?*

Anſwer Ninety yards.

Queſtion 3.

If a yard being 12 *inches in breadth do require* 12 *inches in length to make a foot ſquare; What number of inches in length will make a foot ſquare, when the breadth of the board is* 16 *inches?*

Anſwer 9 inches.

Que-

Question 4.

If the base or end of any solid (as a piece of Timber or Stone) being 144 *inches, do require* 12 *inches in length of that piece to make a solid foot; What number of inches in length will make a solid foot, when the square at the end is* 216 *inches?*

Answer, 8 inches.

I will say no more of this Rule; Neither will I treat of the *Double Rule of Three*, as a rule by it self; but come to the *Rule of five Numbers*, which is an abridgment of the other.

The Golden Rule *Compound of five Numbers.*

Question. 1.

IF *a hundred pound weight* that is 112 pound weight) *carried* 126 *miles cost* 14 s. *how much shall three quarters of a hundred* (that is 84 pound) *cost; being carried* 40 *miles?*

THE RULE.

Multiply the three last numbers one into another, (that is) the third *by the* fourth, *and that product by the* fifth; *the last product shall be the* Dividend.

Again, *Multiply the* two first *numbers together; the product shall be the* Divisor. *This Division being made, the* Quotient *will be the number of shillings desired.*

Example of the former Question.

 First

First, Place your numbers according to the tenor of the question thus:

li.	*miles.*	*s.*	*li.*	*miles.*
112	120	14	84	40
120		12		
2240		28		
112		14		
13440		168 *pence*		
		84		*d.*
		672	13440)	564480 (42
		1344		
		14112		53760
		40		2688
		564480		26880

Your numbers being placed in order, reduce the 14 *s.* into pence, and it is 168 *d.* then multiply 168 by 84, the product is 14112, which multiplied by 40 the later product it produceth 564480 for the dividend.

Then multiply 112 by 120, it produceth 13440 for the Divisor.

Divide 5644800 by 13440, the quotient will be 42 pence which is 3 *s.* 6 *d.* and answers the question.

In this Rule, the *first* number and *fourth*, also the *second* and *fifth*; and also the *third* and *sixth*, are of like denomination and nature.

Question 2.

If 10 li. *for* 6 *months yield* 3 li. *interest*, *what shall* 625 li. *yield for* 36 *months?*

Place

Place them thus; 100, 6, 3, 625, 36.

Multiply the three last, as before is shewed, the later product is 67500 for the *dividend*; and the 2 first multiplied make 600 the *Divisor*, then divide 67500 by 600 the (or 675 by 6, which is all one) the quotient will be 112 whole pounds; and 300 (or 3) remaining, which because it is half the Divisor, signifies the half of a pound; that is 10 shillings. So the answer to the question is 112 *li.* 10*s.*

li.	*m.*	*li*	*li.*	*m.*
100	6	3	625	36
6			3	
600			1875	
			36	
			11250	
			5625	
			67500	

$\cancel{1}\,3$

6) $\cancel{6}\cancel{7}\cancel{5}$ (112

$\cancel{6}\cancel{6}\cancel{6}$

Which might have been given in one denomination, namely 2250 shillings, if before the work the pounds had been turned into shillings, by multiplying them by 20, as hath been shewed before.

But since most questions, except such as are studied for the purpose, are apt to end in some Fraction, I shall next treat of Fractions.

Only first, having spoken of the double Rule of Three, this may let you know, that all questions which are wrought at once by the compound Rule of Five, may be done at twice by the single Rule of Three; and the doing of them so by two Operations, is called, *The Double Rule.*

As in our last question, there are two things considerable,

derable, the difference of money; and the difference of time.

First, for the money.

Say, if 100 *li.* give 3 *li.* what 625 *li*? answer 18 $\frac{75}{100}$ *li.*

Secondly, for the time.

Say, if 6 *mo.* give $\frac{57}{100}$ *li.* what 36 *mo.* answer 112 $\frac{75}{100}$ *li.*

But this will be better understood anon; and then the Reader may use that which he likes best.

Of Fractions.

THe word *Fraction* signifies a *breaking*, or *breach* of any intire thing into parts; and when a number is broken so, the parts (which must needs be every one less than the whole; and the whole is accounted but *One* or *Unity*) being less than Unity, are called Fractions (that is, fragments or pieces) of Unity. Now the Unite, or intire number which is to be broken, may be any thing, as one *pound*, in respect of which, *Shillings* and *Pence* and *Farthings* are *Fractions*; or, one *shilling*, in respect of which, *pence* and *farthings* are *Fractions*; or, one *peny*, in respect of which *farthings* are *Fractions*; and the like of *Weights* and *Measures*, or any other thing to be broken into parts.

In *Fractions*, we shall treat first of *Numeration*, then of *Multiplication* and *Division*, then of *Reduction*; and lastly, of *Addition* and *Substraction*.

The reason of this Order will soon be seen; for *Mul-*

Multiplication and *Division* are here much easier than *Addition*, &c. and therefore ought to be learned before them.

NUMERATION.

NUmeration is nothing else but the way of writing Fractions; and that this may be done, we must consider that any *Unity*, or *Number* representing an *Unite*, may be broken into two parts equal; and then each of the parts is called *one second*, or *half*; or it may be parted into three equal parts, and then each part is called *one third*, and two of them are called two thirds; and the like may be understood if it were parted into 4, 5, 6, 7, 8, 9, 20, 50, or 100, or how many soever.

Now to write these; do thus:

Write	One half	Thus	$\frac{1}{2}$
	One third		$\frac{1}{3}$
	One fourth		$\frac{1}{4}$
	One fifth		$\frac{1}{5}$
	One sixth		$\frac{1}{6}$
	One seventh		$\frac{1}{7}$
	One eight		$\frac{1}{8}$
	One ninth		$\frac{1}{9}$
	One tenth		$\frac{1}{10}$

In every one of these 10 *Fractions*, the Number below the line is called the *Denominator*, and shews into how many parts the *Unite* is broken.

The number above the line shews how many of those parts are taken, or contained in the *Fraction*, and is therefore called the *Numerator*: So in the

Fraction $\frac{3}{5}$: the *Denominator* 5 shews the *Unite* to be broken into 5 parts: and the *Numerator* 3 signifies 3 of such parts to be contained in the *Fraction*; which Fraction therefore is called *three fifths*.

And here it is plain, that, As the *Numerator* is in proportion to the *Denominator*: so is the Fraction to 1, or Unity, for $\frac{3}{3}$ or $\frac{5}{5}$: or any the like, is equal to 1.

And therefore all Fractions are quotients of lesser numbers divided by greater, as $\frac{4}{7}$ signifies 4 to be divided by 7, and as the dividend 4, is to the divisor 7: so is the quotient $\frac{4}{7}$ to Unity.

And therefore this line of separation which is drawn between the *Divided* and *Divisor*, doth properly signifie *Division*.

Hitherto we have spoken only of such Fractions as are less then 1, and those are called *Proper Fractions*: but there are also $2\frac{1}{2}$, $3\frac{3}{4}$, $5\frac{1}{7}$, $6\frac{3}{5}$, and the like mixed Numbers; which so written signifie *two and an half*, 3 *and* 3 *quarters*, *five and a seventh*, 6 *and* 3 *fifths*. These by multiplying the whole Numbers, by the Denominator, and to the product adding the Numerators respectively, are turned to $\frac{5}{2}$, $\frac{15}{4}$, $\frac{36}{7}$, $\frac{33}{5}$, which are called *Improper Fractions*, because every one of them contains more than Unity.

These, nevertheless may be *multiplied*, *divided*, *added*, or *substracted* in the same way as are proper Fractions. And this shall serve for *Numeration of Fractions*.

MULTIPLICATION.

THE RULE.

M*Ultiply all the* Numerators *together*, *the least product shall be the* Numerator *of the product required*: *Likewise*

Likewise multiply all the Denominators *together, the last product shall be the* Denominator *of the product sought.*

Example 1.

If $\frac{3}{5}$ be to be multiplied by $\frac{4}{9}$ Multiply the *Numerator* 3 by the *Numerator* 4, the product is 12, for the *Numerator* of the new product. Also multiplying the *Denominator* 5, by the *Denominator* 9, they produce 45, for the *Denominator* of the desired product, so that product which was required, is $\frac{12}{45}$.

Example 2.

If $\frac{1}{2}$, $\frac{3}{4}$, $\frac{4}{5}$, $\frac{5}{9}$, and $\frac{3}{11}$ were to be multiplied all together, begin with the *Numerators*, saying, ounce 3 is 3, and 3 times 4 is 12, and 12 times 5 is 60, and 60 times 3 is 180, for the *Numerator*: Then multiply the *Denominators*: saying, 2 times 4 is 8, and 8 times 5 is 40; and 40 times 9 is 360, and 360 times 11 is 3960, for the new *Denominator*. So that the product of all these is $\frac{180}{3960}$, that is 1644 to $\frac{1}{22}$, as shall be seen hereafter in *Reduction*.

And thus it appears, that proper *Fractions* being less than *One*, are still made less by *Multiplying*: as here the product $\frac{1}{22}$ is much less then $\frac{3}{11}$, which is the least Multiplier; and the reason hereof is plain, for seeing *Multiplication* is but the taking of a Number, a certain number of times, if that number of times be more than 1, then the Number to be taken is increased by being taken more than once; but if the Number of times be 1, it is not increased nor diminished, but is still the same; Lastly, if that number of times be less than 1, as $\frac{1}{2}$, the number not being taken once, but half of once, produceth a number less by half; that is, the half of the number to be taken; and the like reason is of all others.

Example

Example 3.

Multiply the mixt Numbers, $3\frac{1}{2}$, $4\frac{1}{3}$, and $5\frac{3}{4}$: Firſt (as hath been ſhewn already) turn them to improper Fractions: thus, firſt ſay, 2 times 3 is 6, and 1 is 7. So the firſt is $\frac{7}{2}$. Socondly, 3 times 4 is 12, and 1 is 13: ſo the Second is $\frac{13}{3}$. Laſtly, 4 times 5 is 20, and 3 is 23: ſo the laſt is $\frac{23}{4}$. Now the Fractions to be multiplied are $\frac{7}{2}$, $\frac{13}{3}$, and $\frac{23}{4}$; Firſt, for a new *Numerator*, ſay, 7 times 13 is 91, and 91 times 23 is 2093, for a new *Numerator*.

Then ſay, 2 times 3 is 6, and 6 times 4 is 24. So the new *Denominator* is 24.

And the product of all theſe Fractions is $\frac{2093}{24}$, that is, if real diviſion be made, $87\frac{5}{24}$

DIVISION.

Diviſion, To divide one Fraction by another, is but the croſs multiplication of them; that is, the *Numerator* of the one, by the *Denominator* of the other, and hereby the proportion of one Fraction to another is ſeen.

Example 1.

$$\begin{array}{ccc} & 24 & \\ \frac{3}{4} & \text{X} & \frac{6}{8} \\ & 24 & \end{array}$$

Divide $\frac{3}{4}$ by $\frac{6}{8}$, to do it ſet them thus: and multiply as the croſs leads; Saying, 3 times 8 is 24, which ſet over the Croſs for a new *Numerator*, and 6 times 4 is alſo 24: which ſet under the Croſs for a new *Denominator*; ſo the quotient is $\frac{24}{24}$, that is 1, which ſhews the Fractions to be equal one to another.

Example 2.

Example 2.

27

$\frac{3}{5}$ X $\frac{4}{9}$

20

Divide $\frac{3}{5}$ by $\frac{4}{9}$. Firſt, ſet them thus: And ſay, 3 times 9 is 27, for a *Numerator*, and 5 times 4 is 20, for the *Denominator*: ſo the quotient is $\frac{27}{20}$, and ſo many times is $\frac{4}{9}$, contained in $\frac{3}{5}$ that is, as 27 is to 20, ſo is $\frac{3}{5}$ to $\frac{4}{9}$, and ſo is $\frac{27}{20}$ to 1.

In *Diviſion* it is to be remembred, that the *Numerator* of the *quotient* ever ariſeth of the *Numerator* of the *Dividend*: And the *Denominator* of the *quotient* comes of the *Denominator* of the *Dividend*, each being croſs multiplied as before. And alſo remember alwayes to ſet the Dividend on the left hand of the Croſs.

If a Fraction be to be divided by a whole number; Multiply the *Denominator* by that *number*, the product gives the new *Denominator*, and the *Numerator* remains the ſame. So if $\frac{1}{4}$ be divided by 9, ſay 9 times 4 is 36. So the quotient is $\frac{1}{36}$.

Or if $\frac{1}{4}$ were to be multiplied by 9, the product (by multiplying the Numerator by 9,) will be $\frac{9}{4}$: that is, $2\frac{1}{4}$.

Example 3.

2880

$\frac{320}{8}$ X $\frac{45}{9}$

360

Divide $\frac{320}{8}$ by $\frac{45}{9}$, thus: ſay 320 times 9 is 2880, for a *Numerator*: And 8 times 45 is 360 for a Denominator. So the quotient is $\frac{2880}{360}$, or $\frac{8}{1}$.

For $\frac{320}{8}$ is equal to 40, and $\frac{45}{9}$ equal to 5, but 40 contains 5 eight times.

And ſo in the ſecond example, it may be proved, that as 27 to 20, ſo is $\frac{3}{5}$ to $\frac{4}{9}$. For firſt, multiply the

two

to middle most, then 20 times $\frac{3}{5}$ is $\frac{60}{5}$, that is 12.

Secondly, multiply the first and last, and then 27 times $\frac{4}{9}$ is $\frac{108}{9}$: that is also 12,

Wherefore by that which hath been said in the *Golden Rule*, the four numbers 27, 20 $\frac{3}{5}$, $\frac{4}{9}$, are proportional.

REDUCTION

REDUCTION OF FRACTIONS is threefold.

1. *To reduce one Fraction* (which is not already in the least) *to reduce a Denomination.*

2. *To reduce many Fractions of divers denominations, to one Denomination.*

3. *To reduce any Fraction from one Denomination* (as near as may be) *to any other Denomination desired.*

I. For the first of these, *To reduce a* Fraction *to its least terms.* Divide both the *Numerator* and the *Denominator* by the greatest *Common Division* that you can think of; the two *Quotients* being placed respectively in a Fraction, that Fraction shall be equal to the former Fraction, and in lesser terms.

So (in the 3 Examples of *Division*) to reduce $\frac{2880}{360}$, to $\frac{8}{1}$, divide 1880 by 360, the *Quotient* is 8: then divide 360 by 360, the *Quotient* is 1, and the new Fraction $\frac{8}{1}$ is equal to the former Fraction $\frac{2880}{360}$, and in less terms, as you may see. But to find the greatest common divisor, this is

The Rule.

Divide the greater term *by the* lesser (I mean by terms, the *Numerator* and *Denominator*) *and by the remainder*

mainder (if any be) *divide the* divisor, *and if any thing still remains, by that divide the last* divisor, *continuing this course till nothing remain greater then* Unity) *that* divisor *which is least of all, is the greatest* Common measure *of both terms, by which both being divided, and the* quotient *placed like a* Fraction, *that* Fraction *shall be equal to the former* Fraction, *and in the* least terms.

Example.

Reduce $\frac{148}{16}$ to the least terms; first divide 148 by 16, the quotient is 9, and 4 remains: again, divide 16 by 4, the quotient is 4, and nothing remains; wherefore taking 4, (the last divisor) for the greatest common divisor, by it divide 148, the quotient is 37, and by it divide 16, the quotient is 4. These two last quotients placed orderly in a Fraction, make $\frac{37}{4}$, which is equal to $\frac{148}{16}$, and in the least terms, for no number greater than 1, will divide evenly both 37 and 4.

Other wayes there are of lessening Fractions, as dividing the terms (if they be even numbers) by 2, and the quotients (if even) again by 2, or else by 3, or any other number that will divide them both evenly, that is, leave nothing remaining, but the former Rule being general and easie shall serve for all.

II. Now secondly, *To reduce many* Denominations *to one* common Denominator. Let the Fractions be $\frac{1}{2}$, $\frac{3}{4}$, $\frac{4}{5}$, $\frac{9}{10}$, to be reduced all to one denomination.

The Rule.

Multiply all the Denominators together, and the last Product shall be the common Denominator to all the Fractions. Then multiply every particular Numerator into all the Denominators *except his own, and the last* Product *shall be* Numerator *to that Fraction.*

Thus

Thus to reduce the forementioned Fractions $\frac{1}{2}$, , $\frac{4}{5}$, $\frac{7}{8}$, $\frac{9}{10}$, into one Denomination: Say, 2 times 4 is 8, and 8 times 5 is 40, and 40 times 8 is 320, and 320 times 10 is 3200, this last product 3200 shall be the common *Denominator*. Then to get Numerators for every one of them: As first, for the first, say 1 time 4 is 4, and 4 times 5 is 20, and 20 times 8 is 160, and 160 times 10 is 1160. For the first *Numeration*; so the first Fraction reduced is $\frac{1600}{3200}$. Then for the second *Numerator*: Say, 3 times 2 is 6 and 6 times 5 is 30, and 30 times 8 is 240, and 240 times 10 is 2400. So the second Fraction reduced, is $\frac{2400}{3200}$. After the same manner may the other three be reduced to $\frac{2580}{3200}$ for the third: $\frac{2800}{3200}$: for the fourth: and $\frac{2880}{3200}$ for the last: these are severally equal to the other, the first to the first, &c. as may be proved thus.

Let the Unity be a pound *Sterling*, then

	s.	
The $\frac{1}{2}$ of it is	10	
and $\frac{3}{4}$ is	15	
and $\frac{4}{5}$ is	16	*d.*
and $\frac{7}{8}$ is	17	6.
and $\frac{9}{10}$ is	18	
In all	76 *s.*	6 *d.*

That is 3 whole Unites, and 16 *s.* 6 *d.* over; Turn 16 *s.* 6 *d* all to six pences, it is 33, and because 6 *d.* is the fortieth part of a pound, therefore all the Fractions are equal to 3 $\frac{33}{40}$.

Now add the new Fractions (which being all of one denomination) may be added like whole Numbers: thus,

1600

$$\begin{array}{lr} & 1600 \\ & 2400 \\ & 2560 \\ & 2800 \\ & 2880 \\ \hline \text{In all} & 12240 \end{array}$$

Which divided by the Denominator 3200, the quotient is 3 $\frac{2640}{3200}$. Now $\frac{2640}{3200}$, reduced to the least terms, as hath been shewed how it may, will be $\frac{33}{40}$, so the sum of these also is 3 $\frac{33}{40}$, which is equal to the sum of the Fractions given to be reduced, and therefore they are equal in sum, and might be thus proved equal severally, that is, the first of them propounded to the first reduced. Divide the Numerator 1600 by the Numerator 1, the quotient is 1600. Also divide the Denominator 3200, by the Denominator 2, the quotient is also 1600; and so may any of the rest be proved equal by the equality of quotients. But I leave it as plain enough already.

III. Thirdly, *Any Fraction being given, to change the* denomination *to any other more requisite, retaining still* (as near as may be) *the same value.*

THE RULE.

Multiply the Numerator *given, by the* Denominator *required, and divide the* Product *by the* Denominator *given; the* Quotient *shall be the* Numerator *required.*

Example.

Let the Fraction given be $\frac{7}{13}$ of a pound *Sterling*, what is that in the twentieth parts or shillings? Multiply 7 by 20, the Product is 140, which divided by 13, the quotient 10 $\frac{10}{13}$ that is, 10 *s.* and $\frac{10}{13}$ of a shilling;

ling; which may be brought to pence thus, multiply 10 by 12, Product is 120, which divided by 13 quotient is 9 $\frac{3}{13}$ *d.* And again, multiply 3 by 4, the product is 12, which divide by 13, quotient is $\frac{12}{13}$ of a farthing, so seven thirteenths of a pound is 10 *s.* 9 *d* and almost a farthing.

But he which is resolved to have it in the smallest coin, do it at first work; for seeing a farthing is the 960 part of a pound, multiply 7 by 960, they produce 6720, which divided by 13, the quotient is 516 farthings, and $\frac{12}{13}$ of a farthing: these farthings may be turned to shillings, dividing by 48, or to pence by 4, as in *Reduction.*

This Rule though it be brief and plain is of great use in *Arithmetick*; either for turning Natural and surd Fractions into Decimals; or any other desired *Denomination*, with such facility and speed as may be wished.

IV. *Fractions of Fractions.*

In reduction of Fractions, some make another, or more parts, as *Fractions of Fractions* for one: that is, when there is a part of a *Fraction*; or a part of a part of a *Fraction*, &c. to be valued in one *Fraction.*

THE RULE.

Multiply all the Numerators *together the last* product *shall be the* Numerator *desired: Then multiply all the* Denominators *together, and this last product shall be the* Denominator *sought.*

Example.

Let the *Fractions of Fractions* propounded, be $\frac{4}{5}$ of $\frac{3}{4}$ of $\frac{1}{2}$, for so they are usually written; and let the *Numerators* be multiplied: saying, 4 times 3 is 12, and 12 times 1 is 12, the *Numerator* therefore required

quired is 12: then for the *Denominator*, say, 5 times 4 is 20, and 20 times 2 is 40, for the *Denominator* required; and $\frac{12}{40}$ is equal to $\frac{4}{5}$ of $\frac{3}{4}$ of $\frac{1}{2}$.

The Proof.

Let the Unite be 40 *s.* one fifth of 40 is 8, and therefore $\frac{4}{5}$ is 32, of which one fourth is 8, and $\frac{3}{4}$ is 24, of which one half is 12, and therefore $\frac{12}{20}$ is the just sum of all the Fractions: This needs no farther exemplifying.

ADDITION.

TO add many Fractions into one sum, consider whether they be all of one Denomination or divers: if one, Then add all the *Numerators* together into one Sum, that sum is the new Numerator: and the *Denominator*, in this cause is not altered.

Example.

Let the Fractions to be added be $\frac{2}{4}$, $\frac{4}{4}$ $\frac{5}{4}$, $\frac{1}{4}$, Add the Numerators: saying 2 and 4 is 6, and 5 is 11, and 1 is 12. So the sum of them all is $\frac{12}{4}$, that is 3 Unites.

As, let the Unite be 20 *s.* one fourth is 5 *s.* and $\frac{2}{4}$ is 10 *s.* and $\frac{4}{4}$ is 20 *s.* which added to 10 *s.* is 30 *s.* then $\frac{5}{4}$ is 25 *s* which added to thirty shillings gives 55 *s.* And lastly, $\frac{1}{4}$ is 5 *s.* which added to 55 *s.* makes 60 *s.* that is 3 times 20 *s.* that is 3 *l.* or 3 Unites.

But if the Fractions to be added, be of divers denominations; as let them be $\frac{3}{2}$, $\frac{3}{4}$, $\frac{4}{5}$, $\frac{7}{8}$, then (by the *Reduction* afore-going) they must be turned all into

one denomination, and then they will be $\frac{320}{480}$, $\frac{360}{480}$, $\frac{384}{480}$, and $\frac{422}{480}$. and may be added like those before: thus,

	3 2 0
	3 6 0
	3 8 4
	4 2 0
In all	1 5 8 0

So the ſum of all is $\frac{1484}{480}$, or $\frac{371}{120}$, or that is 3 $\frac{11}{120}$, which if it be money, and the Unite 1 *l.* it is then 3 *l.* 1 *s.* and 10 *d.* as may be tryed thus, Firſt $\frac{2}{3}$ of a pound, is 13 *s.* and 4 *d.* and $\frac{3}{4}$ is 15 *s.* and $\frac{4}{5}$ is 16 *s.* Laſtly, $\frac{7}{8}$ is 17 *s.* 6 *d.* Theſe all added together, the the ſum is 3 *l.* 1 *s.* 10 *d.*

SUBSTRACTION.

IN SUBSTRACION of one Fraction from another, if they be both of one denomination: It is done by taking the *Numerator* of one from the *Numerator* of the other, the remain is the new *Numerator*, and the *Denominator* the ſame as before.

So if $\frac{2}{9}$ be ſubſtracted from $\frac{8}{9}$, the remain is $\frac{6}{9}$, the like of all other.

But if they be not of on Denomination, they muſt firſt be reduced to be ſo; then that which is ſaid before is ſufficient.

Con-

Concerning the Golden Rule in Fractions.

THe *Golden Rule in Fractions* is the same as in whole Nnmbers, I will give you but one instance.

If $\frac{3}{4}$ of a yard of Tape cost $\frac{1}{2}$ of a peny, what shall one Inch, that is, $\frac{1}{36}$ of a yard cost?

Multiply the second by the third, the product is $\frac{1}{72}$, which divided by $\frac{3}{4}$, the quotient is $\frac{41}{216}$ of a penny, for the price of $\frac{1}{36}$ of a yard.

Otherwise.

Seeing $\frac{3}{4}$ of a yard may be turned to 27 inches: Say, if 27 cost $\frac{1}{2}$, what 1? divide $\frac{1}{2}$ by 27, it makes $\frac{1}{54}$ for the answer: which is equal to $\frac{4}{216}$, and in the least terms.

And wheresoever this may be done, to have the first and third Numbers of Fractions of one denomination, the best way is to work with their Numerators, not regarding their Denominators at all: As, *If $\frac{2}{3}$ cost $\frac{3}{4}$, what $\frac{7}{3}$? Instead* thereof write *If 2 cost $\frac{3}{4}$, what* 7? Multiply $\frac{3}{4}$ by 7, it produceth $\frac{21}{4}$, which divided by 2, the quotient is $\frac{21}{8}$, and that is the answer in the least terms.

And all this while it should have been noted that the Fractions are ever writen in a smaller figure then the whole Numbers.

The RULE of PRACTICE.

IN the *Golden Rule*, or *Rule of Three Direct*, I intimated; that if the first of the three Proportional Numbers given were *One*, that then the Product of the second and third numbers gives the fourth Proportional Numbers sought without using of any *Division*; —— Also, that if the second or third of the Proportionals given were *One*, then there was no need of *Multiplication*; but dividing the greater of them by the first, the Quotient shall be the fourth Proportional sought for.

And from hence is framed this Rule of *Practice*, (by some called the *Merchants Rule*) which always hath *One*, an ingredient in the Question, and it is no other but an *Abridgement* or *Compendium* of the *Rule* of *Three*, when *One* is one of the three Proportionals given

And that such Questions that are to be resolved by this Rule may be the more readily and easily answered (Money commonly being one of the three Terms) it is expedient that he which intendeth to make much use of this Rule, should have readily in his mind the *Even* or *Aliquot* parts of a *Pound*, of a *Shilling*, and of a *Peny*. And also to have in *Memory* the several *Products* of 12 (the number of *Pence* in one *Shilling*) multiplied into 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, and 12. All which are set down in the small *Tables* following, which ought first perfectly to be learned by heart, before farther Progress be made into this Rule.

TABLE

TABLE I. *The Aliquot or Even parts of a Pound or 20 Shillings.*

sh.	*d.*			
1	0	is the	One Twentieth $\frac{1}{20}$	of a Pound or 20 *s.*
2	0		One Tenth $\frac{1}{10}$	
2	6		One Eighth $\frac{1}{8}$	
3	4		One Sixth $\frac{1}{6}$	
4	0		One Fifth $\frac{1}{5}$	
5	0		One Fourth $\frac{1}{4}$	
6	8		One Third $\frac{1}{3}$	
10	0		One Half $\frac{1}{2}$	

TABLE II. *The Aliquot or Even parts of a Shilling.*

d.	*q.*			
1	0	is the	$\frac{1}{12}$ One Twelfth	of a Shilling.
1	2		$\frac{1}{8}$ One Eighth	
2	0		$\frac{1}{6}$ One Sixth.	
3	0		$\frac{1}{4}$ One Fourth	
4	0		$\frac{1}{3}$ One Third	
6	0		$\frac{1}{2}$ One Half	

TABLE III. *The several Pence in a Shilling multiplied by* 12

2	Pence multiplied by 12 produceth	24
3		36
4		48
5		60
6		72
7		84
8		96
9		108
10		120
11		132
12		144

For the working of the *Rule of Practice*, when the Price given is of Equal parts of a *Shilling* this is.

THE RULE.

Knowing by your Table what part of a Shilling it is, (whether $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, &c.) *divide the sum propounded by it, and the Quotient shall be the number of Shillings answerings our Question.*

Example.

At 6 *d. the Ounce, what* 7625 *Ounce?* Six Pence is (by your Table) $\frac{1}{2}$ of a Shilling, wherefore take one half of 7625, and it is 3812 *s.* and 1 remaining, which 1 is 6 *d.* So that 7625 Ounces, will cost 3812 *s.* 6 *d.* which is reduced into Pounds, by cutting off the last figure towards the right hand of 3812, and taking the half of the other figures, which will be Pounds and if one remain, in taking of the half it is 10 *s.*—— So the figure 2 being cut off from 3812, the half of 381 is 190 and 1 remaining, which is 190 *li.* 12 *s.* So the price of 7625 ounces will be 190 *li.* 12 *s.* 6 *d.* And so must you do for all others. As if the price be $\frac{1}{3}$, take $\frac{1}{3}$, if $\frac{1}{4}$ take $\frac{1}{4}$, as by the Examples following.

(1) At 6 *d.* the Ounce, what 7625 Ounces?

$\frac{1}{2}$	381|2	——	6 *d.*
	190	12 ——	6 *d.*

(2) At 4 *d.* the yard, what 3621 yards?

$\frac{1}{3}$	120|7	
	60 *li.*	7 *s.* 0 *d.*

(3) At 3 *d.* the Gallon, what 989 Gallons?

$\frac{1}{4}$	24|7		3 *d.*
	12 *li.*	7 *s.*	3 *d.*

(4) At 2 *d.* the Pound, 1760 Pounds?

$\frac{1}{6}$	112|6	8 *d.*
	56 *li.* 6 *s.*	8 *d.*

(5)

(5) At 1 *d.* 2 *q.* the Ell, what 9623 Ells?

$\frac{1}{8}$	1202|2 10 *d.* 2 *q.*
	60 *l.* 2 *s.* 10 *d.* 2 *q.*

(6) At 1 *d.* the Ounce what 672 Onnces?

$\frac{1}{12}$	5|6
	2 *li.* 16 *s.*

Thus have you *Examples* when the price is *even parts* of a *Shilling*, But when they are *uneven parts* of a *Shilling*, as 5 *d.* 7 *d.* or the like, then you must do the work at two or three Operations, though in the same manner, as Pence.

	Pence		*d.* *d.*
If the Price be	5	take for it	3 and 2
	7		4 and 3
	8		4 and 4
	9		6 and 3
	10		6 and 4
	11		6 and 3 and 2

Examples of these uneuen Parts of a Shilling.

(1) At 5 *d.* the Gallon, what 6254 Gallons?

$\frac{1}{4}$ is 3 *d.*	1563	6 *d.*
$\frac{1}{6}$ is 2 *d.*	1042	4 *d.*
5	260|5	10 *d.*
	130 *li.* 5 *s.*	10 *d.*

(2) At 7 *d.* the Ounce, what 9271 Ounces?

$\frac{1}{3}$ is 4 *d.*	3090	4 *d.*
$\frac{1}{4}$ is 3 *d.*	2317	9 *d.*
7	540|8	1 *d.*
	270 *li.* 8 *s.*	1 *d.*

(3) At 8 *d.* the Yard, what 7952 Yards?

$\frac{1}{3}$ is 4 *d.*	2650		8 *d*
$\frac{1}{3}$ is 4 *d.*	2650		8 *d.*
8	530|0		4 *d.*
	265 *li.*	1 *s.*	4 *d*

(4) At 9 *d.* the Ell, what 3769 Ells?

$\frac{1}{2}$ is 6 *d.*	1184		6 *d.*
$\frac{1}{4}$ is 3 *d.*	942		3 *d.*
9	282|6		9 *d.*
	142 *li.*	6 *s.*	9 *d.*

(5) At 10 *d.* the Dozen, what 625 Dozen?

$\frac{1}{2}$ is 6 *d.*	312		6 *d.*
$\frac{1}{3}$ is 4 *d.*	208		4 *d.*
10	52|0		10 *d.*
	26 *li.*	0 *s.*	10 *d.*

(6) At 11 *d* the Pound, what 6952 Pound?

$\frac{1}{2}$ is 6 *d.*	3476		
$\frac{1}{4}$ is 3 *d.*	1738		
$\frac{1}{6}$ is 2 *d.*	1178		8 *d.*
11	637|2		8 *d.*
	318 *li.*	1 *s.*	8 *d.*

(7) At 12 *d.* or 1 *s.* the Ounce, what 9871 Ounces?
$\frac{1}{20}$ of 20 *s.* therefore $\frac{1}{2}$ 987\|2 is
(493 *li.* 12 *s.*

If the Price of the Commodity is in Farthings, or Half pence, bring the Sum into Pence, and work as in the preceding Questions, and according to the following Examples.

(1) At

(1) At 1 *q*. the Pound, what 6392 Pound?

$\frac{1}{4}$	1598	
$\frac{1}{10}$	13|3	2 *d*.
	6 *li*. 13. *s*. 2 *d*.	

(2) At 2 *q*. the Ell, what 3625 Ells?

$\frac{1}{3}$.	1812	2 *q*.
$\frac{1}{10}$	15|1	
	7 *li*. 11 *s*. 0 *d*. 2 *q*.	

(3) At 3 *q*. the Ounce, what 7321 Ounces?

$\frac{1}{2}$	3660	3 *q*.
$\frac{1}{12}$	305	
$\frac{1}{2}$	152	6 *d*.
	45|7	
	22 *li*. 17 *s*. 6 *d*. 3 *q*.	

This is the manner of working for the even parts of a Peny, but if they be uneven parts; As two pence 3 farthings, five pence 1 farthing or the like, *Work first for the even part of a Shilling, and then for the farthings; which added the work is done.* As in these Examples.

(1) At 3 *d*. 3 *q*. the Ell, what 817 Ells?

$\frac{1}{4}$	204	3 *q*.
$\frac{1}{4}$	51	$\frac{3}{4}$
	25|5	
	12 *li*. 15 *s*. 0 *d*. 3 *q* $\frac{3}{4}$.	

(2) At

(2) At 4 *d.* 1 *q.* the Pound, what 7138 Pound?

$\frac{1}{6}$	1189	8
again	1189	8
$\frac{1}{8}$	148	$8\frac{1}{2}$
	252|8	$0\frac{1}{2}$
	126 *li.* 8 *s.*	0 *d* $\frac{1}{2}$

For the even parts of a *Pound*, you must take the parts as you find them expressed in the Table; as for 10 *s.* the $\frac{1}{2}$, for 5 *s.* the $\frac{1}{4}$, for 4 *s.* the $\frac{1}{5}$; as in Example,

(1) At 4 *s.* 6 *d.* the Ell, what 6294 Ells?

$\frac{1}{8}$ 786 *li.* 15 *s.*

(2) At 4 *s.* the Ream, what 735 Reams?

$\frac{1}{5}$ 147 *li.*

If (in this Rule) at any time the Question consists of the part of an *Ell*, *Yard*, *Pound*, *Ounce*, *Cross*, or the like; you must deal with the hole *Ells*, *Yards*, *Ounces*, &c. first, and afterwards add the price of the $\frac{1}{2}$ $\frac{1}{4}$, $\frac{1}{3}$, or what other part soever it be. And thus much shall suffice for this *Rule* of *Practice*.

The RULE of FELLOWSHIP.

THis Rule is useful for *Merchants*, and all such as Trade in *Companies*, with a Joynt stocks; and must share a proportional part of the gains, or loss; every one according to his stock which he laid in.

The Rule is two-fold, with *equal* time; or with *unequal* time.

That which is with *equal time*, is commonly called, the *Rule of Fillowship without time.*

Of this we will first speak. THE

THE RULE.

As the hole Joynt stock is to all the gain or loss: So is each mans particular stocks, to his part of the gain, or loss.

Example 1.

Two Purchasers A. and B. buy 700 *l. a year Land for ever,* (when money is at 8 *per Cent.) for* 14000 *l. of which A. paid* 8000 *l. and B* 6000 *l. after* 5 *years* (money being fallen to 6 *per Cent.) they sell it for* 18700 *l. so there is gained* 4700 *l. how much of this must A. have,*

First for A.

Say, if 14000 gain 4760, what 8000 ? answer, 2685 $\frac{10000}{14000}$.

Then for B.

If 14000 gain 4700, what 6000? answer, , 2014 $\frac{4000}{14000}$. As by the following operation doth appear.

(I) For A.

l. *l.* *l.*

If 14000 gain 4700, what 8000

8

14000) 3760 0000 (2685 $\frac{10000}{14000}$

28000

96000

84000

120000

112000

80000

70000

10000 remainder.

(II) For

(II) For B.

$$\begin{array}{cccccc} & l. & & l. & & l. \\ \text{If} & 14000 & \text{gain} & 4700, & \text{what} & 6000? \\ & & & 6 & & \end{array}$$

$$14000)\,28200000\,(2014\tfrac{4000}{14000}$$

28000
 20000

14000
 60000

56000
 4000 remainder.

Here note, That this Work might have been much abreviated, if from each of the three numbers you had cut off two Cyphers towards the right hand, as hath been formerly shewed in the Compendiums of *Multiplication* and *Division*.

Now for the proof hereof,

If you add $2685\frac{10000}{14000}$

which is the sum that A. gained;

To ——— $2014\frac{10000}{140000}$.

The sum which B. gained; the sum of them is 4700

Which is equal to the total gain.

And according to the proportion of these two Numbers: that is, as 8 to 6, or 4 to 3. So they, ought to have parted the yearly Rent also, all the time they received it: that is, As, ought to have 400*l.* yearly; and B. 300 *l.*

Example 2.

A. B. and C. joyn their moneys to make a stock of 25000 *l. of which A. laid in* 10000 *l. B.* 8000 *l. and C.*

C. put in 7000 l. with this (after a certain time in trade-ing) they gained 7500 l. how must this be parted?

First for **A.**

Say, if 25000 gain 7500, what 10000?

Or shorter, if 25 get 7 $\frac{1}{2}$, what 10? Multiply 7 $\frac{1}{2}$ by 10, it produceth 75, which divided by 25, the quotient is 3, that is, (restoring the three Cyphers) 3000 *l.* for **A.**

Then for B.

Say, if 2500, gain 7500 what 8000?

Or shorter, if 250 get 75, what 80?

Multiply and divide as the *Golden Rule* requires, to the quotient restore the two Cyphers, then it will be 2400 *l.* for B.

Lastly, for C.

Say, if 250 give 75, what 70? answer 21, to which p t the two Cyphers, it makes 2100 for C.

And these three 3000; 2400, and 2100, being added together, make 7500. And have that proportion as the particular stocks had: and therefore the Work is right.

(I) for **A.**

If 25 gain 7 $\frac{1}{2}$, what 10 *s.*

150

$7\frac{1}{2}\ \frac{15}{2}$ X $\frac{10}{1}$

2

1
1 5 0 (75
2 2

1
1 5 (3 3000 *l.* for **A.**
2 5

(II) for

(II) for B.

If 24000 gain 75, what 80

80

250) 6000 (24

500

1000

1000

2400 for B.

(III) for C.

If 250 gain 75, what 70 ?

70

250) 5250 (21

500

250

250

2100 for C.

And if instead of gaining 7500 *l.* whereby every one is supposed to have his stock, and a part of the gains; they had lost 7500 *l.* then their particular stocks had not been due to them, but so much as would be left after their proportional parts of the loss were abated.

Example 3.

A. B. and C. with a joynt stock of 25000 l. *gain* 7500: *of which A. gets* 3000, *B.* 2400, *C.* 2100; *what was their stock?*

This

This is but the Converse of the former, therefore say, if 5700 require 25000, what doth 3000 require? 10000 for A, and so work for the other two.

Many examples are of little use (except to load the Readers memory) where the Rule is so short and plain I will therefore add no more to this part of the Rule but immediately come to the Rule of Fellowship with time.

The RULE of FELLOWSHIP with time.

THis Rule is to be used when the times of the continuance of the particular stocks are unequal, and differ; so that here the difference of time, and also the difference of stock being both to be considered; it can be done no better way then by taking the *Power* of them both to be the particular stock; and all those *Powers* added, to be the whole stock, that which I call the *Power* is the Product of the money of every one multiplied by his time; And then.

THE RULE.

As the sum of those Products, is to the whole gain; so is each particular product, to its part of the gain.

Question 1.

Three Merchants A. B. C. make a stock of 10000 l. *of which A. layes in* 4000 *for* 3 *months, B.* 3000 l. *for* 6 *months; and C.* 3000 l. *for* 8 *months, with this they gain* 2000 l. *what is each mans share?*

First, for A. multiply 4000 by 3, it makes 12000, let that be accounted his particular stock.

Secondly,

Secondly, For B. multiply 3000 by 6, it makes 18000, his particular ſtock.

Laſtly, for C. multiply 3000by 8, it produceth 24000, for his ſtock, add theſe, they make 54000 *l.* for the general ſtock; then ſay,

For A

If 54000 give 2000, what 12000? anſwer, $666\frac{24000}{54000}$.

Then for B.

If 54000 give 2000, what 18000? anſwer, $444\frac{36000}{54000}$.

Laſtly, for C.

If 54000 give 2000, what 24000? anſwer $888\frac{48000}{54000}$.

The three Fractions may be reduced (by dividing each Numerator, and Denominator by 6000) and then the three ſhares will be $444\frac{4}{9}$, $666\frac{6}{9}$, and $888\frac{8}{9}$, which altogether make 2000, as they ought.

Queſtion 2.

Three Farmers, A. B. and C. lay out 1000 l. *to ſtock their grounds with Cattel, of which A. put in* 200 l. *for* 6 *years; B. had* 300 l. *going for* 4 *years; and C* 500 l. *for* 2 *years; at the end (by unſeaſonable times) there was loſt* 200 l. *which made the remain of their ſtock but* 800 l. *what had each man left?*

Multiply 200 by 6, it gives 1200: Likewiſe, 300 by 4, it gives 1200. Laſtly, 500 by 2, the product is 1000: all theſe are 3400 for the joynt ſtock.

Then firſt for A.

Say, if 3400 loſe 200, what 1200? anſwer, $70\frac{2000}{3400}$ for A. to which B. is equal, becauſe the power of his ſtock is ſo.

Therefore for C.

Say, if 3400 loſe 200, what 1000? anſwer, $58\frac{2833}{3400}$. So the 3 ſhares are $70\frac{20}{34}$, $70\frac{20}{34}$ and $28\frac{23}{34}$, equal to 200.

Now becauſe A. put in 200 *l.* and loſt $70\frac{20}{34}$, ſub-

ſtract

ſtract the loſs from the ſtock, remains 129 $\frac{14}{34}$.

And ſo doing for B. his remains will be 229 $\frac{14}{34}$,

And for C. his remain is 441 $\frac{6}{34}$. Now theſe three remains, 129 $\frac{14}{34}$, 229 $\frac{14}{34}$, and 141 $\frac{6}{34}$, make up 800 *l.* which was the whole remain.

Queſtion 3.

A. rents a cloſe for a year, to pay 80 l. *he puts into it* 200 *ſheep: 2 months after B puts* 40 *ſheep in, and 5 months after that C puts in* 100 *ſheep; how muſt every one pay of the rent?*

Multiply 200 by 12, it produceth	2400
And 40 by 10, produceth	400
Laſtly, 100 by 5, (which is C. time) produceth	500
In all	3300

Then for A.

If 3300 pay 80, what 2400? anſwer, 58 $\frac{600}{3300}$.

Then for B.

If 3300 pay 80, what 400? anſwer, 9 $\frac{2300}{3300}$.

And for C.

If 3300 pay 80, what 500? anſwer, 9, $\frac{400}{3300}$

The whole numbers make 79, and the broken numbers make 1. In all 80.

Note.

Whereas, hitherto we have conſidered only difference of *time* and *money*; it may be noted, that there may be difference of other kind, as *perſons* or *place*; but whatſoever they are, the power of all is found like theſe by multiplication; and are to be wrought like theſe, with ſo many Uſes of the *Golden Rule*, as the queſtion requires. I will therefore add but one queſtion more, which is this:

Queſtion.

One leaves a Legacy of 900 l. *among four Kinſfolk, A. B. C. D: ſo as B. may have twice as much as A. and C. thrice as much as B and D. as much and half as much as C; what is every one to have?*

Say, If A. be 1, B. is 2, C. 6, and D. 9, add these Numbers: 1, 2, 6, 9, together, they give 18, then say, If 18 require 900, what 1 ? Answer is 50. So A. is to have 50 *l.* B. 100 *l.* C. 300 *l.* and D. 450 *l.* which are their just parts; and altogether are equal to 900 *l.* and the work right.

BARTER.

TO Barter is to exchange one Commodity for another, the nature whereof will best appear by the resolving of some Questions.

Question 1.

Two Merchants Barter, One hath Sugar at 4 l. *the* C. *ready money, but in Barter he will have* 4 l. 13 s. 4 d. *The other hath French Wine at* 13 l. *the Hogshead ready money; at what price must he rate his Wine to equalize the others advance of his Sugar in Barter?*

Say, by the Rule of Three direct,

If 4 *l.* in Barter require 13 *s.* 4 *d.* advance, what shall 13 *l.* in Barter require ?

l.	s.	d		l.
If 4.———	13	—4	*what*	13 ?

```
      12
     ----
      30
     13
    ----
    160                ~~1~~ d.
     13              (~~1~~ ~~4~~(4
    ----           ~~2080~~(~~5~~ ~~2~~ ~~0~~(4|3
   480              ~~444~~.(~~1~~ ~~2~~ ~~2~~
  160                  ~~1~~2
  -----               l.  s.  d.
  2080  Answer        2   3   4
```

Question

Question 2

Two Barter, one hath 3 C $\frac{1}{2}$ *of Ginger at* 13 d. $\frac{1}{2}$ per *pound. The other hath Sugar at* 15 d. $\frac{1}{4}$ per *pound. How much Sugar must be delivered for the* 3 C. $\frac{1}{2}$ *of Ginger.*

First, by the Rule of Three (or Practice) find what the 3 C. $\frac{1}{2}$ of Ginger comes to at 13 *d.* $\frac{1}{2}$ *per* pound, which will be found to be 22 *l.* 1 *s.* For

If 1 *l.* cost 13 *d* $\frac{1}{2}$, what 3 C. $\frac{1}{2}$ cost?

Answer, 22 *l.* 1 *s.*

Secondly, Say, If 15 *d.* $\frac{1}{2}$ buy 1 *l.* of Sugar, what shall 22 *l.* 1 *s.* buy?

Answer, 347 $\frac{7}{61}$.

Question 3

Two Barter, One hath Tobacco at 14 d. per *pound, which he will Barter for Sugar at* 10d. *per* l. *how much Tobacco must be given for* 8900 l. *of Sugar?*

First, the 8900 *l.* of Sugar at 10 *d.* per pound, comes to 370 *l.* 16 *s.* 8 *d.*

Then, If 14 *d.* buy 1 *l.* of Tobacco, what number of pounds will 370 *l.* 16 *s.* 8 *d.* buy?

Answer, 6357 pound, and so many pounds of Tobacco at 14 *d.* must be given for 8900 pound of Sugar at 10 *d.*

Question 4.

Two Barter, One hath broad Cloth at 15 s. *the yard ready money, for which in Barter he will have* 16 s. 3 d. *The other hath Wool at* 2 s. 10 d. per *pound ready money? what price must his Wool be set at in Barter to equalize the advance which he puts upon his Cloth.*

Say by the Rule of Three direct.

If 15 *s.* ready money require 1 *s.* 3 *d.* in Barter; what shall 2 *s.* 10 *d.* ready money require?

Answer, 2 *d* — 3 *q.* $\frac{1}{3}$

So that he must rate his Wool at 3 *s.* 3 *q.* $\frac{1}{3}$ of a farthing *per* pound.

 Of

Of Interest Simple and Compound.

IN the *Appendix* to the *Second Part* of this *Book*, I have *Tables of Compound interest*, *Rebate* or *Discount* of *Money*, *Purchase* of *Leases* and *Annuities*, whose *Construction* and *Use* are there *Exemplified* by Resolving of *Questions* suitable to each *Table*, as by having recourse thither will appear. But for that *Tables* may not alwayes be at hand, I thought it convenient hear to shew how to resolve *Questions* both in *Simple* and *Compound Interest*, by which *Tables* of that nature may be *Calculated*, were there not enough, already extant.

Question 1.

If 100 *l. in* 12 *months gain* 6 l. *what shall* 625 l *gain in* 3 *years or* 36 *months?*

The Proportion is,

As 100 *l.* is to 6 *l.* in a year,

So is 625 *l.* to 112 *l.* 10 *s.* in a year.

Wherefore multiply 625 *l.* by 6 *l.* and divide the Product by 100, by cutting of two figures, the Quotient will be $37\frac{50}{100}$ that is, 37 *l.* 10 *s.* and this being multiplied by 3, giveth 112 *l.* 10 *s.* as by the Work appears.

l. *l.* *l.*

100 —— 6 —— 625

6

37 | 50

l. *s.* 3

112--10 112 | 50

Question 2.

If 100 l. *in* 12 *months gain* 6 l. *what will* 236 l. 10 s. 5 d. *gain in* 16 *months?*

The Proportion.

As 100 *l.* is to 6 *l.* in a year,

So

So is 236 *l.* 10 *s.* 5 *d.* to 14 *l.* 3 *s.* 9 *d.* 3 *q.* in a year.

l. *s.* *d.*

Multiply 236 — 10 — 5 by 6,

The Product is 1419 — 2 — 6,

This divide by 100, which is done by cutting off two figures of the Integer, leaving 14 *l.* on the left hand of the line. The figures on the right hand multiplied by 20, and the figures (or remains) again by 12; and lastly by 4; shall in all give 14 *l.* 3 *s.* 9 *d.* 3 *q.*

Which divide by 3, and add that third part to 14 *l.* 3 *s.* 9 *d* 3 *q.* the sum will be 18 *l.* 18 *s.* 5 *d.* 0 *q.* as by the work appeareth.

l. *l.* *l.* *s.* *d.*
100 — 6 — 236 — 10 — 5
6

l. 14 | 19 — 2 — 6
20

s. 3 | 82
12

d. 9 {1 | 70
8 | 2

| 90
4

q. 3 | 10

l. ~~1~~ 1 *s.*
2 ~~4~~ ~~3~~ ~~2~~ ~~1~~

l.	*s.*	*d*	*q*	*l.*	*s*	*d.*	*q.*
~~14~~	3	9	3	4	14	7	1

~~3~~ ~~3~~ ~~3~~ ~~3~~ ~~3~~

	l.	*s.*	*d.*	*q.*
In a year	14	3	9	3
In $\frac{1}{3}$ of a year	4	14	7	1
In 16 moneths	18	18	5	0

By this manner of Work, *If* 417 l. 11 s. 8 d. *be put out at Interest for* 2 *years at* 6 l. per *Cent. it will amount unto (I mean the Interest)* 50 l. 2 s. 2 d. *As by the Work appears.*

li.	li.	li	s.	d.
100	6	417	11	08
				6
		25|05	10	00

	li.	s.	d.	
				|20
In one year	25	01	01	|10
			02	|12
In two years	50	02	02	1|20
				|4
				|80

Thus these Questions are wrought by the *Single Rule of Three*, but they may be otherwise wrought by the *Golden Rule Compound of 5 Numbers.* Of which in that *Rule* you have an Example.

Of Compound Interest.

It may be wrought in the same manner that *Simple Interest* was, only add the increase every year as it riseth, to the sum of the year before-going, so continuing this course, till you have gone through the Numbers of Years required

Question.

What will 500 li. *amount unto, if it be forborn* 4 *years, after the rate of* 6 per Cent. *Compound Interest.*

First Work.

li.	li.	li.
100	6	500
		6
		li. 30|00

First year 30 *li.*

Add

Add this 30 *li.* found at this first work to the 500 *li.* it makes 530 *li.* then for the.

Second Work.

li.	*li.*	*li.*
100	6	530
		6
		li. 31|80

Second year 31 *li.*
Which add to 530 *li.* it makes 561 *li.*

Third Work.

li.	*li.*	*li.*
100	6	561
		6
		li. 33|66

Third year 33 *li.*
Which add to 561 *li.* it makes 594 *li.*

Fourth Work.

li.	*li.*	*li.*
100	6	594
		6
		li. 35|64

For the fourth year 35 *li.*

The four *Products* of these four *Multiplications*, being added together (having respect to the figures cut off) do make 131 *li.* which added to the Principal makes 631 *li.* $\frac{1}{10}$, and so much doth the Principal and Interest amount unto, being forborn four years, as here may be seen.

The Principal	500	
1 Product	30	00
2 Product	31	80
3 Product	33	06
4 Product	35	64
	131	10

And obſerving this *method*, you may reſolve any *Queſtion* for any *Number of Years*, and for any *Rate of Intereſt*, and by this *Rule* is the *firſt Table* in the *Appendix* of the ſecond part of this *Book* made. And here it will not be improper to add two of thoſe Tables in that *Appendix* (which are there in Decimal Numbers) reduced into *Pounds*, *Shillings*, *Pence*, and *q*.

The one *ſhewing what any ſum of Money forborn, and number of Years under* 31, *will amount or be increaſed unto.*

The other *ſhewing the preſent worth of any* Annuity, Rent, *or* Portion, *for any number of years to come under* 31.

I. TABLE.

Years	li	s.	d.	q	Years.	li.	s.	d.	q.
1	1	1	2	2	16	2	10	9	2
2	1	2	5	2	17	2	13	10	1
3	1	3	9	3	18	2	17	1	0
4	1	5	9	3 0	19	3	0	6	0
5	1	6		0	20	3	4	1	3
6	1	8	4	2	21	3	7	11	3
7	1	10	0	3	22	3	12	0	3
8	1	11	10	2	23	3	16	4	3
9	1	3	9	2	24	4	0	11	3
10	1	15	9	3	25	4	5	10	0
11	1	17	11	2	26	4	10	11	3
12	2	0	3	0	27	4	16	5	2
13	2	2	7	3	28	5	2	2	3
14	2	5	2	2	29	5	8	4	2
15	2	7	1	1	30	5	14	10	1

The

The first Table, *Sheweth what any sum of Money being forborn any number of years (under 31) will be augmented unto, accounting Interest upon Interest at* 6 per Cent. per Annum.

The Use of the Table.

Question 1.

If 234 li. *be forborn for the term of* 18 *years, how much will it be increased unto, accounting* 6 li. per Cent. *Compound Interest?*

Look in the Table for 18 years, and right against it you shall find 2 *li.* 17 *s.* 1 *d.* And so much will 1 *li.* or 20 *s.* be increased unto in 18 years.

Then say by the Rule of three;

If 1 *li.* give 2 *li.* 17 *s.* 1 *d.* what will 324 *li.* give?

20
57 *s.*
12
115
57
685 *d.*
324
2740
1370
2055 1|*s.*
12) 221940 (1849|5
924 *li.*
12
101
96
59
48
114
108
60
60

li.	*s.*	*d.*
924	15	00

So

So that 324 *li.* being forborn 18 years, will be increased unto 924 *li.* 15 *s.*

Question 2.

If 156 li. 15 *s.* 6 d. *be forborn* 20 *years, to what will it amount.*

Against 20 years in the Table is 3 *li.* 4 *s.* 1 *d.* 3 *q.* Wherefore work by the Rule of Three as followeth.

li.	*li.*	*s.*	*d.*	*q.*	*li.*	*s.*	*d.*
1	3	4	1	3	136	15	6
	20 *s.*				20		
240	64				2735 *s.*		
	12				12		
	129				5479		
	64				2735		
	769 *d.*				32826 *d.*		
	4				3079		
	3074 *q.*				295434		
					229782		
					984780		
					101071254		

q. d.

(1 (3 (6 1

|54 (877|3

0

438,13

438 *li.* 13 *s.* 6*d.* 3*q.*

So much will 136 *li.* 15 *s.* 6 *d.* be increased to in 20 years.

II. TA-

II. TABLE.

Years.	li.	s.	d.	q.	Years.	li.	s.	d.	q.
1	0	18	10	2	16	10	2	1	1
2	1	16	8	0	17	10	9	6	2
3	2	13	5	2	18	10	16	6	2
4	3	9	3	2	19	11	3	2	0
5	4	4	3	0	20	11	9	4	3
6	4	14	4	1	21	11	15	3	1
7	5	11	7	3	22	12	0	10	0
8	6	4	2	1	23	12	6	0	3
9	6	16	0	1	24	12	11	0	0
10	7	7	2	1	25	12	15	8	0
11	7	17	8	3	26	13	0	0	3
12	8	7	8	0	27	13	4	2	2
13	8	17	0	2	28	13	8	1	2
14	9	5	1	3	29	13	11	9	3
15	9	14	3	0	30	13	15	3	2

The ſecond Table, *Shewing what any* Annuity, Rent, *or* Penſion, *being forborn any number of* years *under* 31, rebating *or* diſcounting *yearly, after the rate of* 6 per Cent. *Compound Intereſt is worth in ready money.*

The uſe of the Table.

Queſtion 1.

What is a Leaſe of 25 li. per Annum *payable yearly, and to continue* 2 *years, in preſent money?*

Look in the Table for 21 years, and againſt it you ſhall find 11 *li.* 15 *s.* 3 *d.* 1 *q.* Then ſay by the *Golden Rule.*

If

If 1 *li.* be worth 11 *li.* 15 *s.* 3 *d.* 1 *q.* what 25 *li.*

20

235 Shillings
12

473
235

2823 Pence.
4

11292 Farthings.
25

56460
22584

282300

~~1~~ ~~1~~

~~2~~ ~~2~~~~2~~9~~1~~ (3 *d.*

~~2~~8~~2~~~~3~~~~0~~~~0~~ (~~1~~~~2~~~~2~~~~2~~~~2~~ (588 | 1

~~4~~~~4~~~~4~~~~4~~~~4~~ ~~1~~~~2~~~~2~~~~2~~~~2~~ (294. *s.* 3 *d.*

~~1~~~~1~~~~1~~

294 *li.* 1 *s.* 3 *d.*

And so much is 25 *li.* a year worth in ready Money to continue 21 years.

Question 2.

What is an Annuity *of* 75 li. *a year to continue* 30 *years, and to be paid yearly, worth in ready Money.*

Against 30 years in the Table you find 13 *li.* 15 *s.* 3 *d.* 2 *q.*

Then by the Rule of Three, say,

If 1 *li* 13 *li.* 15 *s.* 3 *d.* 2 *q.*

20

275 *sh.*

12

553

275

3303 *d.*

4

13212 *q.*

75

60060

92484

990900

12 *d.*

53212 154(9

990900 (247728 (2064|3

444444 122222 1032·s.

1111

li. *s.* *d.*

1032—3—9 -- And so much is 15 *li.* a year for 30 years worth in present money.

The Rule of Alligation.

THis hath its name from *binding*, *tying*, or *uniting* many particulars in one *Mass* or *Sum*, the nature of it will be understood in working some *Questions* or *Examples*.

Question

Question. 1.

A Corn Master would mix 4 *sorts of grains together, viz. Wheat at* 4 s. *the bushel, Wheat at* 2 s. 6 d. *the bushel; Rie at* 3 s. *the bushel, and Barley at* 4 s. 8 d. *the bushel; so as to make* 15 *quarters in all, to be sold at* 3 s, 6 d. *the bushel, How much must he take of each?*

$3\frac{2}{1}$	$4\frac{1}{2}$	$\frac{5}{6}$
	4	$\frac{1}{2}$
	3	$\frac{1}{2}$
	$2\frac{2}{3}$	1
		$2\frac{5}{6}$

Place them as in the Margine, so as a greater and lesser may still be together, as $4\frac{1}{2}$, with $2\frac{2}{3}$ and 4 with 3, and place the price required by it self towards the left hand, as here you see $3\frac{1}{2}$; then in a seperated Column, note the difference between the price of a bushel of every one particular given, and a bushel of that required, as the difference betwixt $4\frac{1}{2}$ and $3\frac{1}{2}$ is 1, which must not be placed against $4\frac{1}{2}$, but against that number linked with $4\frac{1}{2}$, that is, against $2\frac{2}{3}$, and so must all the differences be ordered, as is easie to be seen in the Margine: then,

THE RULE.

Multiply the whole Mass *to be made, by any particular* difference; *and divide the product by the sum of all the* differences, *the quotient shall be the just quantity of that* particular kind, *whose price standeth against the difference you wrought with.*

Example.

First turn the quarters into bushels, by saying 8 times 15 is 120, then for the quantity of the first sort at $4\frac{1}{2}$: multiply 120 by $\frac{5}{6}$, the product is 100, which divided by $2\frac{5}{6}$, the quotient is $35\frac{5}{17}$ bushels

of

of that ſort at 4 *s.* 6 *d.* and working ſo for every of the other; they will be found to be thus

At 4 *s.* 6 *d.*	$35\frac{5}{17}$	bushels.
At 4 *s.*	$21\frac{3}{17}$	
At 3 *s.* 6 *d.*	$21\frac{3}{17}$	
At 2 *s.* 8 *d.*	$42\frac{6}{17}$	
In all	120	

Now to prove this right; Firſt, multiply the whole Maſs 120 buſhels, by the deſired price $3\frac{1}{2}$ *s.* omitting the denominations, the ſum is 420 *s.*

Then ſecondly, multiply $38\frac{5}{17}$ by $4\frac{1}{2}$, it is,	$158\frac{14}{17}$
And $21\frac{3}{17}$ by 4, it produces	$84\frac{12}{17}$
And $21\frac{3}{17}$ by 3*s* it makes	$63\frac{9}{17}$
And $42\frac{6}{17}$ by $2\frac{2}{3}$, is	$112\frac{16}{17}$
In all	420

And ſo much all ſhould be worth in ſhillings. And therefore the queſtion is rightly reſolved.

Queſtion 2.

One hath 6 *ſorts of Fruits at ſeveral prices; Dates at* 2 s. *Almonds at* 1 s. 4 d. *Currants at* 10 d. *Raiſins at* 5 d. *Prunes at* 4 d. *and Figs at* 3 d. *the pound; and would take of every ſort ſome to make a mixed quantity of* 30 l. *weight to ſell one with another for* 9 d. *the pound, how much muſt he take of each?*

Having

9	16	6	
	16	5	
	10	4	
			30
	5	1	
	4	7	
	3	15	
	Sum	38	

Having placed the Numbers and their differences, and the ſum of thoſe differences diſtinctly as hath been ſhewed before, and may be ſeen by the Figure in the Margine; the Work is evermore like that in the former queſtion. So 38 is the firſt number in the *Golden Rule*; 30, the ſecond (which that it may not be forgotten, may be ſet at the right ſide of the figure) and every particular difference, as 6, 5, 4, *&c.* is the third in the Rule, to be repeated till all the differences have been employed.

So 30, multiplied by 6, produceth 80, which divided by 38, the quotient is 4 $\frac{22}{38}$ of a pound weight, and ſo much muſt be taken of Dates, at 24 *d.*

Secondly, 5 times 30 is 150, which divided by 38, the quotient is 3 $\frac{36}{38}$ for Almonds. And working after the ſame manner with 4, 1, 7, 15, their reſpective quantities will be ſound to be theſe;

	pounds	38th parts
Dates	4,	28
Almonds	3,	36
Currants	3,	6
Raiſins	0,	30
Prunes	5,	20
Figgs	11,	32
in all	26	152

That is 26 $\frac{152}{38}$, and the reduction of the fraction will make it 30, as it ought to be, and by compa-

ring

ring the prices of these particulars added, with the price of 30 *li.* weight, at 9 *d. per l.* weight, which makes 470 *d.* this may be proved like the former.

But that the Reader may be perfect in it, I will do it here also as followeth:

Say first, 24 times 4 is 96, and 24 times 28 is 672: for the first set them thus: 96, 672

And 16 times 3 is 48 and 16 times 36 is 576	48, 576
And 10 times 3 is 30, and 10 times 6 is 60,	30, 60
And 5 times 30 is 150	00, 150
And 4 times 5 is 20, and 4 times 20 is 80	20, 80
Lastly, 3 times 11 is 33, and 3 times 33 is 96	33, 96
In all	227, 1634

Now this 1634 being the sum of the Numerators of Fractions, whose common Denominator is 38, must be divided by 38, and the quotient will be 43, which added to the whole number 227; the sum is 270. And so much is 30 multiplied by 9, which shews the work to be right.

The Combination or linking of Number may be varied at pleasure, as whereas above I linked 24 and 3, also 16 with 4, and 10 with 3; it might have been 24 with 5, and 16 with 4, and 10 with 3. Or 24 with 4, and 16 with 3, and 10 with 4, of which diversity of linking would follow diversity of solutions, but all true, as the Reader may easily prove by himself.

Likewise, if the Numbers to be linked were 3, 5, 7, or any odd Numbers, one of them may be

linked to two ſeverally, to make the work even.

Example. 3.

If the Numbers were 12, 10, 8, 6, and 4, and the mean or common price required were 9, you might firſt link them as you ſee here, taking 12 twice, or elſe you might take any other twice as you ſhall like; and ſo the work will be every way right, though not the ſame; if the differences be rightly ſet off, and orderly uſed, as is taught before in the firſt queſtion.

9	12	5
	12	3
	10	1
	8	1
	6	3
	4	4

Queſtion 3.

A Goldſmith would mix 3 *ſorts of Silver.* A. B. C. *A. is* 10 d. *weight better;* B. 7 d. *weight better; and* C. 4 d. *weight better, to make an Ingot of* 50 l. *weight, which ſhould be in fineneſs* 8 d. *weight better: How much muſt be taken of each?*

8	10	4	50
	10	1	
	7	2	
	4	2	
		9	

Set them their differences, and the ſum of their differences, as in the Margin.

1. Then

1. Then first 50 multiplied by 4 is $\frac{200}{9}$

and divided by 9, the quotient is

2. 50 Multiplied by 2, $\frac{100}{9}$

and divided by 9, the quotient is

3. 50 multiplied by 2, $\frac{100}{9}$

and divided by 9, the quotient is

4. And again, the same $\frac{100}{9}$

In all $\frac{500}{9}$

Which is equal to 50, the quantity required.

Now the first Fraction multiplied by 10, (omitting the Denominator) is 2000

The second also by 10 gives 500

The third by 7 makes 700

The last by 4 makes 400

In all 3600

That is, $\frac{3600}{9}$, which is equal to 400, and if the whole Ingot 50, be multiplied by the betterness required, namely, by 8, they shall produce 400 also: So this is proved.

In every Alligation, or linking of two numbers, this is evident, that if the sum of the numbers linked be greater than the mean number required taken so many times as there are numbers to be linked, the question would be absurd; and the resolution thereof impossible. And this shall serve for the Rule of *Alligation*.

The Rule of False Position.

THis Rule serves to resolve Questions, which are not presently fit for the *Golden Rule*; and therefore instead of the true Number which is sought: Suppose any number great or small, and make trial of it. whether it resolve the Question without any error; if so, it is the true number: if not, note what error be *too much*, or *too little*; if *too much*, mark it thus ✠, but if *too little*; thus —

Then suppose again another Number, (it imports not whether it be nearer or further off) and try as before, and mark that error also with ✠ or —, according as you find it to be, either *more* or *less*.

THE RULE.

Multiply the first Position *by the second* Error, *and the second* Position *by the first* Error, *and* (if the errors be both ✕ or both —) Substract *the* lesser *product from the* greater, *and keep the remain for a* Dividend, *and the difference of the errors for the* Divisor; *the quotient of that division is the true Number required.*

But if the errors be one ✠, *the other* —, *the sum of the product added together must be the* Dividend: *and the sum of the* Errors, *the* Divisor; the rest of the work is the same as before.

Question. 1.

A man is to drive 48 *young Turkies* 40 *miles, and for every*

every Turky which comes alive to the end of the Journey, he is to receive 3 d. *but for every one which dies by the way, he is to pay* 6 d. *At the end he received* 72 d. *How many dyed by the way?*

Let the first supposition be

That by the way there died 20

For them he was to pay 120 *d.* and for 28 which lived, he was to receive 84 *d.*

so he paid more then he received 36 *d.*

and should have got clear. 72 *d.*

Add 108

Whereof the first error is —108

Let the second supposition be 10

For these he paid 60 *d.* and for the rest he received 114 *d.* the difference is 54, and should be 72: so the second error is —18

Now 20 multiplyed by 18, produceth 360

and 10 by 108 produceth 1080

and the difference is 720 for the *Dividend*, likewise the difference of the errors is 90 for the *Divisor*, and the quotient is 8, which is the true Number of those he lost by the way: As may be proved by tryal.

Question 3.

If it were required to make up a pound Sterling *of Shillings and Groats only; and so as the number of* Groats *may be to the number of* Shillings, as 7 to 1; *How many Shillings must there be?*

First, suppose the Shillings 4

then the Groats must be equal to 16 *s.*

that is 48 Groats; but the Shillings

taken 7 times are 28, to which

48 should be equal, but is ✠20

 Secondly,

Secondly, ſuppoſe the Shillings 2
then the Groats (making 18 *s.*) are 25
which ſhould be equal to 7 times 2, but is ✠ 40

Multiply 4 by 40, product is 160, then

Multiply 2 by 20, the product is 40, which taken from 160, reſts for the Dividend 120
And the difference of errors is 20

Laſtly, 120 divided by 20, the quotient is 6
The number of Shillings therefore is 6
And the number of groats is 42

But as 7 to 1, ſo is 6 times 7 which is 42, to 6 times 1, which is 6: ſo the work is done.

Queſtion 3.

If there be 4 ſeveral weights, A. B. C. D. of which D. is 24 ounces, and C. is double to B. and triple to A. and D. with twice A. is double to C. and quadruple to B. How much doth every one of theſe Weights weigh?

First, ſuppoſe A. to be 8
then D with twice A. is 24, and 16, that is 40,
of which C. being the half is 20, and B 10.

Now thrice A. is 24, to which C. ſhould be equal, but is —4

Secondly, let A. be ſuppoſed 4
then D. more, twice A. is 32, and C. 16.
and B. is 8, but thrice A. is 12, to which 16
ſhould be equal, but is ✠4

Then 8 multiplied by 4, gives 32, and 4 by 4, produceth 16: both theſe Products give 48 for the Dividend: and the ſum of the errors (becauſe the firſt is —, the other ✠) gives 8 for the Diviſor, and the quotient will be 6, to which A. is equal, and twice A. more D. is 36, of which C. being half is 18, and B. is 9, and thrice A. is equal to C. namely 18, and all right.

Whereas the firſt error is equal here to the ſecond, it

it follows that the Positions were equally false; and therefore their difference which is 4, being parted into two equal parts, 2 and 2, if 2 be taken from 8, the remain is the true number 6, or if 2 be added to 4, (which was the second position) the sum will be also 6.

And further, whensover the errors be one ✠, the other —, though they be not equal; yet then if the difference between the positions be parted into two parts, which are in proportion one to another, as the two errors are one to another respectively: then if the first part be taken from the first position (if that be the greater) or add to it (if it be the less) the same number required is thereby had.

As, let the last question be resumed,

And let the first position for A. be	15
Then the first error will be	— 18
Then let the second position be	3
And so the second error will be	✠ 6
And the difference of position is	12

Which divided into two parts 9 and 3, which have that proportion one to another as have the errors 18 and 6, then if the first part 9, be taken from the first position 15, there remains the true Number 6. Or else if the second part 3, be added to the second position 3: thereby also is made the true number 6.

The way of parting 12 (or any other) into two parts proportional with the errors, is easily done by the *Golden Rule*, thus:

As the sum of the errors 24,
is to the difference of position 12;
So is the greater error 18,
to the greater part required, namely 9.

Many other questions are in the other Books exemplified and wrought by this Rule; but seeing I intend

not to write a great Book; and also because some of those questions may be resolved without this Rule, I will add no more: only mention one of those questions.

If there be a Cistern with 4 Cocks, which holds 8 Barrels of Water, and the first cock will run it all out in 6 hours, the second in 4, the third in 3, and the last in 2 hours: in what time shall all of them run it out?

If the first in 6 hours run	8
the second in the same time would run	12
the third	16
the last	24
In all	60

Then say, if 60 require 6, what 8?

The answer $\frac{48}{60}$, that is $\frac{4}{5}$ of an hour; in which time all the 4 Cocks together would run out all the 8 Barrels of water.

The Rule of Ceres and Virginum.

THis is the most uncertain, and unnecessary Rule in Arithmetick; being seldom used except in sporting questions to puzzel young beginnners, with easie problems: such as follow.

Questions 1.

A Caterer bought 8 *birds of two sorts, as Geese and Hens for* 20 s. *the Geese cost* 4 s. *a piece, the Hens* 2s. *a piece; How many did he buy of each sort.*

This

This may be done by the Rule of Falſe; and alſo thus: multiply the whole number 8, into the leaſt price 2, it produceth 16, which taken from the whole price 20, reſts 4 for a *Dividend*; which divided by 2, which is the difference of the particular prices, the quotient is 2, for the uumber of *Geeſe*; and 6 muſt be for the *Hens:* the proof is eaſie.

Queſtions.

If 21 *Perſons*, Men, Women, *and* Children *ſpend* 29 *ſhillings*; *ſo that every* Man *payes* 2 s. *every* Woman 1 s. *every* Child 6 d. *How many is there of each ſort.*

THE RULE

Multiply the number of Perſons *by the* leaſt expence, *and take the product of it from the* whole expence, *the reſt ſhall be the* Dividend; *which divided by the* difference *betwixt the* greateſt *and* leaſt *particular* expences: *the quotient is a* Number, *which the Number of men (or they which ſpend moſt) comes near to; but cannot exceed: or if the ſaid* Dividend *be divided by the ſum of the* greateſt *and* leaſt *expences, the quotient is a Number, then which the Number of men (or thoſe which ſpend moſt) cannot be much leſs.*

So here 21 multiplyed by 6 *d.* that is, by $\frac{1}{2}$, the product is 10 $\frac{1}{2}$, which taken from 26, reſts 15 $\frac{1}{2}$, for the Dividend: and then taking $\frac{1}{2}$ from 2, reſts 1 $\frac{1}{2}$ for the Diviſor, and the quotient is $\frac{31}{3}$, which is ſomething more than 10; the number of Men therefore muſt be but 9.

Then turn the Dividend, and the Diviſor both into whole Numbers, by multiplying them by the common Denominator 2, ſo they reduced will be 31 and 3, as before is to be ſeen in the quotient.

Multiply the Diviſor 3, by 9, (which is the Number

ber of *Men*) the product is 27, which taken from 31, (which is the reduced Dividend) the remain is 4, for the Number of *Women*; and the *Children* must be 8.

Example 1.

9 Men at 2 *s.* each	18 *s.*
4 Women at 1 *s.* each	4
8 Children at 6 *d.* each	4
In all 21	In all 26

But the number of men may be also 8, which multiplied by the reduced Divisor 3; product is 24, which taken from 31, the remain is 7 for the Women; and then the Children must be 6.

Example 2.

8 Men at 2 *s.* each	16
7 Women at 1 *s.* each	7
6 Children at 6 *d.* each	3
In all 21	In all 26

Or the number of men may be 7, which multiplyed by 3, produceth 21, which taken from 31. remains 10 for the Women, and 4 Children.

Example 3.

7 Men at 2 *s.* each	14 *s.*
10 Women at 1 *s.* each	10
4 Children at 6 *d.* each	12
In all 21	In all 26

So is already seen 3 various solutions of this question, which makes this Rule the less to be regarded.

But

But further, the number of Men may be 10, and not more, for if you put them 11, that multiplyed by 3, produceeth 33, which is greater than 31, from which it ſhould be taken, but I ſay it may be 10, and then there is only one Woman, and five Children: this confirms the former part of the Rule.

Now for the later part, if the Dividend 31 be divided by the ſum of the two extream expences (reduced by doubling as the Dividend is) 4, the quotient will be $7\frac{3}{4}$. And the Men may be 7, as hath been ſhewed; but they may be alſo but 6, and fewer they cannot be: as 6 Men, 13 Women, and 2 Children; for if you put them 5, that multiplied by 3, produceth 15, which taken from 31, there remains 16 for the Women, and ſo there ſhould be no Children which is contrary to the ſupoſition.

And further, becauſe the quotient was $7\frac{3}{4}$, the number of Men might be ſo, if pure Arithmetical diviſion be only regarded: and then the Women alſo are in number $7\frac{3}{4}$, and the Children $5\frac{1}{2}$, as may eaſily be tryed; I need not exemplifie it.

Queſtion 3.

If there be an Exhibition of 900 l. per Annum *to* 30 *perſons: ſome Clerks, ſome Meſſengers, and ſome Door-keepers, at* 6 l. *each Clerk,* 40 l. *each Meſſenger, and* 20 l. *each Door keeper; how many muſt there be of each ſort?*

Multiply (according to the Rule) 30 by 20, the product is 600, which taken from 900, remains 300 for the Dividend; and 60 want 20, that is 40, for the Diviſor: and the quotient is $7\frac{1}{2}$, and more the Clerks cannot be; Alſo divide by 60 more 20, that is 80, quotient is $3\frac{3}{4}$, and much fewer the Clerks cannot be.

Not to ſtand upon the Fractions (in this caſe of dividing men) the Clerks may be 7, 6, 5, 4, 3: and the

the Messengers 1, 3, 5, 7, or 9, and the Door-keepers 22, 21, 20, 19, or 18, that the Clerks cannot (in whole numbers) be more than 7, or less than 3, may thus be proved; First, let them be 8, than 8 times 40 is 320, which is more than 300, out of which it should be taken: Secondly, let them be 2, then 2 times 40 is 80, out of 300 remains 220, which divided by 20, gives the quotient 11, for the Messengers; so the Clerks and Messengers being 13, the remain thereof to 30, namely 17, must be Door-keepers.

but,

2	Clerks at 60 *l.* each	120 *l.*
11	Messengers at 40 *l.* each	480
17	Door-keepers at 20 *l.* each	340
In all 30	the work right	In all 940

Which is 40 *l.* too much, therefore the Clerks cannot be two.

Note.

It may be asked, why the remain 220 should be divided by 20: whereas the like remain in the former Example, namely, 16, was taken (without any division) absolutely for the number of Women, or middle number? I answer, although the greatest or first Number being found, (as here to be 2) the residue of 2 to 30, might be rightly parted into two fit parts in the same manner as the first question of this Rule was resolved, or else by the Rule of False: yet to give further satisfaction, the cause of this is, the difference betwixt the two lesser expences, was there $\frac{1}{2}$, which (before the division was reduced to 1, which neither multiplies nor divides any Number, but leaves it the same: whereas, in this last, the middle

expence

expence (or exhibition) being 40, and the least 20, the difference of them was 20, by which dividing the Remain of the last substraction: the quotient is ever the Number of the middle persons. Which may serve as an addition to the Rule, where the sorts things are but three.

Question 4.

If there be 10 *persons of four several Countries,* English, French, Dutch *and* Spanish, *to pay a Debt of* 1000 l. *So that every English Man payes* 50 l. *every French Man* 70 l. *every Dutch Man* 130 l. *and every Spaniard* 150 l. *How many is there of each?*

The Dividend (according to the former Rule) is 500.

Now to make the Divisor, take his sum that pays least (namely 50) out of each of the other three 150, 130, and 70, and the Remains will be 100, 80, and 20.

Add the first and least for the Divisor, it is 120.

And the quotient will be $4\frac{2}{12}$, and the Spaniards cannot be more.

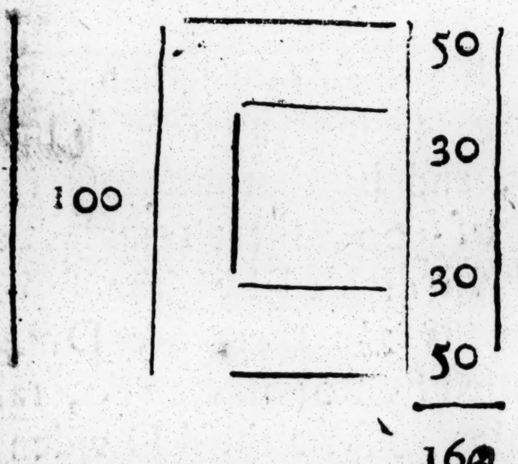
Secondly, add the first and second together for the Divisor, it is 180, and the quotient is $2\frac{14}{18}$, and the Spaniards cannot be less.

I mean, they cannot be much more than 4, or less than 2: and therefore, seeing any one solution will serve, let them be 3, and by that multiply 100, and take the product out of 500, there remains 200 for a second Dividend, which divided by (the second remain) 80, the quotient is $2\frac{1}{2}$: therefore Dutchmen are 2; which multiplied by 80, make 160, take that out of 200, there remains 40 for a third Dividend: which divided by (the third remain) 20, the quotient is 2 for the French Man also; and consequently the English must be 3, because all of them are 10: But the Spaniards may be also 4 or 2.

Example.

4	Spaniards at 150 *l.* each	600
1	Dutchman at 130 *l.*	130
1	Frenchman at 70 *l.*	70
4	English at 50 *l.* each	200
10	In all	1000
2	Spaniards at 150 *l.* each	300
3	Dutch at 130 *l.* each	390
3	French at 70 *l.* each	210
2	English at 50 *l.* each	100
10	In	1000

The reason why the Spaniards and English, as also the Dutch and French are equal in number, is because their payments differ equally from 100, which is the mean sum with which 10 men should pay 1000 *l.* and making it so, this question, and many other of this nature, may be answered by the Rule of *Alligation*: thus,



If 160 give 10, what 50? answer is $3\frac{20}{160}$, (that is in this case 3) for the Spainards, and as many for the English, because their respective differences from 100, the one 50 more; the English 50 less, are equal

And also, because the other two differences 30 and 30 are equal; the Number of the French is equal to the number of the Dutch.

But both those numbers together are 4, because

3 Spa-

3 Spaniards, and 3 Engliſh, taken out of 10, the main muſt be 4.

Wherefore the number of the French is 2, and the Dutch alſo are 2.

Or thus:

Accounting the men in the ſame order, as before.

If 160 require 10, what 30?

Anſwer is $1\frac{160}{160}$, (that is, in this caſe 2) for the Spaniards, and conſequently 2 Engliſh; and therefore the French and Dutch each 3.

100	30	10
	50	
	50	
	30	
	160	

But where any one of the particular ſums is equal to the mean ſum, there this cannot ſo well be done by *Alligation*.

Example.

If one ſhould buy 12 *Loaves of Bread for* 12 pence *ſo that ſome might be* two peny, *ſome* peny, *ſome* halfpeny, *and ſome* farthing *Loaves*: *and it be required to know how many he muſt buy of each?*

Then becauſe of 12 *loaves* for 12 pence, the mean price is 1, but one of the particulars being alſo 1, there ſhould be no peny loaves, becauſe there is no difference betwixt the mean *price*, and a *peny*.

But it may be found by the Rule of *Ceres* and *Virginum*, to be either.

4 Two peny loaves	8 pence
2 Peny loaves	2 pence
2 Half-peny loaves	1 peny
4 Farthings loaves	1 peny
In all 12 loaves.	In all 12 pence.

Or else

3 Two peny loaves	6 *d.*
4 Peny loaves	4
3 Half-peny loaves	$1\frac{1}{2}$.
2 Farthing loaves	$0\frac{1}{2}$.
In all 12	In all 12

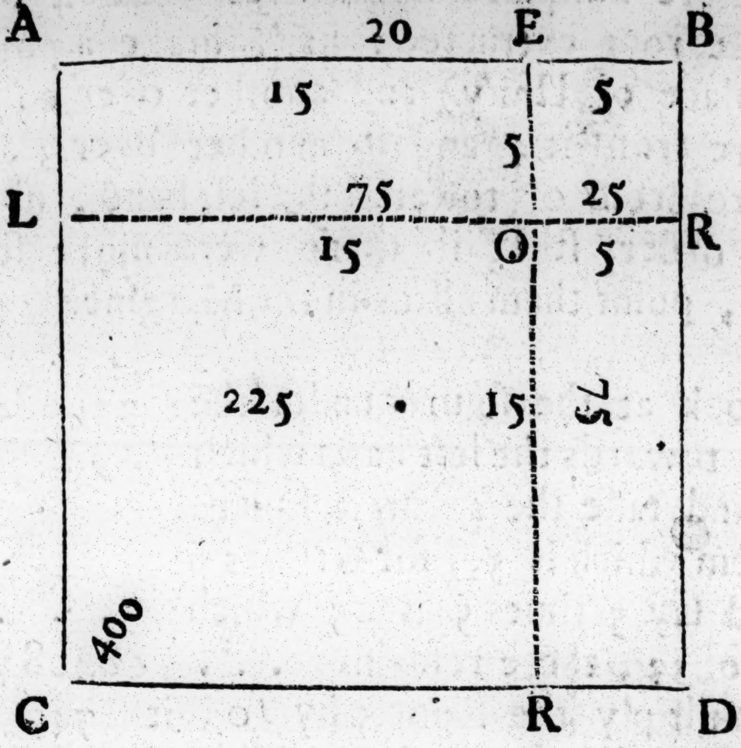
Extraction of Roots.

AND first for the *SQUARE ROOT*, that is a square Number being given, to find the Root or side of it in a Number: which root or side, being multiplied into it self, must therefore produce that square Number.

The doing of this is shewed in (almost) all Books of Arithmetick; and the reason of it (in some) which is taken from the fourth proportion of the second Book of *Euclide*; which saith,

If a right line be divided by chance; the squares made of the parts, together with the Rectangle made of the parts twice, is equal to the square of the whole.

Example.



Let the line A B, be divided by chance in the point E, it is manifest that the square of A B, that is to say, the square ABDC, is equal to the square of LO, that is of A E, and to the square of E B, and to the two rectangle figures AO and DO, that is the rectangle AO (which is made of the parts AE, and BE) twice; according to the Proposition.

Now let AB, be supposed 20, AE 15, and BE 5.

Then the square of AB, (which is made by multiplying the root 20 into it self) is equal to 400

And the square AE, that is, 15 times 15, is equal to	225
And the square of BE is 5 times 5	25
And the rectangle AO, 5 times 15	75
And the rectangle DO, 5 times 15	75
In all	400

which is equal to the square of AB, as before.

If therefore a square number as 334084 be given to have the root extracted: first, make a point over 4, (the place of Unity) and another over o, the second figure from it; and so another over 3, the second figure from o, towards the left hand, observing the like order still, if there were more secondary figures, point them all as in the Margine.

```
. . .
334084
25

. . .
c84084
749

00 9 1 8 4
   9 1 8 4
——————————
   0 0 0 0
```

Then look at the figures under the first point towards the left hand which are 33, and take the greatest square root of them, which is 5, (for 6 times 6 is 36) and say 5 times 5 is 25 which take out of 33, there remains . . .

Then multiply the Root 5 by 20, it gives 100 for a Divisor; and the dividend is 840, namely, the figures which reach to the next point, and so the quotient might be 8; but must be but 7, because the square of the quotient, being now 49, must (together with 7 times the Divisor) be taken out of 840.

And the remainder will be

Add this last quotient 7, being put to the former quotient 5, after the manner used in plain Division, it will be 57, which multiplied still by 20, the product is 1140 for a new Divisor, and the Dividend (because there remains but one point) is all the figures 9184.

And the quotient can be but 8; and must be so much; for 8 times 1140 is 9120
and 8 times 8 is 64

In all 9184

Which taken out of the remain 9184, there now remains

remains nothing: which shews the square 334084 is justly resolved, and putting the last quotient 8 to the two former, 8 and 7, the whole will be 578, which is the true *Root* required.

And the parts of it, if they be 3, are 500, 70, and 8. Or if but two 570 and 8. And the truth may be either way proved, by adding the squares and twice the rectangles of the parts; for the sum of them shall be equal to the whole square 334084, as hath been shewed before.

It is ever certain that there shall be as many figures, or places of figures in the root, as there are points in the square, ordered as before.

And by reading this little seriously, may any one be able to find the root of any other square whatsoever: the first operation in all being to take the greatest square contained under that point next the left hand, and put the Root thereof for the first figure in a quotient.

The Secondary operation, must be repeated so often as there are remaining points; as hath been plainly shewed in the fore-going Example.

I will therefore add no more, but give you two other squares, and their roots, leaving the Reader to extract them himself.

So if there were given the square 8836
The root of it according to former practice may be found to be 94

Or let there be given the square 1522756
The root of that will 1234.

Here followeth a Table of Roots and their Squares from 1 to 1000.

94
94
376
846

1 Cent.		1 Cent.		1 Cent.	
R.	Square	R.	Square	R.	Square
1	1	31	961	61	3721
2	4	32	1024	62	3844
3	9	33	1089	63	3969
4	16	34	1156	64	4096
5	25	35	1225	65	4225
6	36	36	1296	66	4356
7	49	37	1369	67	4489
8	64	38	1444	68	4624
9	81	39	1521	69	4761
10	100	40	1600	70	4900
11	121	41	1681	71	5041
12	144	42	1764	72	5184
13	169	43	1849	73	5329
14	196	44	1936	74	5476
15	225	45	2025	75	5625
16	256	46	2116	76	5776
17	289	47	2209	77	5929
18	324	48	2304	78	6084
19	361	49	2401	79	6241
20	400	50	2500	80	6400
21	441	51	2601	81	6561
22	484	52	2704	82	6724
23	529	53	2809	83	6889
24	576	54	2916	84	7056
25	625	55	3025	85	7225
26	676	56	3136	86	7396
27	729	57	3249	87	7569
28	784	58	3364	88	7744
29	841	59	3581	89	7921
30	900	60	3600	90	8100

1 Cent.		1 Cent.		2 Cent.	
R.	Square	R.	Square	R.	Square
91	8281	121	14641	151	22801
92	8464	122	14884	152	23104
93	8649	123	15129	153	23409
94	8836	124	15376	154	23716
95	9025	125	15625	155	24025
96	9216	126	15876	156	24336
97	9409	127	16129	157	24649
98	9604	128	16384	158	24964
99	9801	129	16641	159	25281
100	10000	130	16900	160	25600
101	10201	131	17161	161	25921
102	10404	132	17424	162	26244
103	10609	133	17689	163	26569
104	10816	134	17956	164	26896
105	11025	135	18225	165	27225
106	11236	136	18496	166	27556
107	11449	137	18769	167	27889
108	11664	138	19044	168	28224
109	11881	139	19321	169	28561
110	12100	140	19600	170	28900
111	12321	141	19881	171	29241
112	12544	142	20164	172	29584
113	12769	143	20449	173	29929
114	12996	144	20736	174	30276
115	13225	145	21025	175	30625
116	13456	146	21316	176	30976
117	13689	147	21609	177	31329
118	13924	148	21904	178	31684
119	14161	149	22201	179	32041
200	14400	150	22500	180	32400

2 Cent.		2 Cent.		2 Cent.	
R.	*Square*	*R.*	*Square*	*R.*	*Square*
181	32761	211	44521	241	58071
182	33124	212	44944	242	58564
183	33489	213	45369	243	59049
184	33856	214	45796	244	59536
185	34225	215	46225	245	60025
186	34596	216	46656	246	60516
187	34969	217	47089	247	61009
188	35344	218	47524	248	61504
189	35721	219	47961	249	62001
190	36100	220	48400	250	62500
191	36481	221	48841	251	63001
192	36864	222	49284	252	63504
193	37249	223	49729	253	64009
194	37636	224	50176	254	64516
195	38025	225	50625	255	65025
196	38416	226	51076	256	65536
197	38109	227	51529	257	66049
198	39204	228	51984	258	66564
199	39601	229	52441	259	67081
200	40000	230	52900	260	67600
201	40401	231	53361	261	68121
202	40804	232	53824	262	68644
203	41209	233	54289	263	69169
204	41616	234	54756	264	69696
205	42025	235	55225	265	70225
206	42436	236	55696	266	70756
207	42849	237	56169	267	71289
208	43264	238	56644	268	71824
209	43681	239	57121	269	72361
210	44100	240	58600	270	72900

2 Cent.		3 Cent.		3 Cent	
R.	Square	R.	Square	R.	Square
271	73441	301	90601	331	109561
272	73984	302	91204	332	110224
273	74529	303	9 809	333	110889
274	75076	304	92416	334	111556
275	75625	305	93025	335	112225
276	76176	306	93636	336	112896
277	76729	307	94249	337	113569
278	77284	008	94864	338	114244
279	77841	009	95481	339	114921
280	78400	310	96100	340	115600
281	78961	311	96721	341	116281
282	79524	312	97344	342	116964
283	80089	313	97969	343	117649
284	80656	314	98596	344	118336
285	81225	315	99225	345	119025
286	81796	3 6	99856	346	119716
287	82369	317	100489	347	120409
288	82944	318	101124	348	121104
289	83521	319	101761	349	121801
290	84100	320	102400	350	122500
291	84681	321	103041	351	123201
292	85264	322	103684	352	123904
293	85849	323	104329	353	124609
294	86436	324	104976	354	125316
295	87025	325	105625	355	126025
296	87616	326	106976	356	126736
297	88309	327	106929	357	127449
298	88804	328	107584	358	128164
299	89401	329	108241	359	128881
300	90000	330	108900	360	129600

4 Cent.		4 Cent.		4 Cent	
R.	Square	R.	Square	R.	Square
361	130321	391	152881	421	177241
362	131044	392	153664	422	178084
363	131769	393	154449	423	178929
364	132456	394	155236	424	179776
365	133225	395	156025	425	180625
366	133956	396	156816	426	181476
367	134689	397	157609	427	182329
368	135424	398	158404	428	183184
369	136161	399	159201	429	184041
370	136900	400	160000	430	184900
371	137641	401	160801	431	185761
372	138384	402	161604	432	186624
373	139129	403	162409	433	187489
374	139876	404	163216	434	188356
375	140625	405	164025	435	189225
376	141376	406	164836	436	190096
377	142129	407	165649	437	190969
378	142884	408	166464	438	191844
379	143641	409	167281	439	192721
380	144400	410	168100	440	193600
381	145161	411	168921	441	194481
382	145924	412	169744	442	195364
383	146689	413	170569	443	196249
384	147456	414	171396	444	197136
385	148225	415	172225	445	168025
386	148996	416	173056	446	198916
387	149769	417	173889	447	199809
388	150544	418	174724	448	200704
389	151321	419	175561	449	201601
390	152100	420	176400	450	202500

4 Cent		5 Cent.		5 Cent.	
R.	Square	R.	Square	R.	Square
451	203401	481	231361	511	261121
452	204304	482	232324	512	262144
453	205209	483	233289	513	263169
454	206116	484	234256	514	264196
455	207025	485	235225	515	265225
456	207936	486	236196	516	266256
457	208849	487	237169	517	267289
458	209764	488	238144	518	268324
459	210681	489	239121	519	266361
460	211600	490	230100	520	270400
461	212521	491	241081	521	271441
462	213444	492	242064	522	272484
463	214369	493	243049	523	273529
464	215296	494	244036	524	274576
465	216225	495	245025	525	275625
466	217156	496	246016	526	276676
467	218089	497	247009	527	277729
468	219024	498	248004	528	278784
469	219961	499	249001	529	279541
470	220900	500	250000	530	280900
471	221841	501	251001	531	281961
472	222784	502	252004	532	283024
473	223729	503	253009	533	284089
474	224676	504	254016	534	285156
475	225625	505	255025	535	286225
476	226576	506	256036	536	287296
477	227529	507	257049	537	288369
478	228484	508	258064	538	289444
479	229441	509	259081	539	290521
480	230400	510	260100	540	291600

5 Cent.		5 Cent.		6 Cent.	
R.	Square	R.	Square	R.	Square
541	292681	571	326041	601	361201
542	293764	572	327184	602	362404
543	294849	573	328329	603	363609
544	295936	574	329476	604	364816
545	297025	575	330625	605	366025
546	298116	576	331776	606	367236
547	299209	577	332929	607	368449
548	300304	578	334084	608	369664
549	301401	579	335241	609	370881
550	302500	580	336400	610	372100
551	303601	581	337561	611	373321
552	304704	582	338724	612	374544
553	305809	583	339889	613	375769
554	306916	584	341056	614	376996
555	308025	585	342225	615	378225
556	309136	586	343396	616	379456
557	310249	587	344569	617	380689
558	311364	588	345744	618	381924
559	312481	589	346921	619	383161
560	313600	590	348100	620	384400
561	314721	591	349281	621	385641
562	315844	592	350464	622	386884
563	316969	593	351649	623	388129
564	318096	594	352836	624	389376
565	319225	595	354025	625	390625
566	320356	596	355216	626	391876
567	321489	597	356409	627	393129
568	322624	598	357604	628	394384
569	323761	599	358801	629	395641
570	314900	600	360000	630	396900

6 Cent		6 Cent.		7 Cent.	
R.	Square	R.	Square	R.	Square
631	398162	661	436921	691	477481
632	399424	662	438244	692	478864
633	400684	663	439569	693	480249
634	401959	664	440896	694	481636
635	403226	665	442225	695	483025
636	404495	666	444556	696	484416
637	405769	667	444889	697	485809
638	407044	668	446224	698	487204
639	408321	669	447561	699	488601
640	409600	670	448900	700	490000
641	410881	671	450241	701	491401
642	412164	672	451584	702	492804
643	413449	673	452929	703	494209
644	414736	674	454276	704	495616
645	416025	675	455625	705	497025
646	417316	676	456976	706	498436
647	418609	677	458329	707	499849
648	419904	678	459684	708	501264
649	421201	679	461041	709	502681
650	422500	680	462400	710	504100
651	423801	681	463761	711	505521
652	425104	682	465124	712	506944
653	426409	683	466489	713	508369
654	427716	684	467856	714	509796
655	429025	685	469225	715	511225
656	430336	686	470596	716	512656
657	431649	687	471969	717	514089
658	432964	688	473344	718	515524
659	434281	689	474721	719	516961
660	435600	690	476100	720	518400

7 Cent		7 Cent.		8Cent.	
R.	Square	R.	Square	R.	Square
721	519841	751	564001	781	609961
722	521284	752	565504	782	611524
723	522729	753	567009	783	613089
724	524176	754	568516	784	614656
725	525625	755	570025	785	616225
726	527076	756	571536	786	617796
727	528529	757	573049	787	619369
728	529984	758	574564	788	620944
729	531441	759	576081	789	622521
730	532900	760	577600	790	624100
731	534361	761	579121	791	625681
732	535824	762	580644	792	627264
733	537289	763	582169	793	628849
734	538756	764	583696	794	630436
735	540225	765	585225	795	632025
736	541696	766	586756	796	633616
737	543169	767	588289	797	635209
738	544644	768	589824	798	636004
739	546121	769	591361	799	638401
740	547600	760	592900	800	640000
741	549081	771	594441	801	641601
742	550565	772	595984	802	643204
743	552049	773	597529	803	644809
744	553536	774	599076	804	646416
745	555025	775	600625	805	648025
746	556516	776	602176	806	649636
747	558009	777	603729	807	651249
748	559504	778	605284	808	652864
749	561001	779	606841	809	654481
750	562500	780	608400	810	656100

8 Cent.		8 Cent.		9 Cent.	
R.	Square	R.	Square	R.	Square
811	657721	841	707281	871	758641
812	659344	842	708964	872	760384
813	660969	843	710649	873	762129
814	662596	844	712336	874	763876
815	664225	845	714025	875	765625
816	665856	846	715716	876	767376
817	667489	847	717409	877	769129
818	669124	848	719104	878	770884
819	670761	849	720801	879	772641
820	672400	850	722500	880	774400
821	674041	851	724201	881	776161
822	675684	852	725904	882	777924
823	677329	853	727609	883	779689
824	678976	854	729316	884	781456
825	680625	855	731025	885	783225
846	682276	856	732736	886	784996
827	683929	857	734449	887	786769
828	685584	858	736164	888	788544
829	687241	859	737881	889	790321
830	688900	860	739600	890	792100
831	690561	861	741321	891	793881
832	692224	862	743044	892	795664
833	693889	863	744769	893	797449
834	695556	864	746496	894	799236
835	697225	865	748225	895	801025
836	698896	866	749956	896	802816
837	700569	867	751689	897	804609
838	702244	868	753424	898	806404
839	703921	899	755161	899	808201
840	705600	870	756900	800	810000

9 Cent.		9 Cent.		9 Cent.	
R.	Square	R.	Square	R.	Square
901	811801	931	866761	961	923521
902	813604	932	868624	962	925444
903	815409	933	870489	963	927369
904	817216	934	872356	964	929296
905	819025	935	874225	965	931225
906	820836	936	876096	966	933156
907	822649	937	877969	967	935089
908	824464	938	879844	968	937024
909	826281	939	881721	969	938961
910	828100	940	883600	970	940900
911	829921	941	885481	971	942841
912	831744	942	887364	972	944784
913	833569	943	889249	973	946729
914	835396	944	891136	974	948676
915	837225	945	893025	975	950625
916	839056	946	894916	976	952576
917	840889	947	896809	977	954529
918	842724	948	898704	978	956484
919	844561	949	900601	979	958441
920	846400	950	902500	980	960400
921	848241	951	904401	981	962361
922	850084	952	906304	982	964324
923	851929	953	908209	983	966289
924	853776	954	910116	984	968256
925	855625	955	912025	985	970225
926	857476	956	913936	986	972196
927	859329	957	915849	987	974169
928	861184	958	917764	988	976144
929	863041	959	919681	989	978121
930	864900	960	921600	990	980100

9 Cent		9 Cent.	
R.	Square	R.	Square
991	982081	996	992016
992	984064	997	994009
993	986049	998	996004
994	988036	999	998001
995	990025	1000	1000000

Extraction of the Cube Root.

A *Cube* is a Solid, or a Body contained within six equal squares; and may be fitly represented by a Die.

When a Cube is given, as is 435519512; point the number as you see in the place of Unity, and every third figure after; then see what is the root of the greatest Cube contained under the first point toward the left hand; that is in 435, it will be found 7 (for 8 times 8. taken 8 times, is 512, which is too much) put this 7 for the first figure in the quotient, having the Cube thereof, which is 343 out of 435: thus

```
435519512
343
---------
092519512
```

And the remain will be 092519512

And so the first work is done.

For

For the ſecond, take the ſquare of the quotient, that is 49, which multiply by 300, the product
is 14700
to which add 30 time 7, that is : : 230

which makes the Diviſor 14910
And the Dividend is 92529
So the quotient might be 6, but muſt be but 5, becauſe the Cube of the new quotient. and 210 times the ſquare of the ſaid quotient muſt be allowed in this work, as followeth.

The remain is 92519512
Multiply the firſt product 147000 by the ſecond quotient 5, and they produce 73500 — 78875
then multiply 210 by the ſquare of 5, it makes :5250
to which add the Cube of 5, : : 125

It makes in all 78875
Which taken from 92519 there remains 13644512
and the quotient is 75, whoſe ſqaure 6625, multiplied by 300
the product is 1687500
ſecondly, 30 times 75 is : : 2250

And the new Diviſor is 1710000
And the third quotient figure is 8,
by which multiply 1687500, it is 1350000
likewiſe, 2250, multipled by the ſquare of 8 (64) is 144000
And the Cube of 8 is : : : 512

Altogether are 13644512

which

Which taken from the ſecond remain, there remains now thirdly nothing. And therefore the laſt quotient 8, put to the former two, it is 758 for the whole Root, as may be tryed by multiplying 758 into it ſelf, and the product again by 758; then the laſt product ſhall be equal to the whole Cube, which was given at firſt to be reſolved.

This one Example is ſufficient for the general underſtanding of the manner of working, wherefore I will add no more Examples, but (as I did in the ſquare Root, ſo in this I will) give you two or three numbers with their Roots, leaving the practice of them to the Learner, Thus,

Of	389017	73	is the Root.
	06440125	405	
	961504803	987	
	12895213625	1345	

Here followeth a Table of Cube Roots, from 1 to 1000.

George Root

M A

William Brooks

1 Cent.		1 Cent.		1 Cent.	
R.	Cube	R.	Cube	R.	Cube.
1	1	31	29791	61	216981
2	8	32	32768	62	238328
3	27	33	35937	63	293047
4	64	34	39304	64	262244
5	125	35	42825	65	274625
6	216	36	48656	66	287496
7	343	37	50653	67	300753
8	512	38	54872	68	314432
9	729	39	55419	69	329199
10	1000	40	64000	70	333000
11	1331	41	68921	71	357911
12	1728	42	74088	72	373348
13	2197	43	79507	73	389017
14	2744	44	85184	74	405224
15	3375	45	91125	75	411875
16	4096	46	97336	76	438976
17	4913	47	103823	77	456533
18	5832	48	110592	78	474522
19	6859	49	117649	79	493039
20	8000	50	125000	80	512000
21	9261	51	135651	81	531441
22	10648	52	140608	82	550408
23	12167	53	148877	83	571787
24	13824	54	157464	84	592604
25	15625	55	167375	85	614125
26	17576	56	175616	86	636056
27	19683	57	185193	87	648303
28	21972	58	195112	88	681472
29	24389	59	205379	89	705669
30	27000	60	216000	90	729000

1 Cent		1 Cent.		1 Cent.	
R.	Cube	R.	Cube	R.	Cube
91	753571	121	1771561	151	3442951
92	778688	122	1815848	152	3511808
93	804357	123	1860867	153	3581577
94	830584	124	1906624	154	3652264
95	857375	125	1953125	155	3723875
96	884736	126	2000376	156	3796416
97	925673	127	2048383	157	3869893
98	941192	128	2097172	158	3944312
99	970299	129	2146689	159	4019679
100	1000000	130	2197000	160	4096000
101	1030301	131	2248091	161	4173281
102	1061208	132	2299968	162	4251528
103	1092727	133	2352637	163	4230744
104	1124856	134	2406104	164	4410944
105	1157625	135	2460375	165	4492125
106	1191016	136	2515856	166	4574296
107	1225043	137	2570353	167	4657463
108	1259712	138	2628072	168	4741632
109	1295029	139	2685619	169	4826809
110	1231000	140	2744000	170	4913000
111	1367631	141	2803221	171	5000211
112	1404928	142	2864288	172	5088448
113	1442897	143	2924207	173	5177717
114	1481544	144	2985984	174	5268024
115	1520875	145	3027525	175	5359375
116	1560896	146	3112136	176	5451776
117	1601613	147	3176523	177	5545233
118	1643032	148	3241792	178	5639752
119	1685159	149	3307949	179	5735339
120	1728000	150	3375000	180	5832000

2 Cent.		2 Cent.		2 Cent.	
R.	Cube	R.	Cube	R.	Cube
181	5929741	211	9393931	241	13997521
182	6028568	212	9528128	242	14172448
183	6128487	213	9663597	243	14348907
184	6229504	214	9800344	244	14526684
185	6331625	215	9938375	245	14705125
186	6434856	216	10077696	246	14886936
187	6539203	217	10218313	247	15069223
188	6644672	218	10360232	248	15252992
189	6751269	219	10503459	249	15438249
190	6859000	220	10648000	250	15625000
191	6967871	221	10793861	251	15813251
192	7077888	222	10941048	252	16003008
193	7189057	223	11089567	253	16194277
194	7301384	224	11239424	254	16387064
195	7414875	225	11390625	255	16581375
196	7529536	226	11543176	256	16777216
197	7645373	227	11697083	257	16974593
198	7762392	228	11852452	258	17173512
199	7882599	229	12008989	259	17473979
200	8000000	230	12167000	260	17576000
201	8120601	231	12326391	261	17779581
202	8242408	232	12487168	262	17984728
203	8365427	233	12649337	263	18191447
204	8489664	234	12812904	264	18399744
205	8615125	235	12977875	265	18609625
206	8741816	236	13144256	266	18821096
207	8869743	237	13312053	267	19034063
208	8999912	238	13481272	268	19248832
209	8129329	239	13651919	269	19465109
210	9261000	240	13824000	270	19683000

2 Cent.		3 Cent.		3 Cent	
R.	Cube	R.	Cube	R.	Cube
271	19902511	301	27270901	331	36264691
272	20123648	302	27543608	332	36594368
273	20346417	303	27818127	333	36926037
274	20571024	304	28094464	334	37259704
275	20796875	305	28372625	335	37595375
276	21024576	306	28652616	336	37933076
277	21253933	307	28934443	337	38232753
278	21484952	008	29218112	338	38614472
279	21717639	009	29503629	339	38958219
280	21952000	310	29791000	340	39304000
281	22188041	311	30080231	341	39651821
282	22425768	312	30371328	342	40001688
283	22665187	313	30664297	343	40353607
284	22906304	314	30959144	344	40707584
285	23149125	315	31255875	345	41063625
286	23393656	316	31554496	346	41421736
287	23539903	317	31855013	347	41781923
288	23893872	318	32157432	348	42144192
289	24137569	319	32461759	349	42508549
290	24389000	320	32768000	350	42875000
291	24642171	321	33076161	351	43243551
292	24897088	322	33386248	352	43614208
293	25143757	323	33698267	353	43986977
294	25412184	324	34012224	354	44361864
295	25672363	325	34328125	355	44738875
296	25934336	326	34645976	356	45118016
297	26198073	327	35456783	357	45499293
298	26353592	328	35287152	358	45882712
299	26730899	329	35611286	359	46268279
300	27000000	330	35937000	360	46656000

3 Cent.		4 Cent.		4 Cent.	
R.	Cube	R.	Cube	R.	Cube
361	47045881	391	59776471	421	74618461
362	47439928	392	60236288	422	75151448
363	47832147	393	60698457	423	75686967
364	48228544	394	61162984	424	76225024
365	48627125	395	61629875	425	76765625
366	49027896	396	62099136	426	77308776
367	49430863	397	62570773	427	77854483
368	49836032	398	63044792	428	78402752
369	50243409	399	63521199	429	78953589
370	50653000	400	64000000	430	79507000
371	51064811	401	64481201	431	80062991
372	51478848	402	64964808	432	80621568
373	51895117	403	65450827	433	81182737
374	52313624	404	65939264	434	81746504
375	52734375	405	66430125	435	82312875
376	53157376	406	66923416	436	82881856
377	53582633	407	67419143	437	83453453
378	54010152	408	67917312	438	84027672
379	54439939	409	68417929	439	84604519
380	54872000	410	68921000	440	85184000
381	55306341	411	69426531	441	88766121
382	55742968	412	69934528	442	86350888
383	56181887	413	70444997	443	86938307
384	56623104	414	70957944	444	87528384
385	57066625	415	71473375	445	88121125
386	57512456	416	71991296	446	88716536
387	57960603	4 7	72511713	447	89314623
388	58411072	418	73034632	448	89915392
389	58863869	419	73560059	449	90518849
390	59319000	420	74088000	450	91125000

4 Cent.		5 Cent.		5 Cent.	
R.	Cube	R.	5 Cube.	R.	Cube
451	91733851	481	111284641	511	133432821
452	92345408	482	111980168	512	134217728
453	92959077	483	112768587	513	135005697
454	93576664	484	113379904	514	135796744
455	94196375	485	114084125	515	136590875
456	94818816	486	114791256	516	137388096
457	95443993	487	115501303	517	138188413
458	96071912	488	116214272	518	138991832
459	96702579	489	116930169	519	139798359
460	97336000	490	117649000	520	140608000
461	97972181	491	118370771	521	141420761
462	98611128	492	119095488	522	142236648
463	99252847	493	119823157	523	143055667
464	99897344	494	120553784	524	143877824
465	100544625	495	121287375	525	144703125
466	101194696	496	122023936	526	145531576
467	101847563	497	122763473	527	146363183
468	102503232	498	123505992	528	147197952
469	103161709	499	124251499	529	148035889
470	103823000	500	125000000	530	148877000
471	104487111	501	125751501	531	149721291
472	105154048	502	126506008	532	150568768
473	105823817	503	127263527	533	151419437
474	106497424	504	128024064	534	152273304
475	107171875	505	128787625	535	153130375
476	107050176	506	129554216	536	153990656
477	108531333	507	130323843	537	154854153
478	109215352	508	131096512	538	155720872
479	109902239	509	131872229	539	156590819
480	110592000	510	132651000	540	157464000

5 Cent.		5 Cent.		6 Cent.	
R.	Cube	R.	Cube	R.	Cube
541	158340421	571	186169411	601	217081801
542	159220088	572	187149248	602	218167208
543	160103007	573	188132517	603	219256227
544	160989184	574	189119224	604	220348864
545	161878625	575	190109375	605	221445125
546	162771336	577	191102976	606	222545016
547	163667323	576	192100033	607	223648543
548	164566592	578	193100552	608	224755712
549	166569149	579	194104539	609	2258665[illegible]9
550	166375000	580	195112000	610	226981000
551	167284151	581	196122941	611	228099131
552	168196608	582	197137368	612	229220928
553	169112377	583	198155287	613	230346397
554	170031464	584	199176704	614	231475544
555	170953875	585	200202625	615	232608375
556	171879616	586	102230056	616	233744896
557	172808683	587	202262003	617	234885113
558	173741112	588	203297472	618	236029032
559	174676879	589	204336469	619	237176659
560	175616000	590	205379000	620	238328000
561	176558481	591	206425071	621	239483061
562	177504328	592	207474688	622	240641848
563	178453547	593	208527857	623	241804367
564	179306144	594	209584584	624	242970624
565	188262125	595	210644875	625	244140625
566	181221496	596	211708736	626	245314376
567	182154263	597	212776173	627	246491883
568	183150432	598	213847192	628	247673152
569	184220009	599	214921799	629	248858189
570	185193000	600	216000000	630	250047000

6 Cent		6 Cent.		7 Cent.	
R.	Cube	R.	Cube	R.	Cube
631	251239591	661	288804781	691	339939371
632	252435968	662	290117528	692	331373888
633	253636137	663	291434247	693	332812557
634	254840104	664	292754944	694	334255384
635	256047875	665	294079625	695	335702375
636	257259456	666	295408296	696	337155396
637	258474853	667	296740963	697	338608873
638	259694072	668	298077632	698	340068392
639	260917119	669	299418309	699	341532099
640	262144000	670	300763000	700	343000000
641	263374721	671	302111711	701	344472101
642	264609288	672	303464448	702	345948408
643	265847707	673	304821217	703	347428927
644	267089984	674	3061820 4	704	348913664
645	268336125	675	307546875	705	350402625
646	269586136	676	308915776	706	351895816
647	270840023	677	310288733	707	353393243
648	272097792	678	311665752	708	354894912
649	273359449	679	313046839	709	356400829
650	274625000	680	314432000	710	357911000
651	275854451	681	315821241	711	359435431
652	277167808	682	317214568	712	360944128
653	278445077	683	318611987	713	362467097
654	279726264	684	320815504	714	363994344
655	281011375	685	321419115	715	365525875
656	282300146	686	3228 8856	716	367061696
657	283593393	687	324242703	717	368601813
658	284890312	688	325660672	718	370146232
659	286191179	689	327082769	719	371694959
660	287496000	690	328509000	720	373248000

7 Cent.		7 Cen.		8 Cent	
R.	Cube	R.	Cube	R.	Cube
721	374805361	751	423564751	781	476379541
722	376367048	752	425259008	782	478211768
723	377933067	753	426957777	783	480048687
724	379503424	754	428661064	784	481890304
725	381078125	755	430368875	785	483736625
726	382657176	756	432081216	786	485587656
727	384240583	757	433798093	787	487443403
728	385828352	758	435519512	788	489303872
729	387420489	759	437245479	789	491169069
730	389017000	760	438976000	790	493039000
731	390617891	761	440711081	791	494913671
732	392223168	762	442450728	792	496793088
733	393832837	763	444194947	793	498677257
734	395446904	764	445943744	794	500566184
735	397065375	765	447697125	795	502459875
736	398688256	766	449455096	796	504358336
737	400315553	767	451217663	797	506261573
738	401947272	768	452984832	798	508169592
739	403583419	769	454756609	799	510082399
740	405224000	770	456533000	800	512000000
741	406869021	771	458314011	801	513922401
742	408518488	772	460099648	802	515849608
743	410172407	773	461889917	803	517781627
744	411830784	774	463684824	804	519718464
745	413493625	775	465484375	805	521660125
746	415160936	776	467288576	806	523606616
747	416832723	777	469097433	807	525557943
748	418508992	778	470910952	808	527514112
749	420189749	779	472729139	809	529475129
750	421875000	780	474552000	810	531441000

8 Cent.		8 Cent.		8 Cent.	
R.	Cube	R.	Cube	R.	Cube
811	533411731	841	594823321	871	660776311
812	535387328	842	596947688	872	663054848
813	537367797	843	599077107	873	665338617
814	539353144	844	601211584	874	667627624
815	541343375	845	603351125	875	669921875
816	543338496	846	05495736	876	672221376
817	545338513	847	607645423	877	674526133
818	547343432	848	609800192	878	676836152
819	549353259	849	611960049	879	679151439
820	551368000	850	614125000	880	681472000
821	553387661	851	616295051	881	683797841
822	555412248	852	618470208	882	686128968
823	557441767	853	620650477	883	688465387
824	559476224	854	622835864	884	690807104
825	561515625	855	625026375	885	693154125
826	563559976	856	627222016	886	695506456
827	565609283	857	629422793	887	697864103
828	567663552	858	631628712	888	700227072
829	569722789	859	633839779	889	702595369
830	571787000	860	636056000	890	704969000
831	573856191	861	638277381	891	707347971
832	575930368	862	640503928	892	709732288
833	578009537	863	642735647	893	712121957
834	580093704	864	644972544	894	714516984
835	582182875	865	647214625	895	716917375
836	584277056	866	649461896	896	719323136
837	586376253	867	651714363	897	721734273
838	588480472	868	653972032	898	724150792
839	590589719	869	656234909	899	726572699
840	592740000	870	658503000	900	729000000

9 Cent.		9 Cent.		9 Cent.	
R.	Cube	R.	Cube	R.	Cube
901	731432701	931	806954491	961	887503681
902	733870808	932	802557569	962	890277218
903	736314327	933	812166237	963	893056347
904	738763264	934	814780504	964	895841344
905	741217625	935	817400375	965	898632125
906	743677416	936	820025856	966	901428696
907	746142643	937	822656953	967	904231063
908	748613312	938	825293672	968	907039232
909	751089429	939	827936019	969	909853209
910	753571000	940	830584000	970	912673000
911	756058031	941	833237621	971	915498611
912	758550528	942	835896888	972	918330048
913	761048497	943	838561807	973	921167317
914	763551944	944	841232384	974	924010424
915	766060875	945	843908625	975	926859375
916	768575296	946	846590536	976	929714176
917	771095213	947	849278123	977	932574833
918	773620632	948	851971392	978	935441352
919	776151559	949	854670349	979	938313739
920	778688000	950	857375000	980	941192000
921	781229961	951	860085351	981	944076141
922	783777448	952	862801408	982	946966168
923	786330467	953	865523177	983	949862087
924	788889024	954	868250664	984	952763904
925	791453125	955	870983875	985	955671625
926	794022776	956	873722816	986	958585256
927	796597983	957	876467493	987	961504873
928	799178752	958	879217912	988	964430272
929	801765089	959	881974079	989	967361669
930	804357000	960	884736000	990	970299000

9 *Cent.*		9 *Cent.*	
R.	*Cube*	*R.*	*Cube.*
991	973242271	996	988047936
992	976191488	997	991026973
993	979146657	998	994011992
994	982107784	999	997002999
995	985074875	1000	1000000000

Some

Some Uses of the Square and Cube Root.

Uses of the Square Root.

WHat the *Square* and *Cube Root* are, and how to *extract* them, hath already been taught; and for more ease and expedition, There are *Tables* ready calculated, both of the *Square* and *Cube Roots*, from 1 to 1000: We come now to shew some Uses thereof, which in some measure will appear in the *Propositions* following:

PROPOSITION. I.

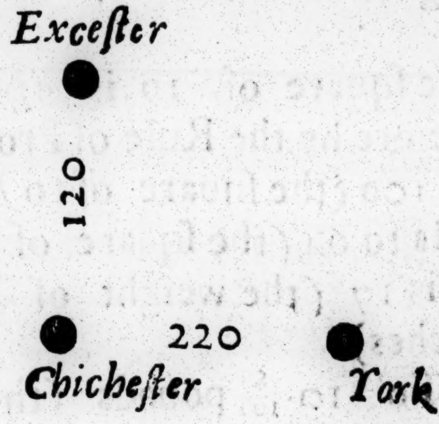
Admit the height of the Wall of a Fort or Castle to be scaled, be 30 *Foot, and the breadth of the Trench about the Fort be* 40 *Foot; I demand of what length a Scaling Ladder shall be, justly to reach from the edge or brow of the Trench, to the top of the Wall?*

By the 47th of the first Book of *Euclids Elements*, it is demonstrated; that, *the square of the Hypotenusal of all right angled plain Triangles is equal to the squares of the* 2 *other sides*; I therefore to resolve this Proposition, square the height of the *Wall*, which is 30, *facit* 900; also I square the breadth of the *Trench* which is 40 *facit* 1600, these two added together make 2500, the square root whereof is 50: and so long must a Scaling Ladder be madde to reach from the edge of the *Trench* to the top of the *Wall*.

PRO-

PROPOSITION. II.

There be two Towns, as Chicester and York, *which lie North and South one from another, and their distance is* 220 *miles, and* Excester *lieth directly West from* Chichester, 120 *miles; I dsire to know the distance of* York *from* Excester?



Square 120, the distance of *Excester* and *Chichester*, it maketh 14400, likewise square 220, the distance of *York* and *Chichester*, *facit*, 48400; these two numbers added together make 62800, whose square root extracted (or found in the Table) will be $250\frac{3}{5}$ near, and so many miles is *Excester* distant from *York*.

Use of the Cube Root.

One chief use of the *Cube Root*, is to find out a proportion between like Solids; such are Spheres, Cubes, and such like; as in the Proposition following.

PROPOSITION I.

If a Bullet of Brass of 4 *inches Diameter, weigh* 9 *pound, what shall a Bullet of Brass weigh, whose Diameter is* 8 *inches?*

Cube 4, the Diameter of the lesser, Bullet, makes 64, likewise Cube the Diameter of the greater Bullet 8, makes 4609. This done, say by the Rule of Proportion; If the Cube 64 give 9 *li*, weight, what shall

shall the Cube number 4608 give? Multiply and divide, you shall 72; and so many pounds will a Bullet of Brass weigh, whose Diameter is 8 inches.

PROPOSITION. II.

If a Fathom of Rope of 10 inches compass about, do weigh 17 pound, how much shall a Fathom of Rope weigh, which is but 8 inches compass about?

The square of 10 is 100, the square of 8 is 64; wherefore by the Rule of Proportion, say

As 100 (the square of 10)

Is to 64 (the square of 8)

So is 17 (the weight of the fathom of Rope of 10 inches)

To $10\frac{88}{100}$ pounds (the weight of the fathom of Rope of 8 inches about.)

PROPOSITION. III.

If a Ship of 100 Tun be 20 foot broad at the Mid ship Beam, of what breadth at the Beam shall a Ship (of the like building) be that shall be 200 Tun?

The Cube of 20 is 8000, then by the Rule of Proportion say,

As 100 Tun (the burthen of the Ship given)

Is to 200 Tun (the burthen of the Ship required)

So is 8000 (the Cube of the given Ships Beam)

To 16000 (the Cube of the required Ships Beam)

Now the Cube Root of 16000 is $25\frac{1}{5}$ almost, and so long at the Mid-ship Beam must a Ship of the same Model be, whose burthen is 200 Tun.

The end of the First Part.

DECIMAL ARITHMETICK

The SECOND PART.

CONTAINING

The Grounds and Reason thereof; The Use of this kind of Artificial (or Decimal) way of working, illustrated by divers Examples, in all the most usual Rules of Arithmetick.

By *Will. Leybourn.*

LONDON,

Printed *Anno Dom.* 1684.

Decimal

ARITHMETICK.

The Second Part.

HAving in the first Part of this Book exemplyfied the Art of *Vuglar Arithmetick* both in *whole Numbers* and *Fractions*; We come now to treat of DECIMAL ARITHMETICK, which teacheth how to perform (as it were) in *Whole numbers*, all that the former did effect by *Fractions*. I will not insist upon the *Antiquity* or *Excellency* of this kind of *Arithmetick*, but come immediately to the *practice* thereof, and shall therefore premise these Propositions following.

Proposition 1.

A vulgar Fraction being given, how to reduce the same into a Decimal.

The Rule.

To the Numerator *of the Fraction given, add what number of* Cyphers *you please; then divide the* Numerator *by the* Denominator, *the quotient shall be the* Decimal Fraction *required.*

Example 1.

Let it be required to reduce $\frac{4}{17}$ *into a Decimal*: First, to the Numerator 4, add five Cyphers, so will it be 400000, divide this number by the Denominator 17, and the quotient will be 23529, which is the Decimal required.

¶ And here Note, that *Decimal Fractions* are not written in a smaller figure with a line between them, as *vulgar Fractions* are, but of the same figure, only there must be a Comma or point put between the whole Number and the Fraction, and that is the distinction.

Example 2. *If you would express* 235 $\frac{4}{17}$ *in a decimal way*, it must be written as followeth.

By the last example you find that $\frac{4}{17}$ reduced to a Decimal, was 23529, therefore 235 $\frac{4}{17}$ must be written thus: 235,23529. In Decimal Fractions the Numerator is only expressed, and the Denominator only intimated; for this RULE is general, *Of how many figures soever the* Numerator *of a decimal fraction doth consist, of so many* Cyphers *with a* Unite *before them, doth the* Denominator *of the same Fraction consist.* So this Decimal 12,625, if it were written in a vuglar way, would be 12 $\frac{625}{000}$ but in a Decimal, only 12,625, the Comma or point between 12 and 625 distinguisheth the

the *whole number* from the *Fraction*, and the fraction 625 consisting of three figures, intimates that the Denominator thereof must consist of three Cyphers and an Unite before them; so the Decimal before expressed, 235, 23529, if it were written in the vuglar way, would be 235 $\frac{23529}{100000}$.

But it sufficeth to express in Decimals the Numerators only, and omit the Denominators, the Denominators of all Decimal Fractions being either 10, 100, 1000, 10000, 100000, &c. according to the number of figures contained in the Numerators.

Acording to this Rule, you shall find that

$\frac{4}{5}$	will be in Decimals by adding 5 Cyphers.	.80000
12 $\frac{3}{7}$		12.42857.
132 $\frac{9}{17}$		132.52941.

And by this means, all manner of Fractions of *Coins*, *Weights* and *Measures*, may be reduced from vulgar Fractions, to Decimal Fractions; as by the next Proposition will appear.

Proposition 2.

How to exprese English Coin *in Decimal Numbers.*

Let it be required to express 9 shillings (which is $\frac{9}{20}$ of a pound *Sterling*) in a Decimal; To the Numerator 9 add two Cyphers, making it 900, which divide by twenty, the quotient is 45, for the Decimal of 9 *s.* So the Decimal of 13 *s.* will be 65, and so for any number of shillings.

¶ Here note, that in the Reduction of Vulgar Fractions into Decimals, that many times the first, second or third places of the Decimal Fractions are Cyphers, as in the following Table the Decimal of one Farthing is .00104167, and the reason is, because if you

reduce $\frac{1}{960}$ into a Decimal (for one farthing is the 960 part of a pound *Sterling*) you shall by adding of Cyphers to the Numerator find the Quotient to be 104167, but two Cyphers must be placed before it; because dividing 1000000 by 960, the place of Unites in the Divisor at the first demand extendeth unto the third Cypher in the Dividend for in reducing of Vulgar Fractions to Decimals, this is

A general Rule.

That if the place of Unites *in the* Divisor, *at the first demand extend but unto the first of the* Cyphers *annexed to the* Numerator *of the Fraction, there must be no* Cypher *put before in the Quotient, but if the place of* Unites *extend unto the second* Cypher *added, then one* Cypher *must be placed before in the Quotient, if unto the third* Cypher, *then two Cyphers must be placed before in the Quotient, &c.*

According to which Rule, if you make tryal you shall find that the Decimal of 7 *s.* will be .35, the Decimal of 5 *d.* will be .0208333, the Decimal of two farthings will be .00208333, as in the Table.

By these Rules last delivered are the ensuing Tables of *English Money, Weight* and *Measure* composed, and the like may be done for a forreign *Coin*, &c. according as every mans occasions shall require.

The

The TABLE of *English* Coin in Decimals.

English Coin.

Sh.	19	·95
	18	·9
	17	·85
	16	.8
	15	·75
	14	·7
	13	·65
	12	.6
	11	·55
	10	·5
	9	·45
	8	·4
	7	·35
	6	·3
	5	·25
	4	.2
	3	·15
	2	.1
	1	.05

D.	11	.04583333
	10	.04166667
	9	.0375
	8	.03333333
	7	.02916667
	6	.025
	5	.02083333
	4	.01666667
	3	.0125
	2	.0833333
	1	.04166667
F.	3	.003125
	2	.00208333
	1	.00104167

Troy Weight in Decimals.

O.	11	.91666667
	10	·83333333
	9	·75

	8	.66666667
	7	.58333333
	6	.5
	5	.41666667
	4	33333333
	3	25
	2	16666667
	1	08333333
P	19	07916667
	18	075
	17	07083333
	16	06666667
	15	0625
	14	05833333
	13	05416667
	12	05
	11	04583333
	10	04166667
	9	0375
	8	03333333
	7	02916667
	6	025
	5	02083333
	4	01666667
	3	0125
	2	00833333
	1	00416667

Gr.	23	.00399395
	22	.00381944
	21	00364583
	20	00347222
	19	00329861
	18	003125
	17	00295139
	16	00277778
	15	00260417
	14	00243056
	13	00225694
	12	00208333
	11	00190972
	10	.00173611
	9	0015625
	8	00138889
	7	00121528
	6	00104106
	5	00086805
	4	.00069444
	3	00052083
	2	00044722
	1	00017311

Aver-

Averdupois great Weight in DECIMALS.

3 *qu.*		.75
2 *qu.*		5
1 *qu.*		25
	27	24107142
	26	23214285
	25	.22321428
	24	21428571
	23	20535714
	22	19642857
	21	1875
	20	.17857143
	19	16964286
	18	16071428
	17	15178571
	16	14285714
	15	13392857
	14	125
	13	11607143
	12	.10714286
	11	09821428
	10	08928571
	9	08035714
	8	07142857
	7	0625
	6	.05357143
	5	04464286
	4	03571428
	3	02678571
	2	01785714
	1	.00892857
Oun.	15	.00837053
	14	0078125
	13	00725446
	12	00669643
	11	00613839
	10	.00558035
	9	00502232
	8	00446429
	7	00390625
	6	00334821
	5	00279018
	4	00223214
	3	.00167411
	2	00111607
	1	00055804
3 *qu.*		00041853
half		00027902
1 *qu.*		00013951

Averdupois little Weight in DECIMALS.

Ounces	15	.937
	14	875
	13	8125
	12	75
	11	6875
	10	.625
	9	5625
	8	5
	7	4375
	6	375
	5	3125
	4	25
	3	1875
	2	125
	1	.0625
Drams.	15	.05859375
	14	.0546875
	13	05078115
	12	046875
	11	04296875
	10	.0390625
	9	.03515265
	8	03125.
	7	02734375
	6	0234375
	5	01953125
	4	015625
	3	01171875
	2	0078125
	1	.00390625
3 qu.		00292969
half		00195312
1 qu.		00097656

Liquid Measures in Decimals.

P.	7	.875
	6	75
	5	625
	4	5
	3	375
	2	25
	1	125
3 qu.		09375
half.		0625
1 qu.		.03125

Dry

Dry Measures in Decimals.

Bushels.	7	875
	6	75
	5	625
	4	5
	3	·375
	2	25
	.1	.125
Pecks.	·3	·09375
	2	0625
		03225
3 q.		.0234375
half.		015625
1 qu.		.0078125
Pints.	3	0058594
	2	0039063
	.1	.0019531

Long Measures, the Integers being Yards and Ells in Decimals.

Qua.	3	:75
	2	5
	1	:25
Nail.	3	·1875
	2	125
	1	·0625
3 qu.		046875
half.		03125
1 qu.		.015625

Time in DECIMALS.

Mo	11	.916667
	10	833333
	9	75
	8	666667
	7	833333
	6	5
	5	416667
	4	333333
	3	25
	2	166667
	1	.085333
Da.	30	082193
	29	097454
	28	07 714
	27	073973
	26	071233
	25	068495

24	.065755
23	063016
22	060274
21	057536
20	054795
19	052055
18	049316
17	.046577
16	043837
15	041097
14	038357
13	035617
12	032877
11	030137
10	027397
9	024657
8	.021918
7	029178
6	016438
5	013698
4	010959
3	0082192
2	0054795
1	.0027397

Dozens in Decimals.

De.	11	.9166667
	10	8333333
	9	75
	8	6666667
	7	5833333
	6	5
	5	4166667
	4	3333333
	3	25
	2	1666667
	1	.0833333
Pa.	11	.076388
	10	0694444
	9	0625
	8	0555555
	7	0486111
	6	.0416667
	5	0347222
	4	0277778
	3	0208333
	2	0138889
	1	.0069444

The

The use of the fore-going Tables.

THe *Tables* preceding are in number nine; The first being of *English Coin*, The second of *Troy Weight*; The third of *Averdupois great weight*. The fourth of *Averdupois little Weight*. The fifth of *Liquid Measures*. The sixth of *Dry Measures*. The seventh of *Long Measures*. The eight of *Time*, and the ninth of *Dozens*. These several Tables are made by the Rules immediatly going before them, and their use is to express in *Decimal numbers* either *Money*, *Weight*, or *Measure*, as by the following *Propositions* will appear.

PROP. I.

How by the Table *to express* English Coin *in Decimals*.

The first of the nine Tables is for this purpose; therefore if you would express either *shillings*, *pence*, or *farthings* in Decimal numbers, you must repair to the first Table, which is of *English Coin*, and there against 13 shillings you shall find .65, which is the Decimal of 13 shillings, also against seven pence you shall find .02916667, which is the Decimal representing 7 pence: Also against 2 farthings you shall find .00208-333, which is the Decimal answering 2 farthings, and the like is to be done for any other number of shillings, pence, or farthings

But if it be required to find the Decimal of divers Denom.

Denominations of Coin in one Sum, as of ſhillings, penee, and farthings together, you muſt add the Decimals of all the particulars together, and the Sum of them ſhall be the Decimal ſought.

Examples.

If you would know the Decimal of 13 *s.* 7 *d.* 2 *q.* in one number, you muſt firſt look in the Table for the Decimal of 13 *s.* which is .65, and ſet that down, then look for the Decimal of 7 *d.* which is .00916667, and ſet that down alſo: Laſtly, ſeek the Decimal of 2 *q.* which is .00208333, ſet that down alſo; then if you add theſe three numbers together, as in common Addition, you ſhall find the ſum of them to be .68125000, which is the Decimal belonging to 13 *s.* 7 *d.* 2 *q.* as by the work in the Margine appeareth.

.65
.02916667
.00208333
.68125000

PROP. II.

How by the Table *to expreſs* Troy-Weight *in Decimals.*

The ſecond Table is of *Troy Weight*, the ſeveral Denominations whereof are *Ounces*, *Penny-weights*, and *Grains*: So that by the Table you ſhall find that the Decimal belonging to five ounces is .41666667, the Decimal belonging to 17 peny-weight is ..07083333, and the Decimal belonging to 13 Grains is. 00225694, and ſo of any other number of ounces, peny weights and grains ſeverally.

But if it were required to expreſs theſe (or an other) ſeveral

ſeveral Denominations in one Decimal Fraction, then you muſt (as before you did for money) take out of the Table the ſeveral Decimals belonging to the reſpective quantities, and add them together, ſo ſhall the Sum of that Addition be the Decimal ſought.

Example.

If it were required to find a Decimal which ſhould repreſent 5 ounces, 17 peny-weight, 13 grains, you muſt firſt look in the Table for the Decimal belonging to five ounces, which is .41666667, and write it down then look the Decimal belonging to 17 grains, which is .07083333, and write that down, then look for the Decimal of 13 grains, which is .00225694, and, write that down, then adding theſe three numbers together, you ſhall find the ſum of them to be .48975694, which is the Decimal repreſenting 5 ounces, 17 grains, 13 peny weight, as by the operation in the Margent appeareth.

.41666667
.07083333
.00225694
.48975694

PROP. III.

How by the Table *to expreſs* Averdupois great weight *in Decimals.*

The third Table is of *Averdupois* great weight, the ſeveral Denominations whereof are *Quarters* of *Hundreds*, *Pounds*, *Ounces*, and *Quarters* of *Ounces*; thus you ſhall find in the Table, that the Decimal of 3 Quarters of a Hundred is .75, the Decimal of 22 pounds is .19642857, the Decimal of 7 ounces is .00390625, and the Decimal of 3 quarters of an ounce

is .00041853, in this manner by the Table you may find the correſpondent Decimal belonging to any number of quarters, pounds, ounces, and parts of ounces ſeverally.

But if it be required to find one Decimal number which ſhall repreſent divers Denominations, you muſt firſt find the Decimal belonging to the ſeveral particulars, and add them together, the ſum whereof ſhall be the entire Decimal required.

Example.

Let it be required to find a Decimal which ſhall repreſent 3 quarters, 22 pounds, 7 ounces $\frac{3}{4}$ of an ounce. Firſt, look in the Table for the decimal of three quarters of a hundred, which is .75, and write it down, then look for the decimal of 22 pound, which is .19642857, and write that down, alſo look the Decimal belonging to ſeven ounces, which is .00390625, and write that down: Laſtly, ſeek the decimal of three quarters of an ounce, which is .00041853, and write that down, then adding theſe four numbers together you ſhall finde their ſum to be. 95075335, which is the Decimal repreſenting 3 qu. 22 *lib.* 7 oun. $\frac{3}{4}$ of an ounc.

.75
.19642847
.00390625
.00041853
.95075335

PROP. IV.

How by the Table *to expreſs* Averdupois little weight *in Decimals.*

The fourth Table is of *Averdupois* little weight, the Denominations whereof are *Ounces*, *Drams* and *Quarters*

ters of drams, so that the Decimal of 11 ounces is. 6875. the Decimal of five drams is .01953125, and the Decimal of one quarter of a dram is .00097656.

But if it be required to find one decimal number, which shall represent 11 ounces, $5\frac{1}{4}$ then you must first look for the decimal belonging to 11 ounces, which is .6875, and set it down; then look for the decimal answering to 5 drams, which is .01953.125, and set that down. Lastly, look the decimal belonging to a quarter of a dram, which is. 00097656, and set that down; these three numbers being added together, produce .70800781, which is the correspondent decimal belonging to 11 ounces, 5 drams, and a quarter of a dram.

$$
\begin{array}{r}
.6875 \\
.01953125 \\
.00097656 \\
\hline
70800781
\end{array}
$$

PROP. V.

How by the Table *to express* Liquid measures *in Decimals*.

Because there is so great variety of Liquid Measures, that hardly any two commodities are sold by the same, the difference of the Gallon continually making alteration, we have therefore in this fifth Table made the greatest denomination to be one Gallon, the next less denomination being Pints and quarters of Pints, so that in the Table you shall find the decimal belonging to three pints to be .375, and the decimal belonging to two quarters, or half a pint, to be .c62. 5, and so for any other.

But for to express Pints and parts of Pints in one entire decimal number, you must add the Decimals of the several denominations together, and their Sum shall be the entire Decimal.

So if you were to express 3 pints and an half in one entire decimal number, add the decimal of three pints, which is .375, to the decimal of two quarters which is .0625, and their sum .4375, shall be the decimal of three pints and an half.

.375
.0625
.4375

PROP. VI.

How by the Table *to express* Dry-measures *in DECIMALS.*

The sixth Table is of *Dry Measures*, the several denominations whereof are *Bushels*, *Pecks*, *quarters of Pecks* and *Pints*, so may you find the decimal of five bushels to be .625 the decimal of two pecks to be 0625, the decimal of three quarters of a peck to be .023437, the decimal of two pints to be 0039063. Thus are the correspondent decimals belonging to the several denominations found.

But if you would have one number to express 5 bushels, 2 pecks, three quarters of a peck, and 2 pints: you must first find the decimal belonging to 5 pecks, which is 625, and write it down, then find the decimal of two pecks, which is .0625, then seek the decimal of three quarters of a peck, which is .0234375, and write that down. Lastly, seek the decimal of two pints, which is .0039063, which numbers being added together, produce 7148438, which is the decimal belonging (or expressing) 5 bushels, 2 pecks, three quarters of a peck,

.625
.0625
.0234375
.0039063
.7148438

PROP.

PROP. VII.

How by the Table *to express* Long measures *in Decimals.*

The seventh Table is of *Long measures*, the Integers being *Yards* and *Ells*: and the lesser denominations are *quarters* of *Yards* or *Ells*, *Nailes*, and quarters of *Nailes*. So you may find in the Table that the decimal of three quarters of a Yard, or an Ell, is 75, the decimal of two Nails, is .125, and the decimal of one quarter of a Naile is 015625.

But if you would have one number to express 3 quarters of a Yard, or an Ell, to Nailes, and one quarter of a Naile; you must seek the decimal of three quarters of a Yard or Ell, which is .75, and write it down, likewise seek the decimal of two Nailes, which is .125, and write that down. Lastly, seek the decimal of one quarter of a Nail, which is .015625, and write that down, these three numbers added together, make .890625, which is the decimal belonging to 3 quarters of a Yard or Ell, 2 Nailes, and one quarter of a Naile.

.75
.125
.015625
.890625

PROP. VIII.

How by the Table *to express the parts of* Time *in Decimals.*

Time is usually divided into *Years*, *Months* and *Dayes*: So the eighth Table which is of *Time*, consisteth of these two denominations, *Months* and *Dayes*, you may

 find

find that the decimal of 5 months is .41667, the decimal of 26 days is .071233. These are the principal decimals, but the compound decimal number representing 5 months, 26 days, is .487900, as you shall find, if you add .071233, which is the decimal of 26 dayes, to .416667, which is the decimal of five Months.

PROP. IX.

How by the Table *to express* Dozens *in* Decimals.

The last Table is of *Dozens*, the Integer being a *Grosse*, and the smaller denominations are Dozens, and parts of Dozens, so may you find the decimal of seven dozen to be .5833333, and the decimal of five parts of a dozen to be .0347222, and these two numbers added together, make .6220555, which is the number which representeth 7 dozen, and $\frac{5}{12}$ parts of a dozen.

In the setting down of Decimal Fractions, to add them together, you must alwayes observe to set *Primes* under *Primes*, *Seconds* under *Seconds*, &c. which the points before the several Fractions will direct you to do.

Hitherto we have shewed the use of the foregoing Tables in expressing of Fractions in decimal numbers. It resteth now to shew the use of them in finding what Fraction either of Money, Weight or Measure, any decimal number given doth represent, and that shall be made evident by the ensuing Proposition.

PROP.

PROP. X.

A Decimal number being given, how to find what Fraction it doth represent.

Let .02916667 be a decimal number, representing some Fraction part of *English Coin*: Because it is required to find the value of this Fraction in *English Coin*, you must therefore repair to the Table of *English Coin*; in the second Column of which Table seek for the number given (*viz.* 02916667) which you shall find to stand against 7 pence, and so much is the value of the decimal Fraction 02916667, in English Coin.

Also if the decimal Fraction .75 were given, you shall find the value thereof to be 15 shillings, and the value of. 003125 to be three farthings.

Likewise in the Table of *Troy weight*, if .41666667 were given, it would signifie five ounces, and .05416-667 would express 13 peny weight and 00173611 will express ten grains, &c.

After this manner may you find the value of any decimal number given, either in *Money*, *Weight* or *Measures*, when the number given may be exactly found in the Table: But if the number given cannnot be found exactly in the Table unto which it is directed at one entrance. Then you must, *Find in the same Table, the nearest number you can, less than the given number, and take the number that answers unto it in the first Column, which will be the greatest Fraction of the number required: then substracting the decimal thus found, out of the decimal given, you shall have a remainder, which remainder seek also in the second Column of the Table, if it may be found, if not, seek the nearest less, and the number answering thereunto in the first Column shall be the next greatest Fraction;*

 then

then substracting this decimal found out of the former remainder, there will be another remainder, which also seek in the Table, and proceed as in the former: An Example or two will make all plain.

Example. 1.

Let .68125000 be a Decimal given, representing some part of *English Coin*, If you look in the Table of English Coin for .68126000, you cannot find it, but the nearest number in the Table less than it, is .65, against which I find .13 *s.* so that 13 *s.* is the greatest fraction part of *English Coin* agreeing to this number.

This done, substract .65 out of .68125000, and there will remain 03 25000, which number also you must seek in the Table of *English Coin*, but being you cannot find it there, you must take the nearest number less than it, which is .02916667, against which I find 7 pence, which is the next greatest Fraction part of *English Coin* agreeing to this number.

Again substract .02916667, out of 03125000, and there will remain .00208333 which number seek in the Table, and you shall find it to stand against 2 farthings, and so much doth this last remainder signifie in *English Coin*, and the whole given number 68125000 doth represent in *English Coin* thirteen shillings seven pence two farthings, as by the operation following doth appear.

.68125000

.68125000 number given.
.65..... the next lesser number in the Table, representing 13 *s.*

.03125000 first remainder.
.02916667 the next lesser number in the Table, representing 7 *d.*

.00208333 second remainder, which represents two farthings.

So doth the whole number represent 13 *s.* 7 *d.* 2 *q.*

Example 2.

Let the Decimal .87426934 representing some fraction of a pound sterling, be given. If you look in the Table of *English Coin* for .87426934 you cannot find it; but the nearest number in the Table less than it, is .85, against which I find 17 shillings, so that 17 shillings is the greatest fraction part of *English Coin*, agreeing to this number.

Then substracting .85 out of .87426934 there will remain .02426934, which number also you must seek in the Table of *English Coin*, but seeing you cannot find it there, you must take the nearest number less than it, which is .02083333, against which I find five pence, which is the next greatest Fraction-part of *English Coin*.

Lastly, substract .02083335, out of .02426934, and there will remain .00343601, which number you must also seek in the Table of *English Coin*; but not finding it exactly there, you must take the nearest number less, which is .003125, against which you shall find 3 farthings, which is the next greatest fraction-part of *English Coin*, and the Decimal .87426934, doth in value signifie 17 shillings 5 pence 3 farthings, and something

more, for .003125 is the decimalof 3 farthings; and the nnmber you are to look for in the Table is .00343=601, greater than the decimal of 3 farthings; wherefore, if you substract .003125 out of. 00343601,there will remain .31101, which is the $\frac{31101}{10000}$ part of a farthing, which is inconsiderable. See the following operation.

87426934 Decimal given.
85 ...... Decimal of ———— 17 *s.*

92426934 First remainder.
02083333 Decimal of ———— 5 *d.*

00343600 Second remainder.
003125 .. Decimal of ———— 3 *q.*

00031101 Decimal part of a Frrthing.

¶ And here note, that whatsoever hatd been here said concerning the uses of the Table of English Coin, the same order is to be observed in the use of the other Tables of *Weight*, *Measure*, *Time*, &c. as by the following Examples (if you make trial) will appear.

Examples.

1 If this decimal 48975694, were given to know the value thereof in *Troy weight*, you shall find it to contain 15 ouuces, 17 peny weights. and 13 grains

2 Also if .95075335 were a Decimal given, and it were required to find the value thereof in *Averdupois great weight*, you shall find it to contain 3 quarters of a hundred, 22 pound, 7 ounces, and 3 quarters of an ounce.

3 Like-

3 Likewise, if .70800781 were a decimal Fraction given, you shall find the value thereof in *Averdupois little weight* to be 11 ounces, 5 drams, and one quarter of a dram.

4 If 4375 were a Decimal, whose value were required in *Liquid Measures* you shall find it to contain 3 pints and an half.

5 Let .7148438 be a Decimal given, whose value is required in *Dry Measure*, you shall find it to contain 5 bushels, 2 pecks, 3 quarters of a peck, and 2 pints,

Thus have I shewed you the use of these decimal Tables in expressing of the fraction-parts of *Money*, *Weight*, *Measure*, &c. But because these Tables may not be alwayes at hand, when there is need of Them, I will here shew you how the value of any decimal given: may be known by multiplication only; and this is

THE RULE.

Multiply the Decimal *given, by the number of known parts of the next inferiour* Denomination, *which are equal to the* Integer, *the* Product *is the value of the* Decimal *proposed in that inferiour* Denomination; *and if there happen to be any* Decimal *in the* Product, *you may in like manner find the value thereof in the next inferiour* Denomination, *and so proceed till you come to the least known parts of the Integer.*

Example.

Let .67395834 be a Decimal given, representing the fraction of a *Pound sterling*. First multiply .67395834 by 20 (the number of shillings in a pound sterling) and the product will be 1347916680, from which cutting of the last eight figures with a point, or dash of

of the pen) becauſe there were eight figures in the given Fraction) there will ſtand before the point (towards the left hand) 13, which are ſhillings, and the remainder .47916680 ſtanding behind the point, will be the fraction-part of one ſhilling ſterling, which number .47916680, you muſt multiply by 12 (the number of pence in one ſhilling) and the Product will be 575000160, from which number cut off the laſt eight figures as before, and there will be 5 left to the left hand, which are 5 pence, aud the figures on the right hand of the point, *viz.* .75000160 are the fraction-part of one peny ſterling, which therefore multiply by 4 (the number of farthings in one peny) and the Product of that multiplication will be 300000640, from which cut off the laſt eight figures to the right hand, and there will be left 3 towards the left hand, which repreſenteth 3 farthings, and the remaining figures towards the right hand are but the fraction-part of a farthing, which we therefore reject. And thus you find by *Multiplication* only, that this fraction .67395834 doth repreſent in the known parts of *Engliſh Coin*, 13 ſhillings, 5 pence, 3 farthings, as by the following operation appeareth.

	.67395834
	20
Shillings	13,47916680
	12
	95833360
	47916680
Pence	5,75000160
	4
Farthings	3,00000640

In

In like manner, if this fraction .94809028 were given, representing some fraction-part of Troy-weight you shall find the value thereof to be 11 ounces, 7 peny weight, 13 grains, as by the operation following appeareth.

	.94809028
	12
	189618056
	94809828
Ounces.	11,37708336
	20
Peny-weights.	7,54166720
	24
	216666880
	208333440
Grains.	13 | 00001280

In this manner may any Decimal given be reduced into the known parts of the Integer by *Multiplication* only. And

¶ Here note, that whereas in the preceeding Tables the Decimal fractions consist of *seven* or *eight* Figures, we shall in the prosecution of our work make use only of *four* or *five* of the first of them, which will be sufficient in ordinary practice, and come near enough to the truth in any ordinary question whatsoever.

So if instead of .02916667, which is the fraction-part of 7 pence, you take out only .02916, it will be sufficient.

Also

Also for	.05833333	take	.05833	In *Troy Weight.*
	.0058594		.0058	In *Dry Measure.*
	.5833333		.5833	In *Time.*

Thus much concerning the construction and use of the decimal *Tables*, we shall now come to the practice of *Decimal Arithmetick*, which shall be taught in the Rules following.

Of Notation of Decimals

NOTATION of Decimals is contrary to that of whole numbers: for whereas in whole numbers the values of figures are increased ten-fold by continual addition of Cyphers towards the right hand: so on the contrary, the values of the places of Decimals do decrease in the same proportion.

And whereas in whole number, Cyphers in the first place towards the left hand are unnecessary, yet in Decimals, they are absolutely necessary to discover the true denominator. Also Cyphers at the end (or towards the right hand) of decimal numbers are of no value, for one single Figure in decimals signifies as much as the same Figure would do, if there were Cyphers placed behind it, so 7 is equivalent unto 70, 700, or 7000, &c. For the denominators of decimal Fractions are alwayes Cyphers with an Unite towards the left hand, as hath been already intimated. So $\frac{70}{100}$ being reduced to its least terms will be $\frac{7}{10}$ and $\frac{7000}{10000}$ will be reduced to $\frac{7}{10}$ also, and so of any other, as by the Table following doth evidently appear.

	987654321	12345678
	100000000	.00000001
	10000000	.0000001
	1000000	.000001
	100000	.00001
	10000	.0001
a thousand	1000	.001 or $\frac{1}{1000}$
a hundred	100	.01 or $\frac{1}{100}$
Ten	10	.1 or $\frac{1}{10}$

Addition of DECIMALS.

IN Addition of Decimals, the same order is to be observed as in Addition of numbers of one Denomination before taught in the first part, in which there is no difficulty: But in Decimal numbers the chief care to be taken is in placing your whole numbers and Fractions in their due order, which you shall easily and certainly do, if you observe this general Rule, *viz.* to *place your whole numbers and fractions one under another, so that the points of separation which* (in decimal numbers) *distinguish the whole numbers from the Fractions, stand directly one under the other*, then are you to proceed in the addition of them in all respects, as you did in whole numbers.

Example 1.

Let it be required to add together in one sum these several sums following, in a decimal way, *viz* 36 *li.* 2 *s.* 8 *d.* 29 *li.* 0 *s.* 2 *d.* 31 *li.* 16 *s.* 9 *d.* and 6 *li.* 2 *s.* 5 *d.* First, set down 36 *li.* and a point or Comma after it, then

then for the fraction-part of 2 *s.* 8 *d.* look in your Table of English Coin, where you shall find the decimal fraction of 2 *s.* 8 *d.* to be, .1333 therefore for 36 *li.* 2 *s.* 8 *d.* set down 36. 1333.

Secondly, for your 29 *li.* 0 *s.* 2 *d.* set down 29. 0083.

Thirdly, for your 31 *li.* 16 *s.* 9 *d.* set down 31. 8375.

Lastly, for your 6 *li.* 2 *s.* 5 *d.* set down 6. 1208 as you see done in the operation following.

	li.	*s.*	*d.*		
	36	02	8		36, 1333
	29	08	2	set down	29, 0083
For	31	16	9		31, 8375
	6	02	5		6, 1208
	103	02	0		103, 0999

Your decimal numbers being thus placed in due order one under another, proceed to the adding of them together, as if they were whole numbers, and you shall find the sum or total of them to be 103. 0999.

Now the 103 which stands towards the left hand, are 103 pounds and the .0999 which stands towards the right hand of the Comma, is the fraction-part of one pound sterling, the value whereof you may find (by the Proposition before going) to be two shillings *fere*, which should be two shillings exact, but it wanteth somewhat, *viz.* the $\frac{2}{1000}$ part of a farthing, which is insensible; for if by the fore mentioned rule you seek the value of the decimal fraction, .0999, you shall find it to be 1 shilling, 11 pence, 3 farthings, and the $\frac{2}{1000}$ part of a farthing, which you may call in all 2 shillings, for decimal numbers will seldom happen to give the exact value of fractions, but will be either greater or lesser

ſer than they ought to be; but in ſuch a ſum as this is, the thouſanth part of a farthing is not to be regarded.

Examples 2.

Let it be required to add togother in a decimal way theſe ſums following, *viz.* 29 *li.* 18 *s.* 7 *d* 3 *q* 63 *li.* 11 *s.* 2 *d.* 1 *q.* 129 *li.* 4 *s.* 0 *d.* 2 *q.* and 3 *li.* 7 *s.* 10 *d.* 1 *q.*

Firſt, for 29 *li.* 18 *s.* 7 *d.* 3 *q.* ſet down 29. 93229.

Thirdly, for 29 *li.* 4 *s.* 2 *q.* ſet down 129. 20208.

Laſtly, for 3 *li.* 7 *s.* 10 *d* 1 *q.* ſet down 3. 39271 as you ſee here down in the Margine.

29. 93226
63. 55937
129. 20208
3. 39371
226. 08645

Your decimal numbers thus placed in order, add them together, as if they were whole nnmbers, and you ſhall find the ſum of them to contain 226. c8645.

Now the 226 which ſtands towards the left hand of the Comma, are 226 pounds, and the other figures towards the right hand, *viz.* 08645 are the fraction parts of a pound ſterling, which if you reduce by the fore-mentioned Propoſition, you ſhall find the value thereof to be 1 ſhilling, 8 pence, 3 farthings, ſo the whole ſum is 226 *li.* 1 *s.* 8 *d*, 3 *q.*

And here note, that what hath been ſaid, as concerning *Money*, the ſame is alſo to be underſtood of *Weight*, *Meaſure*, *Time*, &c. as by the following Examples will appear.

Other Examples for Practice.

Example 1.

In Mony.

135.8833
95.5583
3.2875
234.7291

234 *li.* 14 *s.* 7 *d.*

Example 2.

In Troy weight.

7.97413
6.65330
3.62187
18.24930

103 *C.* 2 *q.* 27 *lb.* 3 *oun.*

Example 3.

In *Averdupois little w.*

12.7227
76.3594
32.625
91.4883
32.8398
246.0398

246 *li.* 00 *oun,* 9 *dr.*

Example 4.

In *Averdup. great weight.*

37.9442
9.3053
33.6786
10.0000
12.8142
103.7423

103 *C.* 2 *q.* 27 *lb.* 3 *oun.*

Subſtraction of DECIMALS.

THE Suhſtraction of Decimals differeth nothing from the Subſtracting of one whole number from another, and the decimal numbers to be ſubſtracted one from another, muſt be placed in the ſame order, as in *Addition* of Decimal numbers, the practice of Subſtraction

ſtraction ſhall be ſeen in the following Examples.

Example. 1.

Let it be required to ſubſtract 31 *li.* 16 *s.* 2 *d.* out of 36 *li.* 2 *s.* 8 *d.*

Firſt, for your 36 *li.* 2 *s.* 8 *d.* ſet down the decimal thereof, which is 36. 1333.

Secondly, for your 31 *li.* 16 *s.* 0 *d.* ſet down the Decimal thereof 31, 8375.

This done draw a line under them, and ſubſtracting the leſſer from the greater, you ſhall find the remainder to be 4. 2958 the 4 on the left ſide of the Comma are four pounds, and the .2958 which ſtandeth towards the right hand, is the fraction-part of a pound, the value whereof being ſought, will be found to be 5 *s.* 11 pence. So that if you ſubſtract 31 *li-* 16 *s.* 9 *d.* out of 36 *li.* 2 *s.* 8 *d.* there will remain 4 *li.* 5 *s.* 11 *d.*

36, 1333
21, 8374
4, 2958

But if divers ſums be to be ſubſtracted out of one greater ſum, then you muſt firſt add all the ſeveral ſmaller ſums together, and ſubſtract the ſum of them from the greater given ſum, ſo ſhall the reſidue be the ſum deſired.

Examples for Practice.

Example 1.
In Money.

Lent	2784. 8375
Paid at several times.	36. 1333 29. 0083 31. 8375 6. 1208
paid in all	103.0999
rests to	2581. 7376
pay	2581 *li.* 14 *s.* 0 *d.*

Example 2.
In *Averdupois* great weight.

Bought	103. 7423
Sold	37. 9442
Unsold	65. 9442
65 *C.*	3 *q.* 5 *l.* 7 *ou.*

Example 3.
In Troy weight.

Delivered to a Goldsmith of old Plate	7. 97413
Received of new Plate	5. 59670
Rests in the Goldsmiths hands	2. 7743
	2 *li.* 4 *oun.* 10 *p. w.* 14 *gr.*

Multiplication of DECIMALS.

MUTIPLICATION *of Decimals* differeth nothing at all from the *Multiplication* of whole numbers, for making the greater number the *Multiplicand*, and the lesser number the *Multiplier*, the number issuing from that Multiplication shall be called the *product*.

Now in the Multiplication of decimal numbers one by another, if there be any Fraction either in the *Multiplicand* or *Multiplier*, or Fractions in both: So many figures

figures as the Fractions contain, so many figures must be cut off from the *Product* towards the right hand, which shall be the Fraction of the *Product*, and the figures towards the left hand of the Comma in the Product, shall be the Integers of the Product.

Example 1.

Let it be required to multiply 34 pounds, five shillings, three pence, by 16 pounds, six shillings, six pence.

First, seek the Decimal of 34 *li.* 5 *s.* 3 *d.* which you shall find to be 34.2625, make this your Multiplicand, then seek the Decimal of 16 *li.* 6 *s.* 6 *d.* which you shall find to be 16. 325, make this decimal number your Multiplier; then draw a line, and Multiply these two numbers together, as if they were whole numbers, and you shall find the Product of them to be 559. 3353125. Now because there are four figures in the Multiplicand which are Fractions, namely, these four towards the right hand, *viz.* 2625, and there are also three figures in the multiplier, which are Fractions, namely, these three towards the right hand, *viz.* 325, that is in all seven figures, representing Fractions, I therefore cut off from the product the seven figures towards the right hand, by making of a Comma there, to distinguish the whole number from the fraction: So is 559 the integer or whole number, and .3353-125, the Fraction of this Multiplication.

Multiplicand	34. 2625
Multiplier	16. 325
	1713125
	685250
	1927875
	2055750
	342625
Product	559. 3353125

Example 2.

If there be Fractions in the multiplicand, and none in the multiplier, yet the work is ſtill the ſame, for you muſt cut off only ſo many figures from the product, as there are Fractions either in multiplicand, multiplier, or both: So if it were required to multiply 5767 yards, and 3 quarters of a yard, by 235 yards, you muſt firſt ſet down 5767.75 for your 5767 yards, and three quarters, which number muſt be your multiplicand: And alſo ſet down 235 yards for your multiplier, then multiplying them together, as if they were whole numbers you ſhall find the product to be 1355421. 25, and becauſe there are only two Fraction figures, both which are in the multiplicand, namely, the two laſt thereof .75, and none in the multiplier. I therefore cut off only two figures of the product, namely, the two laſt, which are .25, ſo is the product of this multiplication 1555421. 25 which is 1355421 ſquare yards, and one quarter of a yard. And ſo if a Garden or other piece of Land, lying ſquare, ſhould contain in length 5767 yards, and three quarters, and in breadth 235 yards, the whole piece would contain 1355421 ſquare yards, and one quarter of a yard.

```
   5767,75
       235
----------
   2883875
  1730325
 1153550
----------
1355421.25
```

Example 3.

If decimal Fractions be to be multiplied by decimal Fractions, you muſt then (as before) multiply them as whole numbers, and from the product cut off ſo many Figures towards the right hand, as there are Figures in

in the multiplicand and the multiplier: So if it were required to multiply .953 by .782, you ſhall find their product to be .745-246, which being but ſix figures in all, I cut them of and that fraction. 745246 is the product of the multiplication of the two given Fractions.

.953
.782
1906
624
6671
.745246

Example 4.

If any two Decimal fractions being multiplied together, the product thereof doth not conſiſt of ſo many places as are required (by the former rules) to be cut off, you muſt then ſupply that defect by prefixing a Cypher, or Cyphers before the product towards the left hand: So if theſe Decimal fractions .063 and .0752 were to be multiplied, their product would be 47376. Now (by the former rules) you ſhould cut of ſeven figures of the product towards the right hand, but this product 47376 conſiſteth but of five figures; whereſore to make it ſeven figures, I prefix two Cyphers before the product on the left hand, making it .0047376, and that is the true product produced by this multiplication.

.0752
.063
2256
4512
.0047376

Example 3.

If you would multiply any Decimal (either Fraction only, or whole number and fraction together) by 10, 100, 1000, &c. You muſt add ſo many Cy-

 phers

phers to the multiplicand, as there are Cyphers in the multiplyer, and cut off ſo many Figures as there are fractions in the multiplicand, and that number ſhall be the product required: So if 7, 856025 were a Decimal given to be multiplied by 100, add two Cyphers to the number given, making it 785602500, then becauſe there were ſix figures of this number towards the right hand, it will be 785, 602500, which is the true product required.

Examples for Practice.

Example 1.

```
   7,432
    2,61
 -------
    7432
  44592
 14864
 -------
1939752
```

Example 2.

```
 22,358
     32
 ------
  44716
 67074
 ------
715.456
```

Example 3.

```
 .352
  .24
 ----
 1408
 604
 ----
.7448
```

Example 4.

```
375.6218
     100
----------
375.621800
```

Diviſion

Division of DECIMALS.

AS *Division* of whole numbers is the hardest of the four *Species* of *Vulgar Arithmetick*, so the *Division* of *Decimals* is the most difficult of the four kinds of *Decimal Arithmetick* but I hope to make it plain, to the understanding of the meanest capacity.

The several varieties that may happen in Division, are principally (if not only these) *four*. Namely, First, *To divide whole numbers and fractions.* Secondly, *To divide whole numbers by mixt, or mixt numbers by whole:* Thirdly, *To divide a greater fraction by a less*; and Lastly, *To divide a lesser fraction by a greater.*

In Division of Decimals this Rule is general, *If the* Dividend *be greater then the* Divisor, *the* Quotient *will be either a* whole number *or* a mixt, *but if the* Dividend *be less then the* Divisor, *the* Quotient *will be* a Decimal. And (for convenience in working, if there be need) any number of Cyphers may be annexed to the Dividend, that thereby the Quotient may extend to as many places as the tenour of the question shall require.

The manner of the working of *Division* in Decimals, is the same with that before delivered in whole numbers in the first part of Vulgar Arithmetick, as will appear by the Examples following, in every of the four premised varieties.

The Rule for the first variety.

The Dividend *and the* Divisor, *being both* mixt numbers, *or one of them being a* whole number *and the other* a mixt

a mixt; *or the* Dividend *being a* Decimal, *and the* Divisor *a* whole number *or* a mixt, *the first figure in the quotient will be of the same* place *or* degree, *with that Figure or Cypher of the* Dividend, *which at the first demand standeth, or (at least) is supposed to stand directly over the places of* Unite *in the* Divisor.

Example 1. *Where the terms given are both mixt numbers.*

Let it be required to divide 659. 3354125 by 16. 325. Here the terms given are both of mixt numbers, which being placed according to the Rules delivered before, for the Division of whole numbers, the figure in the Dividend, which at the first demand, standeth over 6, the place of Unites in the Divisor is 5, and because this standeth in the place of tenths, therefore the first figure in the quotient is in the place of tenths also, and the whole number consisteth of two of the foremost places, and the rest is a Decimal, thus the quotient sought in our present example is 34. 2625, of which 34 the two first figures is the Integer or whole number, 2625 the Decimal fraction.

Divisor

```
Divisor        Dividend          Quotient.
16,325)     559·3353125         (34·2625
              ······
            ——————————
             489·75
              69·585
             ————————
              65· 300
               4· 2853
             ————————
               3· 2650
               1·02031
              ————————
                97950
               040812
              ————————
                32650
                081625
              ————————
                 81625
```

Example 2. *One of the terms given, being a whole number, the other mixt.*

The mixt number 1375421. 26 being divided by the whole number 235, the quotient will be 5767. 75 and the first figure in the place of Thousands, as by the operation doth appear.

```
26)     .35673     (001372
           ...
        -------
         26
         096
        -------
          78
          187
        -------
           182
           0053
         -------
              52
```

The Rule for the second variety.

When the Dividend *is a* whole *or* mixt *number, and the* Divisor *a* Decimal, *add as many Cyphers to the* Dividend *as there are places in the* Divisor; *for the integral part of the quotient will consist of as many places as the* Divisor, *and the places arising from the integral parts of the* Dividend *added together.*

Example 1.

Let 348. 75 be the mixt number given, to be divided by the decimal .25, to the number given, I add two Cyphers, the number of places in the divisor, and then it will be 348. 7500, which being divided by .25, the integral part of the quotient will be 1395. because the whole part of the dividend 348, being divided by .25 giveth two places, and the number of places in the Divisor being two, giveth two more; and so the Integral part consisteth of four figures, as by the operation.

Divisor

Divisor	*Dividend*	*Quotient*
235)	135542125	(5767. 75

```
135542125
   ......
---------
1175
 1804
-------
  1645
   1592
-------
    1410
     1821
--------
      1645
       1762
--------
        1645
         1275
--------
          1176
```

Example 3. The Dividend being a Decimal, and Divisor a whole number.

The Decimal fraction .35673 being divided by the whole numb r 26, the quotient will be .002372, and the first significant figure in the place of thousands, or fourth place from Unity, as by the operation it doth appear.

```
Divisor        Dividend
 .25)          348.7500 (1395
                 . . .
               ----------
               25
                98
               ----------
                75
               237
               ----------
               225
                125
               ----------
                125
```

Example 2.

Let the mixt number 72. 5) be divided by .075, the number of places in the integral part of this Quotient will be 966. because there are 3 places in the Divisor; and but 3, because the integral part of the Dividend is less than the significant figure in the Divisor, as by the operation it doth appear

```
Divisor        Dividend
075        72.  5000 (966
           ---------------
           675
            500
           ---------------
            450
             500
           ---------------
               450
                500
               -----------
                450
```

The Rule for the third Variety

When the Terms given are both Decimals, *the* Dividend *being the greater, the integral part of the quotient will consist of as many places as the* Divisor *doth.*

Example.

Let the Decimal .73658 be divided by the Decimal .32 the integral part of the Quotient will be 23, because the Divisor doth consist of two places, as by the operation in the Margine doth appear.

Divisor *Dividend.*

.32) .73958 (23.11
....
―――
64
99
―――
96
35
―――
32
38
―――
32
6

The Rule for the fourth variety.

When the terms given are both Decimals, *consisting of equal places, the* Dividend *being the* lesser term, *place the* Dividend *as a* Numerator, *and the* Divisor *as* Denominator; *so is such vulgar fraction the quotient sought: But if the terms given consist not of equal places, supply the place or places wanting in either of the terms, by annexing a* Cypher

a Cypher or Cyphers on the right hand, and then proceed as before. Thus if .27 be given to be divided by .93, the quotient will be $\frac{17}{93}$. Also if .35 be given to be divided by .78563, the quotient by annexing 3 Cyphers to .35, the lesser decimal given, will be $\frac{35000}{478563}$, which vulgar fractions may be reduced into decimals if need be, by the first Proposition in this Second part of decimal Arithmetick.

Examples for Practice.

```
44).35673(.0081, &c.        .25)2481.00(9924
--------                    ----------
     352                        225
      47                        0231
   -------                     -------
      44                        225
       3                         60
                               -------
                                 50
                                100
                               -------
                                100
                                000
```

Having given you Examples of the four foregoing Rules in the several cases of Division in Decimals, according to the third way of Division taught in the First Part: I will now bring all the forementioned four Rules into one general Rule, and give you Examples of all the Varieties, that can possibly arise in Decimal Division, ready wrought according to the first (or most common) way of Division, taught in the First Part.

The general Rule is this.

When the Dividend and Divisor are placed orderly one under another (as in the First Part is directed) Observe this general Rule.

When the place of Primes in the Divisor, of any Decimal Fraction, comes to be under the place of Primes in the Dividend; The demand that is then made shall be the first figure of the whole number (or Integer) that is to be placed in the Quotient: all after it being Primes, Seconds, Thirds, &c.

Examples of all Varieties

Example. 1.

To divide a mixt number by a mixt number, as 172. 5 by 3. 746.

```
        1 3 (4
        2 6 7 5
      4. 4 8 4 8(4
    2 2 6 2 6 1 0
    5 4 9 6 4 4 6(6
  1 7 2. 5 0 0 0 0 0   (46. 049
    3. 7 4 6 6 6 6 6
      3. 7 4 4 4 4
          3 7 7 7
            3 3
```

Example 2.

Of a mixt Number by a mixt Number; the Divisor being greater than the Dividend. As 2. 34 by 52. 125.

(1
3(0
8 1
4 9 2 (8
6 5 0 9
4 7 6 2 1
5 5 0 0 0 (7
2 6 6 2 2 4 2 (5
2. 3 4 0 0 0 0 0 0 (.04489
5 2.1 2 5 5 5 5
5 2.1 2 2 2
5 2.1 1
5 2.

Example 3.

To divide a Whole Number by a Decimal Fraction, As 82, by .056.

3 (2
2 6 3 2 5
3 6 6 5 0 (2
8 2. 0 0 0 0 (1465 .3
.0 5 6 6 6 6 6
5 5 5 5

Exam-

Example 4,

To divide a Decimal Fraction by a Decimal Fraction, As. 0 25 by .5.

$$\begin{array}{l} .0\cancel{1}\cancel{2}\cancel{5}\ (.025 \\ \ \ 0\,.\cancel{5}.\,\cancel{5} \end{array}$$

Example 5.

A Decimal Fraction by a Decimal Fraction, As .8564 by .008.

$$\begin{array}{l} 000.\cancel{8}\cancel{5}\cancel{6}\cancel{4}\cancel{8}\ (107.05 \\ .00\cancel{8}\cancel{8}\cancel{8}\cancel{8}\cancel{8} \end{array}$$

Example 6.

A Decimal Fraction by a Decimal Fraction. As .73952 by .32.

$$\begin{array}{l} \ \ \cancel{1}\cancel{9}\cancel{3}\cancel{3} \\ 0.\cancel{7}\cancel{3}\cancel{9}\cancel{5}\cancel{2}\ (2.311 \\ 0.\cancel{3}\cancel{2}\cancel{2}\cancel{2}\cancel{2} \\ \ \ \cancel{3}\cancel{3}\cancel{3} \end{array}$$

Example 7.

A Mixt Number by a whole Number. As 32.959 by 27.

$$\begin{array}{l} \ \ \cancel{1}\cancel{1} \\ \cancel{1}\cancel{5}\cancel{5}\ (1 \\ \cancel{3}\cancel{2}.\cancel{9}\cancel{5}\ (9\ \ (1.220 \\ \cancel{2}\cancel{7}.\cancel{7}\cancel{7}\cancel{7} \\ \ \ \cancel{2}\cancel{2}\cancel{2} \end{array}$$

Example 8.

A Mixt Number by a whole Number. As 367.875 by 243.

$$
\begin{array}{l}
1 \\
2 \\
6 \\
1\ 3\ 7\ 2 \\
2\ 8\ 1\ 0\ 1 \\
6\ 3.\ 7\ 8\ 7\ 5\ (0.2625 \\
2\ 4\ 3\ 3\ 3\ 3 \\
2\ 4.\ 4\ 4 \\
2\ 2
\end{array}
$$

The

The Rule of Three in Fractions *Vulgar* and *Decimal*.

WHat the Rule of Three is, and the manner of working, is already ſhewed in the firſt part, that which we here intend is only to add ſome Examples in Fractions Vulgar, as well as Decimal; that by comparing the work in both, the Excellent uſe of Decimal Arithmetick might the better appear.

And how to convert the known parts of *Money*, *Weight*, or *Meaſures Engliſh*, into *Decimals* hath been already ſhewed, both *Arithmetically* and by *Tables*; yet to prevent the ſeveral *additions* and *ſubſtractions* in thoſe *Tables*, I have here annexed another *Decimal Table*, for the more ſpeedy Reduction of *Engliſh money* under two ſhillings, all ſums of money above, not having pence or farthings annexed, being as eaſily reduced by *memory* as by *Tables*, and this I have the rather done, becauſe the ſame *Table* will alſo reduce the *Coins* of *France*, and the parts of *Troy-weight*, if an ounce be made the Integer which in point of practice is much more uſeful than the pound.

The Table of REDUCTION

Peny		Gr.	Peny		Gr.
	001042			026014	
	002083	1		027083	13
	003125			028125	
1	004166	2	7	029166	14
	005208			030208	
	006250	3		031250	15
	007291			032291	
2	008333	4	8	033333	16
	009375			034375	
	010410	5		035406	17
	011458			036458	
3	012500	6	9	037500	18
	013541			038541	
	014583	7		039583	19
	015625			040625	
4	016666	8	10	041666	20
	017708			042708	
	018750	9		043750	21
	019791			044791	
5	020833	10	11	045833	22
	021874			046875	
	022916	11		047916	23
	023958			048958	
6	025000	12	12	050000	24

The Table of REDUCTION.

	051082			076014	
	052083	G. 1		077083	13
s. d.	053125			078125	
1. 1.	054160	2	7	079165	14
	055208			080208	
	056250	3		081250	15
	057292			082921	
2	058333	4	8	083333	16
	059375			084375	
	060416	5		085416	17
	061458			086458	
3	062500	6	9	087500	18
	063542			088541	
	064583	7		089583	19
	065625		10	090625	
4	066666	8		091666	20
	067708			092708	
	068750	9		093750	21
	069792		1	094791	
5	070833	10		095833	22
	071874			098675	
	072916	11		0 9716	23
	073958		12	099858	
6	075000	12		000000	24

Thefe things premifed, we will now fhew the ufe of the Table in fome practical queftions belonging to the Rule of Three direct.

1 *Queftion.*

If $\frac{7}{8}$ of a yard of Cloth, coft $\frac{9}{12}$ of a pound: what fhall 17 *yards coft at the fame rate?*

If $\frac{7}{8}$ coft $\frac{9}{12}$, what fhall 17 coft? Anf. 14 *li.* $\frac{4}{7}$.

Firft, multiply $\frac{9}{12}$ by $\frac{17}{1}$ the product is $\frac{153}{12}$, then divide $\frac{153}{12}$ by $\frac{7}{8}$, the quotient is $\frac{1224}{84}$: again, if you divide 1224 by 84, the quotient is 14 $\frac{8}{84}$, or in the leaft terms 14 pound $\frac{4}{7}$ of a pound.

And the value of this fraction $\frac{4}{7}$ of a pound, will be found by the third Rule of Reduction of Fractions to be 11 fhillings 5 pence, and $\frac{4}{7}$ of a penny, which is fomewhat above two farthings: for it is 2 farthings, and $\frac{2}{7}$ of a farthing.

The fame Queftion in Decimals.

If $\frac{7}{8}$ of a yard of Cloth coft $\frac{9}{12}$ of a pound, what fhall 17 *yards coft at the fame rate?*

To anfwer this queftion $\frac{7}{8}$ of a yard, and $\frac{9}{12}$ of a pound muft firft be reduced into decimals, either by Divifion, or by the Tables of Reduction: by both which ways of Reduction the decimal of $\frac{7}{8}$ will be .875, and the decimals of $\frac{9}{12}$ will be .75, and then the terms of the queftion will ftand thus;

If ·875 parts of a yard coft. 75 parts of a pound, what fhall 17 yards coft at the fame rate?

If 0875——0.75———17. Here if you multiply the fecond term 0.75 by 17 the third term given, the product will be 12.75 and this product divided by 875 gives in the quotient 14.57142, that is, 14 pound 57142 parts of a pound, or 145 Decades, that is 14 pound

pounds 10 ſhillings, and .7142 parts of a Decade (or two ſhillings) which by the preceding Tables is 1 *s.* 5 *d.* 2 farthings, and .0059 parts of a farthing.

2 *Queſtion.*

If a piece of Gold plate weighing 19 *ounces* 3 *peny weight and* 5 *grains, be worth* 62 *pounds* 10 *ſhillings* 6 *pence; what is one ounce of the ſame Gold worth?*

This queſtion in vulgar fractions muſt be expreſſed thus.

If 1 *l.* $\frac{3437}{5760}$ Troy-weight, coſt 62 *l.* $\frac{126}{140}$, what ſhall $\frac{1}{21}$ of a pound Troy coſt at the ſame rate?

To anſwer this queſtion, the fractions 1 *l.* $\frac{3437}{5760}$ and 62 *l.* $\frac{126}{240}$, muſt be firſt reduced into improper fractions and then the fraction $\frac{1}{12}$ into the leaſt known parts of a pound Troy, and then the queſtion will ſtand thus.

If $\frac{9197}{5760}$ give $\frac{15006}{240}$, what ſhall $\frac{480}{5760}$ give?

Now becauſe it is neceſſary the terms given be reduced into their leaſt *Denominations*, before the queſtion be reſolved, therefore the anſwer may be found, by uſing the terms given thus reduced as whole numbers, not having any regard to the Denominators of theſe fractions; Saying thus.

If 9197 grains, coſt 15006 pence, what ſhall 480 grains coſt.

And here if you multiply 15006 by 480, the product will be 7202880, which being divided by 9197, the quotient will be 783 pence $\frac{1629}{9197}$ parts of a peny, and dividing 783 by 12, it will be 65 ſhillings 3 pence $\frac{1629}{9197}$, or 3 *l.* 5 *s.* 3 *d.* $\frac{1629}{9197}$. And although this queſtion is thus more eaſily anſwered than it would have been, if the terms had been wrought as vulgar fractions, yet the ſame terms being reduced to Decimals, the anſwer of the queſtion will yet be found with more eaſe, as ſhall appear by the operation following.

The ſame queſtion in Decimals.

If a piece of Gold plate weighing 19 *ounces* 3 *peny weight and* 5 *grains, be worth* 26 l. 10 s. 6 d. *what is an ounce of the ſame Gold worth?*

The Decimal of 19 ounces 3 peny weight and 5 grains, making an ounce the Integer, is by this Table 19.16041, for that 19 ounces are 19 Integers, 2 peny weight is one tenth of an ounce. and the Decimal of 1 *d.w.* 5 grains is by this Table .06041; and the decimal of 62 *l* 10 *s.* 6 *d.* by the ſame Table is 62.525, and becauſe an Unite or Integer is the third term given, there needs no multiplication, if therefore you divide 62.525 the ſecond term, by 16.16041 the firſt term propounded, the quotient will be 3.2632, that is 3 pounds 5 ſhillings 3 pence, and ſomewhat more, as by the operation in the margine it doth appear.

19.16041) 62.52 500000 (3.2632

5748123
5043770

3.832082
1.2116880

1.1496146
6206340

5748823
4512170

3732082
750088

3 *Question.*

If 5 Ells and a quarter of linnen Cloth cost 2 l. 16 s. 8 d. 3q. *what shall* 278 *Ells and a half cost at the same rate?*

If you would work this Question by whole numbers, your easiest way is first to reduce all the terms into their least Denominations, that is to say, the Ells into quarters, and the pounds, shillings pence and farthings, all into farthings, so shall your 5 Ells and a quarter be 21 quarters, and your 278 Ells and an half will be 1114 quarters, and your 2 *l.* 16 *s* 8 *d* 3 *q* will be 2723 farthings and then will your question stand thus in whole numbers.

quarters	*farthings*	*quarters.*
If 21 cost———	2723—what will——	1114 cost?

Then multiplying the second number by the third, that is, 2723 by 1114, the product will be 3033422, which divided by 21, the quotient will be 144448 farthings, which being again reduced into pounds, shillings, and pence, giveth 150 *l.* 9 *s.* and 4 pence, as by the operation following doth appear.

The

The Operation.

li. s. d. q.

$5\frac{1}{4}$ —— 2 —— 16 —— 8 —— 3 —— $278\frac{1}{2}$

21   20   4

40   1112

16 *shillings.*   2

56   1114

12

112

56

672

8 *pence*

680

4

2720

3 *farthings.*

2723

1114

10892

2723

2723

2723

3033422

21) 3033422 (144448
......

21
93

84
93

84
94

84
102

84
182

168
14

d.
2 (4   1   s. d.
144448   (368112   (300   (9(150
44444   12222   222   0
111

But if you would work the ſame queſtion by Decimal numbers, you may ſave the labour of reducing the terms to their leaſt Denominations, for 5 Ells and a quarter is in decimal numbers 5.25, and 278 Ells and an half is 278.5, and 2 pound 16 *s.* 8 *d.* 3 *q.* is in decimals 2. 8364, and then your queſtion in decimals will ſtand thus:

Ells

Ells. *pounds.* *Ells.*
If 5.25 cost 2. 836, what 278. 51.

If you multiply (according to the Rule) the second term by the third, that is 2. 8364 by 278. 5, the product of that multiplication will be 789. 93400, which divided by the first term 5. 25, the quotient will be 150.4642, which decimal representeth 150 *l.* 9 *s.* 4 pence, and so much in money will 278 Ells and a half cost.

The Operation.

Ells. *pounds.* *Ells.*

5.25 ——————— 2.8364 ——————— 278.5

```
        2.8364
         27.85
        ------
         14182
       226912
      198548
     56728
   -----------
   789.93740
```

```
5.25) 789.937400 (150.4642
      ......         or
      ----------
        525          (150li. 9s.4d.
        2649
        ----
        2625
         2437
        ----
         2100
          3374
        ------
          3150
           2240
        ------
           2100
            1400
        ------
            1050
             350
```

I have been the larger in this Rule, and especially in this Example, which is incumbred with fractions sufficient, because I would have the Reader the better discern the difference between the *Vuglar* and the *Decimal* way, and also to see how expeditious the one is over the other. Now this example being thus largely explained, I shall with the more brevity pass over the Rules following, giving one Example or two at the most in each Rule. And thus much shall suffice for the Golden Rule, or Rule of Three direct in Fractions.

The Rule of Three Reverſe.

A *Lends* B. 233 l. 6 s. 8 d, *for a year without Intereſt, upon condition that* B. *ſhould do the like courteſie for* A. *when required.* A. *hath occaſion for money 7 months; how much money ought* B. *to lend* A. *to requite his courteſie, and ſave himſelf harmleſs.*

If will not in this place tell you what the Rule of Three reverſe is, nor the manner of working thereof, that being already ſufficiently declared in the firſt part, but give you the Example, and the working thereof which take as followeth: So will the Queſtion be thus, ſtated:

months	*li.*	*s.*	*d.*	*months.*
12	233	6	8	7

Which in Decimals ſtands thus,

months	*li.*	*months.*
12	333·33	7

```
   333·33
       12
  --------
    46666
   23333
  --------
  2799·96
    6666(3
  27999 (399993/4
  7777
```

Here you ſee that 12 months and 7 months are whole numbers, and ſo we let them alone without any Reduction,

Reduction, but the Decimal of 233 *l.* 6 *s.* 8 *d.* will be found by the fore-mentioned Tables and Rules to be 233.33, which is the middle term in the question, and of the same quality with that, must the fourth term sought be, therefore if (according to the Rule delivered in the first part) you multiply 233.33 by 12, the product will be 2799 96, which divided by 7, giveth in the quotient 39999, which is the Decimal of 400 *l.* and so much money ought *B.* to lend *A.* for 7 months.

The Rule of Proportion, consisting of five Numbers.

Question 1.

If 100 li. *in* 12 *moneths yields* 6 li. *interest, what interest shall* 264 li. 16 s. 5 d. *yield in* 15 *months at the same rate?*

Set down your numbers in Decimals, as in the Example following appeareth, so shall you find the Decimal of 264 *li.* 16 *s.* 5 *d.* to be 264.8208 all the rest being whole numbers, having no fractions joyned with them we neglect, and work with them as they are, so will the several numbers of your question (if rightly disposed) stand as followeth:

li.

li.	*mo.*	*li.*	*li.*	*mon.*
100 ——	12 ——	6 ——	264.8208 ——	15

100 × 12 = 1200

264.8208 × 6 = 1588.9248

1588.9248 × 15:

79.46240
15889248
————
23833.8720

1200) 23833.8720 (19.8615

1200
11833
————
10800
10338
————
9600
7387
————
7200
1872
————
1200
6720
————
6000
720

or *li.* 19 *s.* 17 *d.* 2 *q.* 3 *fere.*

Your numbers being thus orderly diſpoſed, you muſt according to the Rule before delivered in the firſt Part, multiply the firſt and ſecond terms together, which in this Example are 100 and 12, whoſe product

duct is 1200, which is your Divisor; Then multiply the three last terms one into another, as 264, 8208 (which is the Decimal of 264 *li.* 16 *s.* 5 *d.*) by 6, and the product thereof will be 1588 9248, which number again multiplied by 15, (which is the last term) the product will be 23833.8720 which is your Dividend, and this number being divided by your former product, giveth in the quotient 19 8615, which is the Decimal of 19 *li.* 17 *s.* 2 *d.* 3 *q. fere*, and so much doth the simple Interest of 264 *li.* 16 *s.* and 5 *d.* amount unto in 15 months, after the rate of six *per centum* for a year.

Question 2.

If the carriage of 23 *hundred and* 3 *quarters of any thing* 127 *miles, cost* 4 li. 13 s. 6 d. *what shall the carriage of* 47 *hundred and an half of such like commodity cost, being carried* 381 *miles.*

Place your numbers in order as in the following Example doth appear, then multiply the first and second terms together for your Divisor, and the three last one into another for your Dividend, and so will the quotient of this division answer the question demanded, and the work will stand as followeth.

```
  C.        miles        li.        C.        miles
23.75 ——— 127 ——— 4675 ——— 47.50 ——— 381
  127                   47.50
 ————                   —————
 16625                  23375
 4750                  32725
2375                   1870
———————                ——————
3016.25               222.06250
                           381
                      ——————————
                       22206250
                     177650000
                     66618750
                    ——————————
         3016.25)84605.81250(28.050
                     .....
                 ——————————
                  603250
                 2426081
                 ————————
                  2413000
                  1508250
                  ————————
                  15081250
                  00000000
```

Here you ſee that the firſt and ſecond terms multiplied together produced 3016.25, which muſt be your Diviſor, and the three laſt terms being multiplied one into another, produce 84605.81250, which number, divided by 3016.25, giveth in the quotient 28.050, which Decimal repreſenteth 28 *l.* one ſhilling, and ſo much will the carriage of 47 hundred and a half coſt being carried 381 miles.

Thus

Thus have I shewed the use of decimal Arithmetick in such questions as concern the *Golden Rule*, or *Rule of Three*; both Direct, Reverse, and Compounded, by an Example or two in each Rule, and those compounded of Fractions sufficient, I should now proceed to questions in *Fellowship*, with and without Time, as also *Barter*, *Alligation*, the Extraction of the Square and Cube Roots, &c. but forasmuch as these last mentioned Rules depend only upon the *Rule of Three*, as by Examples in the first part doth plainly appear, I shall therefore save that labour, and spare my Reader the pains of practising questions which wholly depend upon that which I (by this time) suppose him perfect in; Yet if the Reader be desirous to make tryal of any such question for his own satisfaction, he may either make tryal of those questions in the former part of this Book in those several Rules, reducing the numbers there given into Decimals, or if he please, he may frame questions according to his own fancy. And thus I shall conclude this Second Part.

The End of the Second Part.

AN APPENDIX,

To the *Second Part*

Section 1.

Of Exchange *of the* Coins, Weights, *and* Measures *of one* Country, *with the* Coins, Weights *and* Measures *of another* Country.

TO perform this work, there is nothing required more than the *Golden Rule*, if first the *Rate* or *Proportion* between the *Coins*, *Weights*, and *Measures* of any two *Cuntrys* be first known, which is best obtained by experience, rather than taken upon trust, all that in this place I shall do, is only to instruct the ingenious in the manner of Work, and make use of such Rates or Proportions as I find set down by Mr. *Lewis Roberts* Merchant, in his *Map of Commerce*.

Question 1.

How many Riders (each Rider containing 1l. 1 s. 2 d. 2 q. *Sterling) shall I receive for* 125l. 6 s. 4 d. 2 q *Sterling?*

Facit 237 Riders.

l. s. d. *Rider* *li: s. d.*
If 1 1 2½ *give* 1 *how many Riders shall* 251 6 4½ *give?*

Here if you reduce your numbers to their least Denominations, or set them down in Decimals, and multiply and divide according to the Golden Rule, you shall find in your quotient 237, and so many *Riders* ought to be received for 251 *l.* 6 *s.* 4 *d.* 2 q. Sterling.

Question 2.

How many French-Crowns)*each French Crown being valued at* 6 s. *Sterling*) *shall I receive for* 492 l. 18 s. *Sterling?*

Facit 1643 French Crowns.

s. *F. C.* *li.* *s.*
If 6 *give* 1, *what shall* 492 18 *give.*

Multiply and divide acording to the Golden Rule, and you shall have in your quotient 1643, and so many French Crowns are to be received for 492 *l.* 18 *s.* Sterling.

Question 3.

A Merchant delivered at Paris 1643 *Crowns of* 6 s. *Sterling the piece, how many pounds Sterling ought to be received at* London?

Answer 492 *l.* 18 *s.* *Sterling.*

Crown *s.* *Crowns*
If 1 *give* 6, *what shall* 1643 *give?*

Multiply and divide, and you shall have in your quotient 492 *l* 18 *s.* and so much Sterling Money ought to be delivered at *London*, for 1643 French Crowns, of 6 *s.* the Crown Sterling.

Question 4.

If 3 *yards at* London, *be* 4 *Ells at* Antwerp, *how many yards at* London *make* 84 *Ells at* Antwerp?

Ells Antwerp	*Yards* London,	*Ells Ant.*
4	3	84

Facit 63.

And so many yards at *London*, are equal to 84 Ells at *Antwerp*.

Question 5.

How many yards of London *make* 27 *Ells of* Antwerp *when* 100 *Ells of* Antwerp *make* 60 *Ells of* Lions, *and* 20 *Ells of* Lions *make* 25 *yards of* London?

The first Work.

Ells Lions	*Yards* London	*Ells* Lions
20.	25	60
	60	
	15010	
	75 *facit* 75	

That is 75 Yards of *London* is equal to 100 Ells of *Antwerp*.

The Second Work.

Ells Antwerp,	*Yards* Lon.	*Ells* Antwerp.
100	75	27
	27	
	525	
	150	
	20|25	*Yards of London facit* 20. 25.

Question 6.

If 100 li. *Sterling be* 104 li. 6 s. 4 d. *Flemish, what is one pound Sterling worth?*

li. Sterl.	*li.*	*s.*	*d.*	*li. Ster.*
100	134	9	4	1
	20			
	2686			
	12			
	5372			
	26864			
	322|36			

Facit 322 *pence* $\frac{39}{100}$ of a peny.

Question 7.

How many Ells of Franckford *make* 42 $\frac{1}{4}$ *Ells of* Vienna *in* Austria, *when* 35 *Ells of* Vienna *make* 24 *at* Lions, 3 *Ells of* Lions, 5 *Ells of* Antwerp, *and* 100 *Ells of* Antwerp, 125 *Ells at* Franckford.

First Work.

Ells Ant.	*Ells* Franck.	*Ells* Ant.
100	1.25	5

$$\begin{array}{r} 1.25 \\ 5 \\ \hline 625 \end{array}$$

Facit 6. 25 or 6 Ells and a quarter of *Franckford* e-qual to 3 Ells of *Lions*.

Second Work.

Ells Lions,	*Ells* Franck.	*Ells* Lions.
3	6.25	24

$$\begin{array}{r} 6.25 \\ 24 \\ \hline 2500 \\ 1250 \\ \hline 3)150\,00(50 \\ 15 \\ 0 \end{array}$$

Facit 50 Ells of *Franckford*, equal to 35 Ells of *Vienna*.

Third

Third Work.

Ells Vien.	Ells Franck.	Ells Vien.
35	50	42.2

$$
\begin{array}{l}
35)\ 2112.50\ (60.35 \qquad\qquad 2112.50 \\
\hline
210 \\
125 \\
\hline
105 \\
200 \\
\hline
175 \\
\ \ 25
\end{array}
$$

Facit 60. 35. or $6\frac{5}{14}$ Ells of *Frankford*, equal to 35 Ells of *Vienna*.

Thus have I given you a few Examples of Exchanges, I will now insert some few Tables derived from Mr. *Lewis Roberts* his Map of Commerce afore-said, of the truth of which I am not a competent Judge, but shall leave that to the scrutiny of such as have occasion to trade into Forreign Countries.

A TABLE shewing what one pound of *Averdupois* Weight at *London* maketh in divers eminent Cities, and other remarkable places.

		lb.
One pound of Averdupois weight at London maketh at	Antwerp	.9615
	Amsterdam	.9
	Abeville	.91
	Ancona	1.282
	Avignon	1.12
	Burdeaux	.91
	Burgoyne	.91
	Bolonia	1.25
	Bridges	98
	Calabria	1.3698
	Calais	107
	Constantinople	.8474 Loder ;
	Deep	.91
	Dantsick	1.16
	Ferrara	1.3333
	Florence	1.282
	Flanders in general	1.06
	Geneva	.9345

Genoa

		lb.
One pound of Averdupois *weight at* London *makes at*	Genoa	1.4084 suttle; 1.4285 grosse
	Hamburg	.92
	Holland	.95
	Lixborn	.881
	Lions	1.07 common weight; .98 silk weight; .9 customers weight
	Legorn	1.3333
	Milan	1.4285
	Mirandola	1.3333
	Norimberg	.88
	Naples	1.4085
	Paris	.89
	Prague	.83
	Placentia	.13888
	Rochel	.112
	Rome	.127
	Rovan	.875 by Vicont; .9017 common weight
	Sivil	.108
	Tholousa	.112
	Turin	.12195
	Venetia	1.5625 suttle; 9433 gross
	Vienna	.813

The

The uſe of the preceding Table.

How much weight at Bolonia, *will* 655 l. *Averdupois make ?*

Look in the Table for *Bolonia*, and right againſt it you ſhall find 1. 25, which ſheweth that one pound Averdupois at *London* is equal to 1. 25 *l.* at *Bolonia* ; Therefore ſay by the Rule of Three :

If 1 *l.* Averdupois give 1. 25 *l.* at *Bolonia*, what ſhall 655 *li.* Averdupois give ? *Anſwer* 818. 75. As by the operation following doth appear.

li.	li.	li.
1	125	655
	655	

625
625
750

81875 or 818 $\frac{3}{4}$

A TABLE shewing what one pound Weight in divers Forreign Cities, and remarkable Places, maketh at *London* of *Averdupois* Weight.

One pound weight in	makes at London of Averdupois weight. lb.
Antwerp	1.04
Amsterdam	1.1111
Abeville	1.0989
Ancona	.78
Avignon	8928
Burdeaux	1.0989
Burgoyne	1.0980
Bolonia	.8
Bridges	1.0204
Calabrio	.73
Calais	.9345
Deep	1.0989
Dantsick	.862
Ferrara	.75
Florence in general	.78
Geneva	.9433
Genoa subtle	1.07
Genoa	.71
Genoa gross	.7

Ham-

One pound weight in		*makes at* London *of* Averdupois *weight* lb.
Hamburg		1.0865
Holland		1.0526
Lisbone		1.135
Lions	common weight	.9345
Lions	ſilk weight	1.0204
Lions	cuſtom weight	1.1111
Legorn		.75
Milan		.7
Mirandola		.75
Norimberg		1.136
Naples		.71
Paris		1.1235
Prague		1.2048
Placentia		.72
Rochel		.8928
Rome		.7874
Rovan	by Vicount,	1.1428
Rovan	common weight,	1.1089
Sivil		.9259
Tholouſa		.8928
Turin		.82
Venetia	ſuttle,	.64
Venetia	groſs:	1.06
Vienna		1.23

The use of the foregoing Table.

In 7652 *li. weight at* Mirandola, *how many pound Weight of* Avercupois?

Look in the Table for *Mirandola*, and right against it you shall find .75, which sheweth that one pound *Averdupois* is equal to 75 or $\frac{3}{4}$ of a pound at *Mirandola*, wherefore say by the Rule of Three,

If 1 *l.* at *Mirandola*, gives .75 or $\frac{3}{4}$ of a pound *Averdupois*, what shall 7652 *l.* of *Mirandola* give? *Answer* 5739, as by the operation following doth appear.

li.		
1	.75	7652
		75
		38260
		53564
		573900

A Table

A TABLE reducing English Ells to the Measures of diver forreign Cities and remarkable places.

One Ell at London makes at

Amsterdam	1.6949	Ells.
Antwerp	1.6666	Ells.
Bridges	1.64	Ells.
Arras	1.65	Ells.
Norimberg	1.74	Ells.
Colen	2.08	Ells.
Lisle	1.66	Ells.
Mastricht	1.57	Ells.
Frankford	2.0866	Ells.
Dantsick	1.3833	Ells.
Vienna	1.45	Ells.
Paris	.95	Aulns.
Rovan	1.03	Aulns.
Lions	1.0166	Aulns.
Calais	1.57	Aulns.
Venice Linnen	1.8	Braces.
Venice Silk	1.98	Braces.
Lacques	2.	Braces.
Florence	2.04	Braces.
Milan	2.3	Braces.
Legorn	2.	Braces.
Madera Isles	1.0328	Braces.
Sivill	1.35	

Lisbone

One Ell at Lon. makes at		
Lisbone	1.	Vares
Castilia	1.3875	
Andoluzia	1.4625	
Granado	1.3625	
Genoa	4.8083	Palmes
Saragosa	.55	Canes
Rome	.56	
Barselona	.7225	
Valentia	1.2125	

The use of this Table.

In 632 *Ells at* London, *how many Braces at* Florence?

Look in the Table for *Florence*, and right against it you shall find 2. 04, which sheweth, that one Ell at *London*, maketh at *Florence* 2. 204 Braces; wherefore say by the Rule of Three.

If one Ell at London *give* 2. 04 *Braces at* Florence, *how many Braces shall* 632 *Ells give*? Answer 1289.28, as by the operation following doth appear.

```
1 ——————— 2. 04 ——————— 632
           632
          ————
           408
          612
         1224
        ——————
        1289|28
```

A TABLE reducing the Measures of divers Forreign Cities and Remarkable places, to *English* Ells.

One Ell at	Amsterdam	*Makes at London*	.59	*Ells*
	Antwerp		.6	
	Bridges		.6097	
	Arras		.606	
	Norimberg		.5474	
	Colen		.4807	
	Lisle		.6024	
	Mastricht		.6369	
	Frankford		.4792	
	Dantsick		.7228	
	Vienna		.9896	
One Auln at	Paris		1.0526	
	Rovan		.9708	
	Lions		.9836	
	Calais		.6369	
One Brace at	Venice — Linnen		.5555	
	Venice — Silk		.5102	
			.5	
	Lucques			
	Florence		.4901	
	Milan		.4347	
	Legorn		.5	
	Madera		.9681	

Sivill

One Vare at	Sivil	*makes at* London	·7409
	Lisbone		1.
	Caſtilia		·7207
	Andaluſia		·7339
	Granado		·7339
One Palm at Genoa			.2079
One Cane at	Saragoſa		1.8181
	Rome		1.7857
	Barſelona		1.4035
	Valentia		.8247

The uſe of this Table.

In 5727 *Braces at* Legorn, *how many Ells* Engliſh.

Look in the Table for *Legorn*, and right againſt it you ſhall find .5, which ſheweth that one Brace at *Legorn* maketh at *London* .5 or half an Ell, wherefore ſay by the Rule of Three.

If one Brace at Legorn *give* .5 *Ells at* London, *what ſhall* 5727 *Braces give*? Anſwer 2863.5, as by the work appeareth.

1 ——— ·5 ——— 5727

·5

———

2863.5

Section 2.

Concerning Interest and Annuities.

The First TABLE, shewing what One Pound being forborn any number of Years under 31, will amount unto, accounting Interest upon Interest, after the Rate of 6 *per cent*.

Y.	6 *per cent.*	Y.	6 *per cent.*	Y.	6 *per cent*
1	1,06	11	1,89829	21	3,89956
2	1,1236	1	2,01219	22	3,60353
3	1,19101	13	2,13292	23	4,81975
4	1,26247	14	2,260 0	24	4,04893
5	1,33822	15	2,39655	25	4,29187
6	1,41851	16	2 54039	26	4,54938
7	1,50363	17	2,69277	27	4,82234
8	1,59384	18	2 85433	8	5,11168
9	1,68947	19	3 02559	29	5,41838
10	1,79084	20	3,20713	30	5,74349

The

The first Column of this Table having Y at the top thereof, beginning at 1, and so proceeding to 30, signifies years, and the number in the next Column answering thereunto does shew what one pound is worth being forborn any number of years under 31, which Table is made according to this proportion.

As 100 to 106, so is 1 to 1. 06

and again,

As 100——106——106——1.1236

and thirdly.

As 100———106——1.1236——1.19101

Et sic ad infinitum.

The use of this Table.

What 136 l. 15 s. 6 d. *will amount unto, being forborn* 20 *years, after the rate of* 6 per centum, *interest upon interest.*

Look in the Table for 20 years, and right against it in the broader Colume, you shall find 3.20713; which shews that one pound being forborn 20 years will be augmented to 3.20713. Then if you reduce your 136 *li.* 15 *s.* 6 *d.* into a Decimal, either by the Tables in the Second part, or by the Scales in the Third Part of this Book, you shall find it to be 136.775. Wherefore say by the Rule of Three Direct.

If one pound being forborn 20 *years will be augmented to* 3. 20713, *to how much will* 136.775 li. *be augmented in the same time.* Answer, *to* 438 *li.* 13 *s.* 1 *d.* 1 *q.* as by the operation following doth appear.

li.	*li.*	*li.*
1	3.20713	136.775

```
      3.20713
      136.775
   -----------
      1603565
     2244991
    2244991
   1924278
  962139
 320713
 -------------
 438.65520575
      Or
438 li. 13 s. 1 d. q.
```

The

The Second TABLE ſheweth, what one pound will amount unto, being forborn any number of Years under 31, at 6 *per cent*. Intereſt upon Intereſt, the *Annuity* being to be paid Yearly.

Y.	6 *per cent.*	*Y.*	6 *per cent.*	*Y.*	6 *per cent.*
1	1,00000	11	14,97164	21	39,99272
2	2,06000	12	16,86994	22	43,39228
3	3,18360	13	18,88213	23	46,99582
4	4,37461	14	21,01506	24	50,81557
5	5,63709	15	23,27596	25	54,86451
6	6,97531	16	25,67252	26	59,15638
7	8,39383	17	28,21287	27	63,70576
8	9,89746	18	30,90565	28	68,52810
9	11,49131	19	33,75999	29	73,63979
10	13,18079	20	36,78559	30	79,05818

The uſe of this Table.

What will an Annuity of 20 li. *payable Yearly, be augmented unto in* 12 *Years, being all that time forborn, accounting intereſt upon Intereſt at* 6 per cent. per annum.

 Look

Look in the first column of the Table for 12 years, and right against it in the next column you shall find 16, 86994, which shews that 1 *li.* Annuity, payable yearly, being forborn 12 years, will amount unto 16, 86994, wherefore say by the Rule of Three Direct.

If 1 *pound Annuity forborn* 12 *years, give* 16.86994; *what shall an Annuity of* 20 *pound a year give, being forborn the same term of* 12 *years?* Answer, 337 *li.* 7 *s.* 11 *d.* 3 *q. fere*, as in the operation doth appear.

——————16.86994———— —— 20

20

337.39880

or

337 *l.* 7 *s.* 11 *d.* 3 *q. fere*

The

The Third TABLE sheweth, what one pound being forborn any number of Years under 31, is worth in ready Money, rebating yearly, after the rate of *6 per cent.* Interest upon Interest.

Y.	6 *per cent.*	Y.	6 *per cent.*	Y.	6 *per cent.*
1	,943396	11	,526787	21	,294155
2	,809996	12	,496989	22	,277505
3	,839619	13	,468839	23	,261797
4	,792093	14	,442300	24	,246978
5	,747258	15	,417263	25	,232998
6	,704960	16	,393646	26	,219810
7	,665057	17	,370364	27	,207367
8	,627412	18	,351343	28	,195630
9	,591898	19	,330512	29	,184536
10	,558394	20	,3[illegible]804	30	,174110

The making of the Table

As——106——100——.943396——889996.

and again,

As ——106——100——.889996——839619

Et sic ad infinitum.

If

If 356 li. *be payable at the end of* 7 *years, what is it worth in ready mony, discounting or rebating after the rate of* 6 per cent. *interest upon interest.*

Look in the Table for 7 years, and against it you shall find 665057, being the ready money which 1 *li.* is worth payable at 7 years end, wherefore say by the Rule of Three.

If 1 li. *in years rebate or decrease to* .665057, *to what will* 356 li. *rebate or decrease in the same?* Answer *to* 170 *li.* 5 *s.* 1 *d.* at by the operation doth appear.

1 ———— .665057 ————
256

3990342
3325285
1330114

170.254592
or
170 *l.* 5 *s.* 1 *d.*

The

The Fourth TABLE, ſheweth the preſent worth of one pound Annuity, to continue any number of Years under 31, and payable yearly after the rate of *6 per cent.* Intereſt upon Intereſt.

Y.	*6 per oent.*	*Y.*	*6 per cent.*	*Y.*	*6 per cent.*
1	0,94337	11	7,88687	21	11,76305
2	1,83339	12	8 38384	22	12,04158
3	2,67301	13	8,85268	23	12,30337
4	3,46510	14	9,29498	24	1[illegible],55035
5	4,21236	15	9,71224	25	12,78335
6	4,9[illegible]732	16	10,10589	26	13,00316
7	5,58 38	17	10 47725	27	13,01053
8	6,20979	18	10,82-60	[illegible]8	13,40616
9	6,80169	19	11,15811	29	13,59071
10	7,36008	20	11,46902	30	13,76482

The uſe of this Table

What is the preſent Rent or Annuity of 25 *pound* per annum *worth payable yearly, for* 22 *years, accounting Intereſt upon Intereſt at* 6 per centum.

Look

Look in the Table for 21 years, and right against it you shall find 11. 76407, which is the present worth of one pound Annuity for 21 years, wherefore say by the Rule of Three.

If an Annuity of 1 li. per annum *for* 21 *years be worth* 11. 76407 *ready money, what is an Annuity of* 25 li. per annum *worth in ready money for the same*? Answer 293 *li.* 2 *s.* 0 *d.* 1 *q.* as by the operation following doth appear.

1 ——— 11. 76407 ——— 25

25

5882035

2352814

294. 1075

or

294 *l.* 2 *s.* 0 *d.* 1 *q.*

The

The Fifth TABLE ſheweth what Annuity payable Yearly, one pound will purchaſe for any number of Years under 31, after the Rate of 6 *per cent.* compound Intereſt.

Y.	6 per cent.	Y.	6 per cent.	Y.	6 per cent
1	1.06000	11	.12679	21	.08500
2	.54363	12	.11926	22	.08304
3	.37411	13	.10297	23	.08127
4	.28859	14	.10758	24	.07967
5	.23739	15	.10296	25	.07822
6	.20336	16	.09895	26	.07690
7	.17913	17	.09544	27	.07569
8	.16103	18	.09235	28	.07459
9	.14702	19	.08962	29	.07357
10	.13586	20	.08718	30	.07264

The uſe of this Table.

What Annuity to begin perſently, and to continue 28 *years, payable at yearly payments, will* 640 li. *purchaſe, accounting compound intereſt after the rate of* 6 per cent.

Look

Look in your Table for 28 years, and right against it in the ne t column you shall find 07459, which shews that one pound ready money will purchase an Annuity worth. 07459, and to continue 28 years, wherefore say by the Rule of Three.

If one pound ready money will purchase an Annuity worth 07459 *to continue* 28 *years, what Annuity shall I purchase for the same time, paying* 640 li. *ready money?* Answer, 47 *li.* 14 *s*, 19 *d.* as by the operation doth appear.

1 —— .07459 —— 640

640

29836

44754

47·73760

Or

47 *l.* 14 *s.* 9 *d.*

FINIS.

INSTRUMENTAL
ARITHMETICK.
The THIRD PART.

TEACHING,

By a new Artifice (not heretofore Publiſhed, to my Knowledge, in any Language) The manner how to ſet down any *Decimal Fraction* required: Or a *Decimal Fraction* being given to find the value thereof in *Engliſh Money*, *Weight* or *Meaſure*; by inſpection only. By certain SCALES contrived, ſuitable to the *Coins*, *Weights* and *Meaſures* now uſed in *England*.

By *Will. Leybourn.*

LONDON,

Printed *Anno Dom.* 1684.

$\frac{15}{75}$ $\frac{16.3}{813}$ $\frac{2.10}{1.42}$ Page 115th Quest. 4 in Bart

Decimals

75 " ·813 — ·142

·142

1626

3252

813

75) ·115446 (1539

75

404

375

294

225

·696

675

·21

3)2780

·9360

4

3)7440

500 600 700 800 900 1000

11 12 13 14 15 16 17 18 19 20 21 22 23 24

...wo shillings being the Integer

500 600 700 800 900 1000

PW 3 6 9 12 15 18 21 PW

...peny weight being the Integer

500 600 700 800 900 1000

13 14 15 16 17 18 19 20 21 22 23 24 25 26 27 28

...uarter of an hundred being the Integer

500 600 700 800 900 1000

7 8 9 10 11 12 13 14 15 16

... or one pound being the Integer

9 1 2 3 4 5 6 7 8 9 10

5 6 7 8 9 10

2 3 4 5 6 7 8 9 10

...Walter Hayes Fecit.

500 600 700 800 900 1000

4 5 6 7 8

...ushels being the Integer

500 600 700 800 900 1000

20 25 30 35

...ons, or one Barrell being the Integer

500 600 700 800 900 1000

0 2 3 4

...e Yard being the Integer

500 600 700 800 900 1000

6 7 8 9 10 11 12

... or 12 Inches being the Integer

place this at the beginning of the third part to fold out.

Instrumental ARITHMETICK.

The Third Part.

THe *Arithmetick* of which we now come to treat, and which I call *Instrumental Arithmetick*, is not any new kind of *Arithmetick*, but is indeed the ſame with *Decimal Arithmetick* before taught; only, whereas in *Decimal Arithmetick* there were certain *Tables* made of *Money*, *Weight* and *Meaſure*, by help of which the Decimal of any Fraction of *Money*, *Weight* and *Meaſure*, might be ſet down (as it were) in whole numbers; here in this *Inſtrumental part*, we have contrived certain *Scales* of *Money*, *Weight* and *Meaſure* equally divided into the ſeveral Denominations into which the *Weights* and *Meaſures* for which they are contrived, may be equally divided. Unto all which Scales there is joyned a Scale of 100, 1000, or 10000, equal parts, according to the length of the Scale, ſo that by inſpection only you may readily and exactly without Addition (as in uſing the forementioned *Tables* you muſt neceſſarily do) ſet down the *Decimal*

Fraction of any part of *Mony*, *Weight* or *Measure*, with great celerity and exactneſs, if the Scale be any thing well divided, and be but of a reaſonable length.

Now the Scales which I have chiefly made choice of in this Work; as being of moſt uſe with Engliſh men; (though other Scales may be made for the *Coins*, *Weights* or *Meaſures* of any other Country as well, and upon the ſame ground) are chiefly theſe, *viz.*

1 Of Money.
2 Of Troy Weight.
3 Of Averdupois great Weight.
4 Of Averdupois little Weight.
5 Of Liquid Meaſures.
6 Of Dry Meaſures.
7 Of Long Meaſures.
8 Of Dozens.

Unto every of theſe Scales, is joyned another Scale of 100 or 1000 equal parts, theſe Scales are made to face one another, ſo that if you look upon any one Diviſion in the one, you ſhall alſo diſcern plainly what Diviſion or part of a Diviſion anſwereth thereunto in the other.

Theſe Scales being thus diſpoſed, as they may eaſily be upon any *Ruler* of *Silver*, *Braſs* or *Wood* ; but beſt of all upon a *ſquare Ruler*, made in form of a *Parallelepipedon*, will by inſpection only give you any Decimal fraction required without Addition, or (or on the contrary) reduce the fraction into the known parts of the Integer, by inſpection alſo, without *Subſtraction*.

Let thus much ſuffice for a general deſcription of what I mean by Scales, the particular deſcription of them will more plainly appear, when we treat of Numeration upon the Scales, unto which we ſhall now proceed. But firſt take a view of the Scales as they are here

here disposed, and as they may be set upon such a Ruler as I have here mentioned.

Numeration upon the Scales.

THe Scales here to be described are in number eight' as hath been already shewed, and as by the figure of them appears. *Now* Numeration *upon a* Scale, *is to find upon what part of the* Scale *any number upon the same* Scale *will fall.*

We will begin with the first, and so proceed till we have given an Example in every one,

1 The first Scale is of *English Money*, and is divided into 24 equal parts, which represent 24 pence or 2 shillings, these parts are numbred with Arithmetical figures, from the beginning thereof, by 1, 2, 3, 4, 5, &c. to 24, each division representing one peny, and the whole 24 divisions representeth 24 pence, or 2 shillings; So that where the figure 1 standeth, that part of the Scale representeth one peny, where the figure 2 standeth, it representeth two pence, where the figure 18 standeth, it representeth 18 pence, or one shilling six pence, and so of any other figure of the same Scale.

Then because there are four farthings contained in a peny, each of these pence (or divisions) is sub-divided into 4 other equal parts by short lines, every one of these representing one farthing, so is the whole Scale divided in all in 96 equal parts, which are the number of farthings contained in two shillings. Thus if you look into the Scale of mony for 8 pence 3 farthings, you shall find it at the letter *a*, which letter is here put onely for example sake. Also if you would find in the Scale the place of 18 *d.* half peny, you shall find it at *c*,

and thus may you find the placeofany number of pence and farthings under two shillings upon the Scale.

Unto this Scale of mony (as to all the rest of the Scales) there is joyned another Scale, which I shall alwayes hereafter call theScale of1000,theuse of which Scale is this. When you have found any number of pence or farthings upon the Scale of mony, you shall find upon the Scale of 1000, what parts of a thousand is the Decimal of those pence and farthings: Thus when in the Scale of mony you find at the letter *a* 8 pence 3 farthings,if you cast your eye directly cross to the Scale of 1000, you shall find 364 to stand directly against 8 *d.* 3 *q.* which 364 is the Decimal of 8 *d.* 3 *q.* Also, if you find upon the Scale of mony 18*d.* half peny, which is at the letter *c*, you shall find against itin the Scale of 1000, this number .771, which is the Decimal of 18 *d.* half peny. And in this manner may the Decimal of any number of penceor farthings under two shillings be most easily and exactly obtained.

Now on the contrary, suppose a Decimal Fraction were given, representing some part of *English Coin*; if you look in the Scale of 1000 for your number given right against it in the *Scale of Mony*,you shall find what number of pence and farthings is represented thereby. As for Example, Suppose .364 were a Decimal given, and it were required to find what part of Coin it doth represent. Look in the Scale of 1000 for the number 364,and right against it you shall find 8 pence 3 farthings. Also if .771 were a Decimal given, if you look in the Scale of 1000 for.771, you shall find against that number 18 pence 2 farthings. And thus of any other.

By what hath been already said, it may be easily discerned of what exceeding expedition these Scales thus disposed are of, for I dare affirm, that I will set down 2 (if not 3) numbers, by the Scale, as soon as one by

the

the Tables, and if the Scale be but of any reaſonable length, altogether with as much exactneſs, but if I ſhould vary an unite in the laſt place, in my eſtimation in the Scale of 1000, it is not any thing material.

I have been very tedious in ſhewing the uſe of theſe Scales to find the fraction-parts of *Money*; but the reaſon is, becauſe I intend to be the briefer in the reſt, for *Weight* and *Meaſure*; the manner of working (when the Diviſion of the Scale is known) being the ſame in all reſpects without the leaſt alteration.

2 The ſecond Scale is of *Troy Weight*, two peny weight being the Integer, which Scale is divided into 48 equal parts or diviſions, each of which diviſions contains one grain, and are numbers by Arithmetical figures at every three grains by 3,6,9,12,&c. to 24, and at the place where 24 ſhould ſtand, there ſtandeth P W, which ſignifieth one peny weight, or 24 grains, this P W, ſtandeth in the middle of the line. Then is the ſame Scale continued farther by Arithmetical figures. 3,6,9,12,&c. as before to 24, and there is written P W, again, repreſenting two peny weight, or 48 grains.

The Scale being thus divided, it is eaſie to find the place where any number of Grains under 48 ſhall be upon the Scale; As for example, if it be required to find where 8 peny weight ſhall fall, look upon the Scale of Troy weight, from the beginning thereof, and count the figures 3 and 6. then count alſo two of the ſmaller Diviſions, and that makes 8 grains, which you ſhall find to ſtand at the letter *d*. which is the place of 8 grains; Alſo if upon the Scale you would find the place of one peny weight 10 grains, you ſhall find it at the letter *e*, and ſo of any other number of grains under 48, or two peny weight.

But if you had a Decimal given, and would know what number of grains it repreſenteth, if you ſeek your

Decimal given in the Scale of 1000, right against it in the Scale of Troy weight, you shall find the number of grains represented thereby.

Example. Let. 167 be a Decimal fraction given, If you look in the Scale of 1000, for 167, right against it in the Scale of *Troy weight.* you shall find 8 peny weight.

Also if 708 were a Decimal given, if you seek 708 in the Scale of 1000, right against it you shall find 1 peny weight 10 grains.

3 The third scale is of *Averdupois great weight*, 28 pounds, or one quarter of an hundred, being the Integer, this Scale is numbred by 1, 2, 3, 4, &c. to 28, which 28 representeth 28 *l.* or a quarter of a hundred, and each of those is sub-divided into four small parts, each representing one quarter of a pound.

Now if you would know what is the Decimal of any number of pounds or quarters under 28, if you seek the number of pounds in the Scale of *Averdupois great weight*, right against it in the Scale of 1000, you shall find the Decimal thereof.

Thus if it were required to find the Decimal of 8 pound and an half, if you look upon the Scale for 8 pound and an half, you shall find it at the letter *g*, and right against it in the Scale of 1000 you shall find. 304, which is the Decimal of 8 pounds and an half.

On the contrary, suppose. 304 were a Decimal given, and it were required to find what part of *Averdupois great weight* were represented thereby, if you look in the Scale of 1000 for. 304, right against it in the Scale of *Averdupois great weight*, you shall find 8 pound and an half.

4 The fourth Scale is of *Averdupois little weight*, 16 ounces or one pound being the Integer; This Scale is first divided into 16 equal parts, and numbred by 1, 2, 3, 4. &c. to 16, each Division representing one Ounce.

Then

Then again, each of these ounces is sub-divided into 8 other smaller parts or divisions, each of which division representeth two Drams; but if your Scale be large enough, you may have each ounce divided into 16 equal parts or divisions, each division representing one Dram.

Now to find the Decimal belonging to any number of Ounces and Drams, repair to the Scale of *Averdupois little weight*, and on it the quantity of ounces and drams required, and right against it in the Scale of 1000, you shall have the Decimal thereof.

Thus if it were required to find the Decimal of 6 Ounces and 6 Drams, if you look this in the Scale of *Averdupois little weight*, you shall find it at the letter *h*, and right against it in the Scale of 1000, you shall find 398, which is the Decimal of 6 ounces and 6 drams.

But if .398 were a Decimal given, and it were required to find the value thereof in *Averdupois little weight*, if you look for .398 in the Scale of 1000, right against it in the other Scale you shall find 6 ounces 6 drams.

5 The fifth Scale is of *Dry Measures*, one *Quarter* or 8 *Bushels* being the Integer; This Scale is first divided into 8 equal parts, and numbred by 1,2,3, &c. to 8, each of which divisions representeth a Bushel, and each of those parts is again sub-divided, first into 4 equal parts or divisions, each representing one *Peck*, and then those again sub-divided into 4 other smaller parts, representing *Quarters*, *Halves*, and *Three Quarters* of a Peck.

Now if you would know the Decimal belonging to any number of Bushels (under 8 Bushels or one Quarter) Pecks and parts of a Peck, if you seek the number of Bushels, Pecks, and parts of a Peck in the Scale of *Dry Measures*, right against it in the Scale of 1000, you shall have the Decimal required.

As for example, if it were required to find the Decimal belonging to 5 Bushels 2 Pecks, and half a Peck, if you look into the Scale of Dry Measures you shall find 5 Bushels, 2 Pecks, and an half to stand at the letter *k*, and right against it in the Scale of 1000. you shall find .702, which is the Decimal answering to 5 bushels, 2 Pecks and a half.

But if .702, were a Decimal given, and it were required to find what number of Bushels, pecks and parts of a peck it representeth, if you look in the Scale of 1000 for .702, you shall find against it in the Scale of *Dry Measures*, 5 bushels, 2 Pecks and a half.

6 The sixth Scale is of *Liquid Measures*, the Integer being 36 Gallons or one Barrel, this Scale is divided first into 36 equal parts or divisions, and numbred by 5, 10, 15, &c. to 36, then every of these divisions, is again sub-divided into 4 other small divisions, each representing a quart, but (if the Scale be large enough) you may sub-divide each Gallon into 8 parts so will every part represent one piont.

Now to find the Decimal belonging to any number of *Gallons* (under 36 Gallons or one Barrel) quarts or pints, repair to the Scale of *Liquid Measures* and seek there upon the Scale, the number of gallons, quarts or pints, and against it in the Scale of 1000, you shall find the Decimal thereunto belonging.

So if it were required to find a Decimal representing 10 gallons and two quarts, or 4 pints, which is all one if you seek in the Scale of Liquid measures for 10 gallons, 2 quarts, you shall find it at the letter *m*, against which in the Scale of 1000, you shall find .292, which is the decimal of 10 gallons 4 pints.

Likewise if .292 were a decimal given, and it were required to find what number of gallons, quarts or pints were represented thereby, if you look in the Scale of 1000 for .292, right against it in the Scale of

Liquid

Liquid Measures, you shall find 10 gallons, 2 quarts, or 4 pints.

7 The seventh Scale is of *Long Measure's*, the Integer being *Yards* or *Ells*, this Scale is divided into 4 equal parts, and numbred by 1,2,3,4, representing 1 quarter, 2 quarters, 3 quarters, or 4 quarters of a Yard or, Ell, these are again sub-divided, first into 4 other equal parts, representing *Nailes*, and those may be again sub-divided at pleasure if need be.

Now if you would know what decimal belongeth to any number of *quarters or nailes of a yard or Ell*, if you seek the number of quarters and nailes in the Scale of *Long Measure*, the Scale of 1000 will give you the decimal thereof.

Thus if it be required to find the decimal belonging to 1 quarter or 3 nailes, if you seek this in the Scale of *Long Measure*, you shall find it stand at the letter *o*, against which in the Scale of 1000 you shall find .437, which is the Decimal answering to 1 quarter and 3 nails of a *Yard or Ell*.

Also if .437 were a Decimal given, and it were required to find what quantity of *yards* or *ells* were represented thereby, if you look in the Scale of 1000 for .437, you shall in the Scale of *Long Measure* find against it one quarter and 3 nails.

8 The eight and last Scale is of *representative inches*, the whole Scale being divided into 12 equal parts, and numbred by 1, 2, 3, &c, to 12. and those parts are again sub-divided into halves, quarters, and half quarters as Carpenters-Rules are usually divided.

Unto this Scale (as unto all the other) there is joyned a Scale of 1000, this Scale will readily discover what is the Decimal belonging to any number of Inches, halves or quarters, and the use is the same with the Scales before mentioned.

Thus I have given you a brief description of these

Scales

Scales, and the uses of them, and do now suppose my Reader to be perfectly acquainted with the way of numbring or counting upon them; wherefore I intend onely to give you a question or two in the most usual Rules of Arithmetick, and so conclude; for Decimal Arithmetick being already sufficiently explained, I shall not need to repeat the Rules (or the manner of working them) again, but give you one example, by which the exactness and expedition of these Scales may the more evidently appear, for when we work by Scales, it is supposed that we do not use *Vulgar*, but *Decimal Arithmetick* and addition, Substraction, Multiplication, Division, the Rule of Three, and indeed, all the other Rules of Arithmetick, are to be performed, as is before taught, the Scale serving only to avoid Reduction.

ADDITION.

WHat *Addition* is, and the manner of working of it hath been already taught, both in the first and second Parts, we will now come to an Example, which let be in *Addition of English Coin*, and let the sums to be added be 36 *l.* 8 *s.* 8 *d.* 29 *l.* 0 *s.* 2 *d.* 31 *l.* 16 *s.* 9 *d.* and 6 *l.* 2 *s.* 5 *d.*

Frist, set down 36 *l.* 29 *l.* 31 *l.* and 6 *l.* one under another, in such order as you see herein the margine, drawing a line by the side of them as you see here done, and also a line under them.

36
29
31
6

This done, seeing that your first number to be set down to 36 *l.* is 8 *s*, 8 *d.* you must for the 8 *s.* (because two shil-

lings, which we called a Decade, or the tenth part of a pound, is made the Integer, in the Scale of money) ſet down 4, which is done by memory, and after it make a comma; Then your next number to be ſet by 29 *l.* being 0 *s.* 2 *d.* for the 0 *s.* ſet down a Cypher; Thirdly, for your number to be ſet by 31 *l.* being 16 *s.* 9 *d.* for the 16 *s.* ſet down 8 Decades, with a comma after it, and laſtly, the number to be ſet by 6 *l.* being 2 *s.* 5 *d.* for the 2 *s.* I ſet down 1 Decade with a comma after it, and then will your work ſtand, as here you ſee.

36	4,
29	0,
31	8,
6	1,
—	——

Then take your Scale in hand, and ſeeing your firſt number of pence are 8 *d.* look in your Scale of money for 8 *d.* and againſt it in the Scale of 1000, you ſhall find 333, which ſet 36 *l.* 8*s.* behind the comma, then your next number of pence being 2 *d.* look in your ſcale for 2 *d.* and againſt it in the ſcale of 1000, you ſhall find 083, which ſet to 29 *l.* 0 *s.* behind the comma. Then your third number of pence being 9 *d.* look in your Scale for 9 *d.* and againſt it in the Scale of 1000, you ſhall find 376, which ſet to 31 *l.* 16 *s.* and laſtly, your laſt number of pence being 5 *d.* look in your Scale for 5*d.* and againſt it you ſhall find 208, which ſet to 6*l.* 2 *s.* and then will your whole work ſtand, as here you ſee.

36	4,333
29	0,083
31	8,376
6	1,208
—	——

Your ſums being thus ſet down, which is done with more facility than you can imagine, tell you make trial and be ſomething perfect therein, you muſt then add all the numbers together, as in Addition of Decimals and you ſhall find the ſum of them to be 103|4,000, Now to know this in mony, is as eaſie as it was to ſet ſeveral ſums down, for the figures 103, which ſtand behind

behind the down right line, are 103 *l.* and the figure 4 which ſtands between the down right line and comma are, 4 decades or 8 *s.* and being the reſt to the right hand are all Cyphers, they ſignifie neither pence nor farthings, ſo is the total of this Addition 103 *l* 8 *s* o *d.*

li:	
36	4,333
29	0,083
31	8,376
6	1,208
103	4,000

That the manner of working may appear more plain, I will give you another ſhort Example as difficult as I can invent, which I performed by a Scale of wood but of 8 inches long. Let the ſums to be added together be theſe following;

li.	*s.*	*d.*	*q.*
332	17	4	1
159	6	8	1
217	5	3	3
709	9	4	1

First ſet down your ſeveral ſums of pounds one under another as before, and draw a line by the ſide of them, and another under them. So will they ſtand as here you ſee.

332	
159	
217	

1 Your ſums of pounds being thus orderly placed and lines drawn, repair to your Scale, and ſeeing your firſt number of ſhillings, pence and farthings is 17 *s.* 4 *d.* 1 *q.* for your 17 *s.* ſet down 8 Decades which is 16 *s.* with a comma after it, then will there reſt to be ſet down 1 *s* 4 *d.* 1 *q.* or 16 *d.* 1 *q.* which if you ſeek in your ſcale of mony, you ſhall find to ſtand againſt it in the ſcale of 1000 this number 677, which is the Decimal of 1 *s.* 4 *d* 1 *q.*

2 Your ſecond number of ſhillings, pence and farthings is 6 *s* 8 *d.* 1 *q.* for your 6 *s.* ſet down 3 decades, which

which is 6 *s.* and then there will remain 8 *d.* 1 *q.* which if you ſeek in your ſcale of mony, you ſhall find to ſtand againſt it in the ſcale of 1000, this number.344, which is the Decimel of 8 *d.* 1 *q.*

3 Your third number of ſhillings, pence, and farthings, is 5 *s.* 3 *d.* 3 *q.* for your 5 s. ſet down 2 decades, which is 4 *s.* with a comma after it, then will there reſt to be ſet down 1 *s.* 3 *d.* 3 *q.* or 15 *d.* 3 *q.* which if you ſeek in your ſcale of money, you ſhall find to ſtand againſt it in the ſcale of 1000, *l.* this number .656, which is the Decimal of 15 *d.* 3 *q.* or 1 *s.* 3 *d.* 3 *q.* and the three ſums to be added together will ſtand as here you ſee.

332	8,677
159	3,344
217	2,656
709	4,677

Theſe ſums being added together according to the Rule for Addition of Decimals, you ſhall find the ſum of them to be 709'4,677 now to know what this is in mony, take notice that the 709 which ſtands to the left hand of the down right line are 709 pounds, and the figure 4. which ſtands between the down right line and the comma, are 4 decades or 8 *s.* but (becauſe the firſt figure next after the comma is above 5, *viz.*6) you muſt add 1 *s.* to the 4 decades, making them 9 *s*, then will there remain 177, wherefore if you look in the ſcale of 1000 for 177, you ſhall find againſt it in the ſcale of Mony 4 *d.* 1 *q.* So is the whole ſum of this Addition 709 *l.* 9 s. 4 *d.* 1*q.* as by the preceeding work doth appear.

¶Here note, that when you had ſet down your 709 *l.* 4 decades or 8 *s.* there remained beyond the comma 677, which if you had ſought in your ſcale of 1000 you ſhould have found againſt it in the ſcale of Money 15 *d.* 1 *q.* or 1 *s.* 3 *d.* 1 *q.* (which is all one) as before; for it appeareth plainly by the Scale, that 500 in the line of 1000 is equal to one ſhilling.

I might proceed farther in giving you Examples in *Weight* and *Measure* answerable to the *Scales*, but that would only make the Reader spend his time to little purpose; for being before acquainted with *Decimal Arithmetick*, and (as by this time I suppose he is) with *Numeration* upon the *Scales*, he cannot be deficient in the applying of the other *Scales* of *Weight and Measure* to the same purpose for which they were contrived, I having so largely exemplified the use of the *Scale* of *Money*.

SUBSTRACTION.

S*Ubstraction* (as hath been before said) is the taking one or more smaller sums out of one greater; I shall only give you an Example or two, as I have taken the numbers from a Scale.

Example.

Delivered to a Gold-smith of old Plate 297 *ounces*, 13 *peny-weight*, 19 *grains.*

Received of the same Gold-smith first 165 *ounces*, 11 *peny-weight and* 7 *grains, and after that received of the same Gold-smith* 32 *ounces* 19 *peny-weight and* 32 *grains, what Plate remaines in the Gold-smiths hands?*

Take your Numbers out of the Scale of Troy-weight, and set them down as you here see.

ounces

	ounces
Delivered	297|6,896
Received first	1655,646
Received more	329,979
Received in all	198|5,625
Rests in the Gold-smiths hands	99|1,271

or

ounces	*peny-w.*	*gr.*
99	2	13

Then add the several weights of Plate received, together, and they make 198|5,625, or 198 ounces 11 peny-weight, 6 grains, which if you substract from 297|6,896, or 257 ounces, 13 peny-weight, 19 grains, which was the quantity of Plate delivered, there will remain 99|1,271, or 99 ounces, 2 peny-weight, 13 grains, and so much Plate is still in the *Gold-smiths* hand. And let thus much suffice for *Substraction.*

Now we should proceed to *Multiplication and Division*, but when the numbers are taken from the Scale and set down, the manner of working doth not at all differ from *Multiplication* and *Division* of *Decimals* before taught. But before I teach you how to Multiply and Divide Instrumentally, I shall shew you,

The

The farther use of the Decimal Scales, *and how by them to find the* Square *or* Cube Root *of any* Number. *And*

2. *Any* Root *being given, to find the* Square *or the* Cube number *of that Root.*

And both these by inspection only, without the help of either *Pen*, *Compasses* or any other *Motion*.

For the effecting hereof, there is now inserted, among the forementioned *Decimal Scales* of *Money*, *Weight*, *Measure*, &c. namely, between the Scales of *Averdupois little weight*, and that of *Dry Measures*, Two other *Scales*, one having written at the beginning thereof the word *Square*, and to the other there is added the word *Cube*, and between them, there is a third line, which hath written upon it, the word *Root*. And by these three *Scales* thus united, the *Square* and *Cube Roots* of any number may be extracted by inspection only. For,

If you find any number whose *Square Root* you require, in the *Line* or *Scale* of *Squares*, right against it, in the *Line* of *Roots*, you shall have the *Square Root* of that number. Thus,

If the number 64 *were given, and it were required to find the* Square Root *thereof.*

Find the given number 64, upon the *Line* or *Scale* of *Squares* (which you may do at the letter *a*) and right against it, in the Scale of Roots stands the figure 8, which shews, that 8 is the *Square* Root of 64. And in the same manner you may find the *Square Rooot* of any other number.

for

For, against	in the Line of *Squares*, you shall find	in the *Scale* of *Roots*, which is the *Square Root* thereof,
81		9
64		8
49		7
36		6
25		5
16		4
9		3
4		2

In like manner,

If the number 64 *were given, and it were required to find the* Cube Root *thereof.*

Find the given number 64 in the *Scale* of *Cubes*, (which you may do, by counting the ſame number between the ſecond and third figures of 1 upon the Scale, at the letter *b*) and right againſt it, in the *Line* or *Scale* of *Roots*, ſtands the figure 4, which ſhews, that 4 is the *Cube Root* of 64. And in the ſame manner you may find the *Cube Root* of any other number.

For, against	in the Scale of Cubes, you ſhall find	in the Scale of Roots, which is the Cube Root thereof.
729		9
512		8
343		7
216		6
125		5
64		4
27		3
8		2

And by this *Artifice*, not only the *Roots* of direct *Square* and *Cube* numbers may be found, but in numbers that be not directly *Square* or *Cube*, the *Fraction* part of the *Root* is nearly diſcovered alſo.

I have hitherto given you Examples in ſuch *Square* and *Cube* numbers, as are common and familiar, and that any man may compute almoſt by memory; but by

these the *demonstration* of the Artifice is discovered, the *Lines* of *Squares*, and *Cubes* being only *Square* and *Cube* numbers transferred to Lines. And now let us proceed to greater Numbers. And

1. *For the* SQUARE ROOT.

In the *Extraction* of a *Square Root*, it is usual to set a *Prick* under the first figure, the third, the fifth, the seventh, and so forwards, beginning from the right hand towards the left. And as many pricks as fall under the *Square number* given, of so many figures will the *Root* of that number consist: So that if the number given be less than 100, the *Root* shall be onely of one figure, if less than 10000, it shall be but two figures, if less than 1000000, it shall be three figures, and so forward.

Hence it is, That the *Line* or *Scale* of *Squares*, is divided first into 100 parts, and if the number given be greater than 100, the first division (which is the place where the first figure of 1 standeth, and which before did signifie *One*) must now signifie 100, and the whole line shall be 10000. If farther, the number be greater than 10000, you must count or esteem the first figure of 1 to be 10000, and then will the wole line be 1000000 parts; and if this be too little to express the number given, you may proceed farther, and call the first 1 1000000, and so increase the Line 100 times more; but this is sufficient.

Thus when any Square number is given, if you set it down, and prick it, (or imagine it so to be) If the last prick to the left hand shall fall under the last figure, (which will always be when the figures in the given number be odd) you must find all such numbers upon the line, between the two figures of 1.——— ——But if the

the laſt prick ſhall fall under the laſt figure but one of the given number, (which it will always do, when the figures of the number given are even) then you muſt find the number given in the line of *Square*, between the ſecond figure of 1 and 10 at the end of the line.

This being conſidered, find the number given, whoſe Square Root is required, in its proper place upon the Line of Squares, and againſt it in the Scale of Roots you ſhall have its Square Root deſired. Thus,

The *Square Root* of	36 360 3600 36000	will be found to be	6 19 60 189.

And in this manner may the neareſt Root of any number not exactly Square be obtained. For

The nearſt *Root* of	725 7250 72500 725000	will be found to be about	27 85 269 851

And thus on the contrary, a Number may be *Squared*, as may partly appear by what hath been before delivered ; for if you find the *Root* in the *Scale* of Roots, you have its *Square* in the line of *Squares* and, ſo

Againſt	27 85 269 851	in the Scale of *Roots*, you ſhall find	725 7250 72500 725000	the Square thereof.

Thus much for the Square Root. Now

2. *For The* Cube Root.

In the *Extraction* of the *Cube Root*, it is usual to set Pricks under the first figure, the fourth, the seventh, the tenth, and so on, pricking always the 3 figure from the right hand towards the left. And as many pricks as fall under the *Cubick* number given, so many figures shall be in the *Root*. So that if the number given be less than 1000, the Root shall consist only of one figure; If less than 1000000, it shall be only of two figures; if it be above 1000000, and less than 1000000000, it will be only three figures.

Hence it is, That the *Line of Cubes* is divided first into 1000 parts; And if the number given be greater than 1000, the first figure of 1 (which before did signifie only *One*) must now signifie 1000, and the second figures of 1, must now signifie 10000, and the third 1, must signifie 100000, and the whole line must be estemed to be 1000000. Farther, If the number given be greater than 1000000, the first 1, must signifie 1000000, the second 10000000, the third 100000000, and the whole line 1000000000 parts. And if these be yet too little, you may proceed farther; but let these suffice.

Thus when any Cube number is given, if you set it down, and prick it; If the last prick to the left hand shall fall under the last figure, the number shall be reckoned between the first and second figures of 1, and the first figure of the Root shall be alwayes either 1 or 2 ——— If the last prick fall under the last figure but one, the number given must be rekoned between the second and third figures of 1, and thefirst figures of the Root shall alwayes be either 2, 3, or 4. ——— But if the

the laſt prick ſhall fall under the laſt figure but two then the number given muſt be reckoned betwen the third figure of 1, and 10 at the end of the line.

This being conſidered, find the number given, whoſe *Cube Root* is deſired, in the proper Section upon the *Line* or *Scale* of *Cubes*, and right againſt it in the *Scale* of *Roots*, you ſhall have its *Cube Root* deſired. Thus

849	The Cube Root of	8490000	will be found to be about	204
849		84900000		439
849		849000000		947

And the like of any other.

On the Contrary, a number may be *Cubed*; for if you find the number in the line of *Roots* you ſhall have the *Cube* thereof right againſt it in the Scale of *Cubes*, giving the true denomination to the Cube, according as the part of the Line againſt which the *Root* ſtandeth doth require.

Thus have you by this Inſtrumental way of working, theſe things, which in the ordinary courſe are moſt hard and intricate, rendred very familiar and eaſie; And although at all times you do not make uſe of them, yet they are ready helps to confirm you in your working without the tedious way of proving by Reverſe working. And here by the way take this

Advertiſement.

The forementioned *Decimal Scale*, and theſe *Lines* of the *Square* and *Cube Roots*, as alſo *Nepairs Bones*, whoſe uſe followeth, are made in Silver, Braſs or wood, by Mr. *Walter Hayes* at the *Croſs-Daggers* in *Moor-Fields*, who alſo maketh all ſorts of *Mathematical Inſtruments*.

Section I.

Concerning the

Fabrick and Inscription

Of these

RODS.

IN the foregoing Argument I told you, That the Author and Inventer of this kind of Instument, of which I intend to shew the Use, called it *RABDOLOGIA*, and the word he thus defines:

RABDOLOGIA est Ars Computandi per Virgulas numeratrices. That is, *RABDOLOGIA* is the Art of Counting by Numbering Rods.

I. *Of the Fabrick of these Rods, according to the Inventor's Description of them.*

These Rods may be made either of Silver, *Brass*, *Box*, *Ebony*, or *Ivory*, of which last substance I suppose they were at first made, for that they are (for the most part) by all that know or use them, called *NEPAIRS-BONES*.

But let the matter of which they are made be what it will, their form (according to this description) is exactly a square Parallelepipedon, the length being about three Inches, and the breadth of them about One tenth

tenth part of the length. But the length of these Rods are not confined to three Inches, but let the length be what it will, the breadth must be a tenth part thereof, but that may be accounted a competent breath that is capable of receiving of two numerical Figures, for there is never upon one Rod required more to be set on the breadth thereof.

The breadth of these Rods being exactly one tenth part of the length thereof, when 10 of these are laid together they do exactly make a Geometrical square, and if 20 of them be tabulated or laid together, they will make aright-angled Parallelogram, whose length is double to its breadth. If 30 be tabulated, the Figure will be still a Parallelogram, whose length will be three times the breadth, and so if 40, four times the length, & *sic* 650.

The Rods being thus prepared of exact length and breadth, let each of them be divided into 10 equal parts, with this *Proviso*, that Nine of the Ten parts stand in the middle of each Rod, and the other tenth part must be divided into two parts, half whereof must be set at the one end, and the other half at the other end of the same Rod. Then from side to side draw right Lines from division to division, so is your Rod divided into Squares on every side thereof. Lastly, from corner to corner every of these Squares draw a Diagonal Line, and that will divide every Square into two Triangles. The Rods being thus prepared and lined, first into squares, and then into Triangles, they are then fit to be numbred.

The figure 1. at the beginning of the Book shews the Form of one of these Rods lined as it ought to be.

SECT. II.

How theſe Rods are to be Numbred.

IN the two half Squares which are at the ends of each Rod on every ſide, there are ſet one ſingle Figure, on each ſide of every Rod one, in the diviſion at the end thereof, ſo every Rod containing four ſides, Ten Rods will contain 40 ſides, and ſo conſequently will have 40 ſingle Figures at the ends of every of them; that is, there will be upon the Ten Rods amongſt them, four Figures of each kind, that is, four Ones, 1111. four twos, 2222. four threes, 3333. four fours, 4444. four fives, 5555. four ſixes, 6666. four ſevens, 7777. four eights, 8888. four, nines, 9999. four Cyphers, 0000.

And here it is to be noted, That what Figure ſoever it be that ſtandeth at the top of the Rod alone; the Figure that ſtandeth alone on the other ſide of the ſame Rod, maketh that figure up the number 6. As for Example, If 1 ſtand on one ſide, 8 will ſtand on the otherſide, ſo 2 and 7, &c. As in this Table, where,

	1		8	
	2		7	
	3	ſtands alone	6	ſtandeth on
	4	at the top of	5	the other
If	5	any ſide of	4	ſide of the
	6	any of the	3	ſame Rod.
	7	Rods , then	2	
	8		1	
	9		0	

This

This also is to be observed in the figuring of every Rod, that what figure soever standeth alone at the top or superiour part of the Rod, the figure or figures that stand in the two Triangles next underneath it, is double to the figure which standeth at the top. And the figures which stand in the next two Triangles below, that is three times as much as the figure above. And that in the fourth place, or Triangles, is four times as much as the figure above, *&c.* till you come to the lowest Triangles in that Rod, and then the figure or figures that stand in those Triangles are nine times as much as the figure which standeth at the top of the Rod.

So if a Rod have 4 at the top thereof, in the two Triangles which are just and next under it, hath 8 in them, which is double to 4: in the next two Triangles below there is 12, which is treble to 4; in the two Triangles below them is 1 and 6, which together make 16, which is four times as much as the 4 at the top; the next Triangles have in them 20, which is five times as much; the next 24, which is six times as much; the sixth hath 28, which is seven times as much; the next Triangles have in them 32, which is eight times 4; the last hath 36, which is nine times as much. All which is visible by the Figure 2 at the beginning of the Book.

And is evident enough by this little Table following, which is the Table of Multiplication, commonly called *Pythagoras* his Table.

Figures

Figures at the top of each Rod.

The Figures in the		0	1	2	3	4	5	6	7	8	9	Square, which are		Times as much as the Figure at the top.
	First	0	1	2	3	4	5	6	7	8	9		1	
	Second	0	2	4	6	8	10	12	14	16	18		2	
	Third	0	3	6	9	12	15	18	21	24	27		3	
	Fourth	0	4	8	12	16	20	24	28	32	36		4	
	Fifth	0	5	10	15	20	25	30	35	40	45		5	
	Sixth	0	6	12	18	24	30	36	42	48	54		6	
	Seventh	0	7	14	21	28	35	42	49	56	63		7	
	Eighth	0	8	16	24	32	40	48	56	64	72		8	
	Ninth	0	9	18	27	36	45	54	63	72	81		9	

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to fold out.

Fig: 1.

Fig: 2.

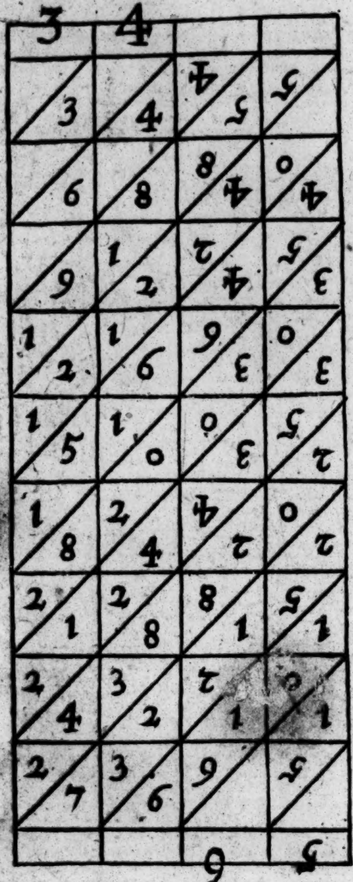


Fig: 3.

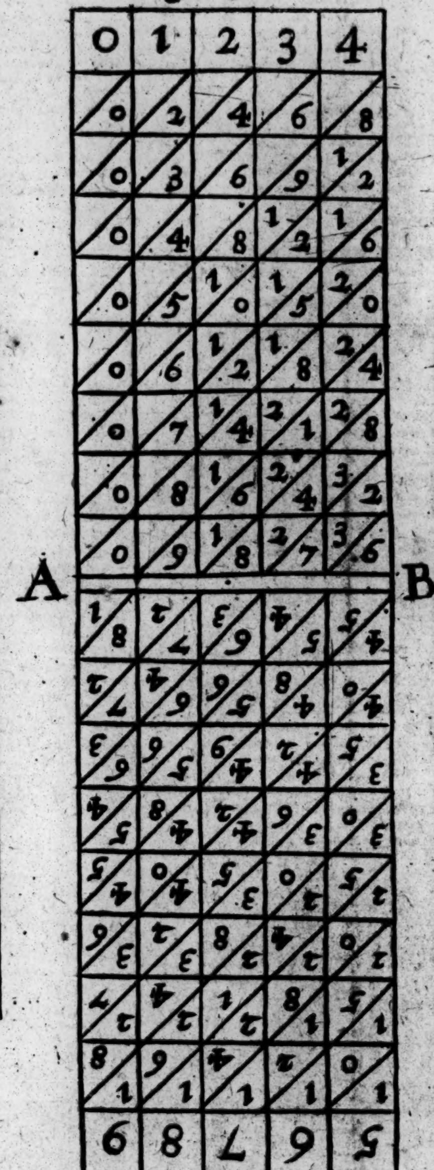
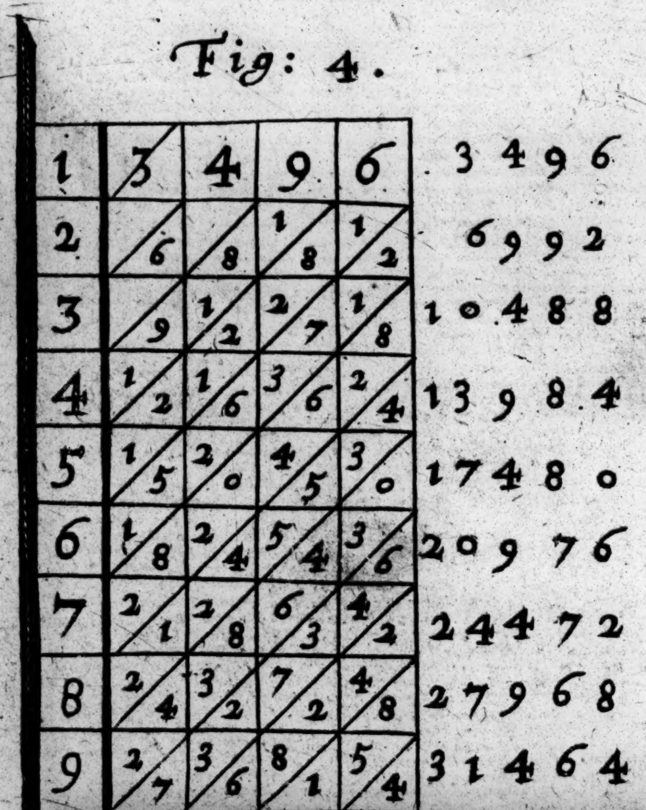


Fig: 4.



The Tabulat with Rods

Thus have you the *Fabrick*, *Inscription and number-ing* of these Rods, according to the Inventors contrivance of them : He makes mention of Ten of them, and hath in his Book set the figure of the said Ten, of one of which Ten I have given you a Scheme at the beginning of the Book, which is *Figure* 2. I will now proceed to give you the description of these Rods in another more commodious form.

SECT. III.

A Description of these Rods according to their best and latest Contrivance.

THe Description which I shall here give of these Rods, varies not at all from that before delivered in the matter of which they are made, for these may be made either in Silver, Brass, wood, Ivory, *&c.* Neither do they differ in their dividing, nor yet in their numbering : Only, whereas my Lord *Nepair* maketh them square, each Rod to contain four sides, these are made flat, consisting each Rod but of two sides, and contain in length about 2 Inches.$\frac{2}{10}$
and in breadth $\frac{1}{4}$ of an Inch.
and in thickness $\frac{1}{12}$ of an Inch.

One set of these Rods consisteth of five pieces, and therefore hath but ten Faces or sides, whereas those of the Lord *Nepair*'s consisted of 40 Plaines or sides.

Upon one of these five pieces (a Figure whereof is at the beginning of the Book, noted with *Figure* 3) you have a Cypher at the head of the first piece, and 9 at the bottom thereof. Upon the second of them you have

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to fold out.

Fig: 1.

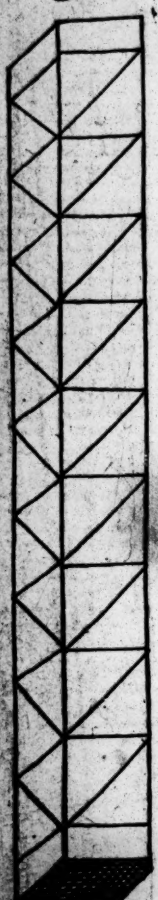


Fig: 2.

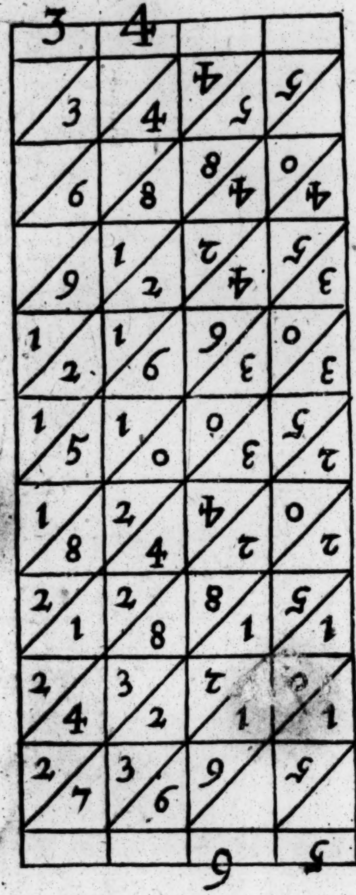


Fig: 3.

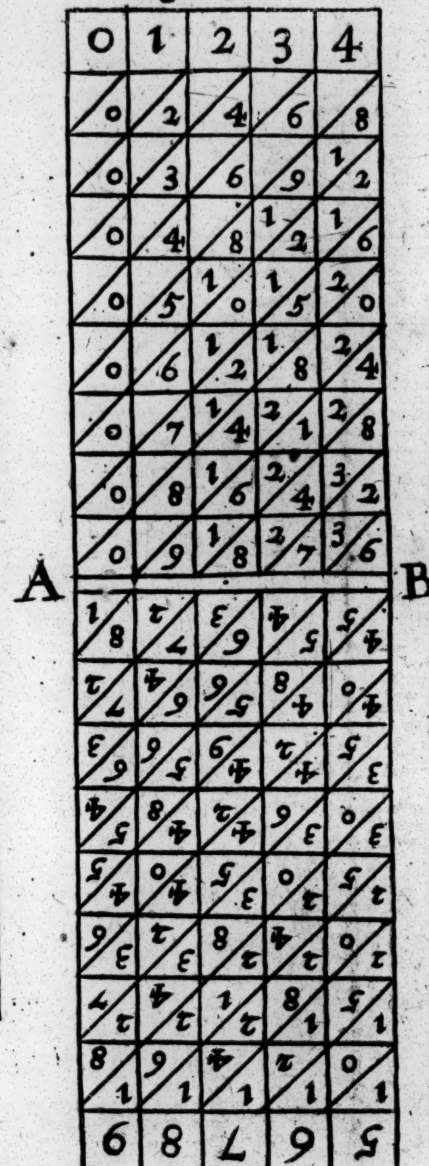
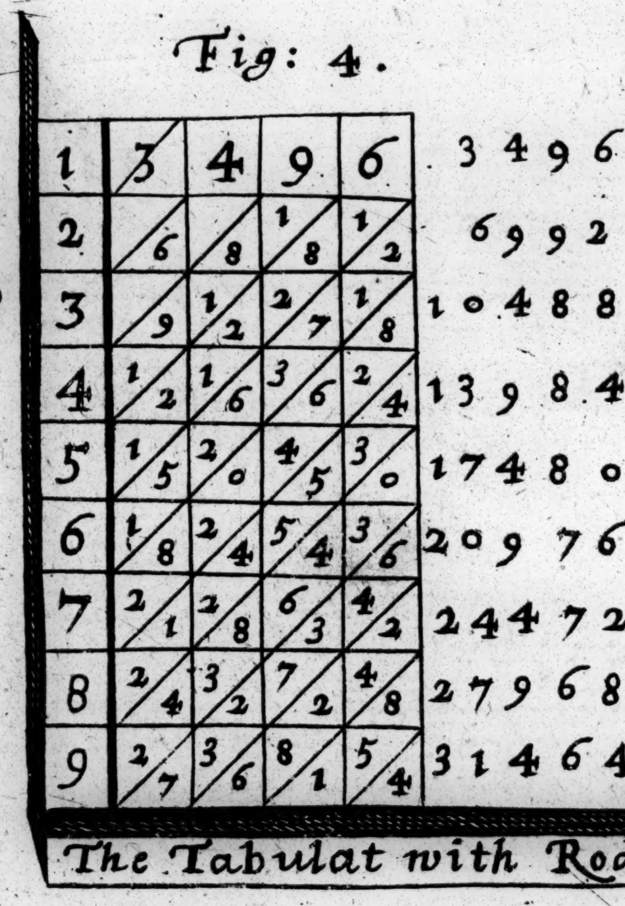


Fig: 4.



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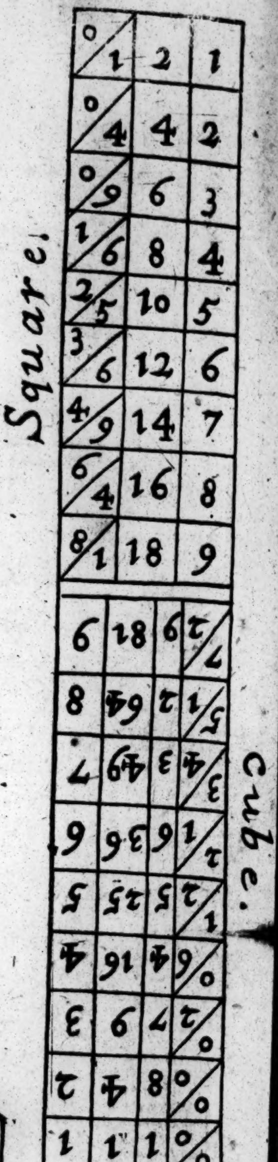

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and in thickness $\frac{1}{12}$ of an Inch.

One set of these Rods consisteth of five pieces, and therefore hath but ten Faces or sides, whereas those of the Lord *Nepair*'s consisted of 40 Plaines or sides.

Upon one of these five pieces (a Figure whereof is at the beginning of the Book, noted with *Figure* 3) you have a Cypher at the head of the first piece, and 9 at the bottom thereof. Upon the second of them you have

have 1 at the head, and 8 at the bottom: upon the third you have 2 at the head, and 7 at the bottom, upon the fourth 3 at top, and 6 at bottom; and upon the fifth you have 4 at the top, and 5 at the bottom. Every of the two Figures at the top and bottom together make 9; as 0 and 9 is 9, 1 and 8, 2 and 7, 3 and 6, 4 and 5. And here obſerve, that the Figures 9 8 7 6 5, which ſtand at the bottom of the Scheme, ſtand with their heels upwards in this manner, 6 8 L 9 S, and ſo do all the other figures under them, till you come to the double Line which is in the middle of the Scheme, noted with *A* and *B*, at which Line, if the Scheme were cut into two pieces, and folded or paſted on the back-ſide of the other half, ſo that the 9 at the bottom were placed upon the Cypher at the top, and ſo 8 upon 1, 7 upon 2, 6 upon 3, and 5 upon 4, and then the Scheme cut again into five little ſlippets by the down-right Lines; theſe five ſlippets would exactly repreſent one ſet of theſe Rods, for upon one ſide of one of theſe pieces, you ſhould have a Cypher upon one ſide, and 9 on the other: Upon the next 1 and 8, upon another 2 and 7, on another 3 and 6, and on the other 5 and 4; both the Figures on ether ſide making 9, as before was deſcribed.

Theſe 5 ſlippets do now contain the whole Table of *Pythagoras* before mentioned, but ſo few are not of ſufficient uſe, neither are the Ten before mentioned of the Lord *Nepair*'s order; for there can be but four Figures of one kind, which in all caſes is not ſufficient.

Therefore, as theſe Rods are made now a days, they do commonly make ſix ſets of them, that is, 30 pieces, which contain 60 faces, and theſe will be of good uſe, and there will ſeldom be found a want, which in thoſe of the Inventor's there will often be, except you have a great quantity, which will be far more cumberſom

than

than these here described, for there is required as much Metal or Wood in one of his, as in four of these, and then for his Four sides we have here Eight.

Concerning a Case *for these Rods.*

For the orderly keeping and ready finding of these Rods, I have often (for my self and others) had a Box made of walnut-tree or Pear-tree, with five partitions in it, each partition to hold five or six sets of these Rods, or more if more Rods were required. Every of these partitions being figured on the side thereof next the Eye, with such figures as the Rods in such a partition had figures at the top, so that the party that was to use them, could take them as readily out of his partition, as a Printer can take his Letters out of his respective Boxes to make any word.

In this Box there is also convenient Room made for one other Rod, double in breadth to these here described, but of the same length and thickness; upon the one side whereof there is a Table or Plate useful in the Extracting of the *Square Root*, and on the other side another for the Extracting of the *Cube Root*, the Figure whereof is at the beginning of the Book, noted with *Square, Cube:* But I shall forbear to say any thing of them, till I come to shew you how to Extract the *Square* and *Cube Roots* by the help of them and the *Rods*.

Of a Board with a Frame, upon which to lay your Rods, when any Operation is to be wrought by them, known by the name of a TABELLET.

In the uſing of theſe Rods, care is to be had firſt of the orderly laying of them, and then ſecondly, for the keeping of them in that poſition till your work be ended. For the effecting whereof, both neatly and certainly; there is a little Table or frame contrived, containing in breadth $\frac{1}{20}$ of an Inch more than the length of the Rods, and in length at pleaſure, but it may well be about one and a half the length of the breadth.

It ought to be made of a thin peice of Pear or walnut-tree, or of ſuch matter as your Box or Caſe is made of, and it may very commodiouſly be contrived to be put into the Box as ever I had them made to do, for that I found it convenient to carry looſe.

Upon the Superficies of this board, cloſe to one of the edges thereof, muſt be glewed or otherwiſe faſtened with Pins, a ſmall piece of the ſame matter, and alſo of the ſame length, breadth, and thickneſs of one of your Rods, which muſt be divided into 9 equal parts, and Lines drawn croſs the piece, ſo will there be 9 Squares, in which you muſt Grave or Stamp the nine Digits, beginning with 1 at the top, and ſo deſcending by 2 3 4 to 9 at the bottom thereof: And it were neceſſary that theſe Figures (as alſo thoſe which are at the head of every of your Rods) were Graven or Stamped of ſomething a bigger Figure than the other figures of your Rods are.

Under the end of this ledge beginning at the Figures, and ſo continuing the whole length of the Board, muſt another ledge of the ſame matter and thickneſs, be glewed, or pinned, and then is your *Tabellet* finiſhed. A Figure whereof you have at the beginning of the Book,

Book, noted with *Figure* 4, it is called a *Tabellet*, for that, when the Rods are laid thereon, for any Operation to be wrought by them, we usually say, the Rods are Tabellated.

Being thus prepared with Rods and *Tabellet*, you are ready for the work intended by them, and for which chiefly they were invented.

Thus much for the *Fabrick, Inscription, and Numbering* of these Rods; let us now come to shew the Uses of them.

SECT. IV.

To what Use these Rods generally serve.

THe chief Uses which these small Rods serve unto, are all manner of Multiplications and Divisions, as also of the Extraction of both the Roots, either Square or Cube; all which are so facilly and expeditiously performed, and that by the help of Addition and Substraction only, that it is inconceivable; for here is no charge at all required of the Memory, and you shall assuredly take your Quotient-Figure in Division always certain; neither too great nor too little, an inconvenience so prejudicial, that I leave it to the censure of such as have found it, to their great loss of time, and other vexation which it hath put them to. But ceasing to say more of their properties, I will now come to shew their Use.

SECT. V.

How to apply or lay down any Numbers by the Rods.

PROP. I.

Any Number being given, how to Tabulate or lay down the ſame by the Rods.

LEt it be required to Tabulate or lay down this Number 3496.

Firſt, from among your Sets of Rods, (or out of your Caſe) take four of them, of which let one of them have the Figure 3 at the top thereof, and lay it upon your Tabellet cloſe to the edge thereof, then,

Secondly, Take another Rod from your Caſe, which hath the figure 4 at the top of it, and lay that alſo upon your Tabellet cloſe by the ſide of the other.

Thirdly, Take another Rod which hath the Figure 9 at the top of it, and lay that upon your Tabellet cloſe by the other two.

And laſtly, take a fourth Rod, having the figure 6 at the head thereof, and lay that alſo upon your Tabellet cloſe by the reſt.

Theſe four Rods thus taken, and laid upon the Tabellet, you ſhall ſee in the uppermoſt Row (which ſtandeth againſt the Figure 1 on the ſide of your Tabellet) theſe four Figures, 3 4 9 6, that is 3496, equal to your given Number. In the ſecond Row (againſt the figure 2 of your Tabellet) you ſhall find the double thereof.

thereof. In the third (against the figure 3) you shall find the triple thereof. In the fourth the Quadruple thereof. In the fifth the Quintuple; and so on the ninth and last, in which you shall find the Noncuple of the Number given.

PROP. II.

How these Rods will appear when Tabulated, and being Tabulated, how to read the Multiplication of that Number so Tabulated, by any of the Nine Digits.

The Four Rods being Tabulated according to the Precepts delivered in the preceding *Proposition*, they will appear exactly as they are represented in *Figure* 4 at the begining of the Book, which Figure lively represents the four Rods lying upon the Tabellet, which mind well, for upon the true Tabulating, and right reading of the Rods so tabulated, depends the whole work.

The Rods thus Tabulated, and as you see them in the *Figure* 4, do to the Eye appear in the form of a Glass-window, every Pane thereof representing a Rhomboyades or Diamond-form: In the reading of the Figures which are in these several Rhomboyades or Diamond-form, observe these few Directions following, which will fully illustrate the wole business intended, and therefore especially to be minded.

Note,

I. *That the Figures upon the Rods are to be read, beginning at the right hand and reading towards the left;* which is contrary to our common course of reading and writing, which is from the left hand towards the right.

II. *That in every Rhomboyades or Diamond, there are either One Figure, or Two Figures, but never more than Two.*

III. *If there be but one Figure in a Rhombus, then that Figure is the Figure to be set down alone* (be it either a Figure or a Cypher) *but if there be two Figures in a Rhomboyades* (as for the most part there is) *then add them two Figures together, and set down their sum in one Figure.*

IV. *But if the sum of the two Figures in one Rhomboyades or Diamond do exceed Ten, then you must set down the overplus above Ten, and keep One in mind, which One you must carry to the next Rhomboyades.*

V. *Note that the first towards your right hand, and the last towards your left hand are but half Rhomboyades or Diamonds, and never have in them more than one Figure only, but all between them are whole ones, and for the most part have two Figures in them.*

VI. *If in either Rhomboyades or half Rhomboyades, you find no Figures but Cyphers, you must not neglect but set them down as if they were Figures.*

¶ These Rules being rightly understood, all that follows will be familiar and easie, and these I shall explain by Example following.

Example.

For the illustration of the preceeding Rules, we will make use of those Rods which were before tabulated, therefore have recourse to *Figure* 4 at the beginning of the Book, where this Number 3496 is tabulated.

The

The Figures at the top of the Four Rods are these: 3,4,9,6. which signifie the former given number 3496, and this number stands against the figure 1 on the side of the Tabellet. Then I say, that the figures in the next row standing against the figure 2 of the Tabellet are double thereunto, which I thus prove.

Repair to the Rods as they lie upon the Tabellet, and in that row which lieth against the figure 2, you shall find in the first half Rhomboyades towards your right hand (where *by Rule* 1 you must begin) the figure 2, wherefore set down with your Pen upon Paper the figure 2. In the next Rhomboyades in the same row you shall find 8 and 1, which added make 9, set down 9 on the left hand of 2: In the next Rhombus you shall find 8 and 1 again, which is 9 also, set down 9 on the left hand of the other, and in the last Rhomboyades you shall find only 6, wherefore set down 6 on the left hand of 9, so have you in all 6992, which is double to 3496.

Again, the figures in the row which stands against the figure 3 in the Tabellet, are triple to 3496; for in the first half Rhomboyades towards your right hand, you have 8, set down 8.— In the next Rhom. you have 7 and 1, which is 8, set down 8 again.— In the next you have 2 and 2, which is 4, set down 4.— In the next Rhom. you have 9 and 1, which makes 10, set down 0 and carry 1, but it is the last Rhom. and because there is never another to carry the 1 unto, you must therefore set it down, so have you this number 10488, which is triple to 3496.

Again, the figures standing against 4 in the Tabellet, are Quadruple to 3496,—for in the half Rhom. you have 4, set it down: in the next 6 and 2, which is 8, set that down: In the next 6 and 3, which is 9, set that down: In the next 2 and 1, which is 3, set that down: and in the last half Rhom. you have 13

which allo ſet down: ſo have you 13984, which is Quadruple to 3496.

Alſo, the figures againſt 5 in the Tabellet: the firſt is a Cypher, therefore put down 0; the next is 5 and 3, which is 8, ſet down 8; the next is 0 and 4, ſet down 4; the next is 5 and 2, that is 7, ſet down 7; and the laſt is 1, therefore ſet down 1, ſo have you in all 17480, which is Quintuple to 3496.

Againſt 6 in the Tabellet, you have in the firſt place 6, ſet it down; then in the next 4 and 3, that is 7, ſet that down; in the next 4 and 5, that is 9, ſet 9 down; in the next you have 8 and 2, that is 10, ſet down 0 and carry 1 to the next Rhom. where you find only 1, to which add the 1, which you carried from the Rhom. before, and it makes 2, ſet down 2: ſo have you 20976, which is ſix times 3496.

Againſt 7 in the Tabellet, you have firſt 2, ſet it down; then 3 and 4, which is 7, ſet 7 down; in the next 8 and 6, which is 14, which being 4 above 10, ſet down 4, and carry 1 to the next Rhom. where you have 2 and 1, which is 3, and 1 which you carried makes 4, ſet down 4; then in the laſt place you have only 2, which ſet down, ſo have you in all 24472, which is Septuple to 3496, or ſeven times as much.

Againſt 8 in the Tabellet, you have firſt 8 which ſet down; then 2 and 4, which is 6, ſet 6 down; then 2 and 7, which is 9, ſet 9 down; then 4 and 3, which is 7, ſet 7 down; and laſtly 2, ſet that down; ſo have you 27968, which is Octuple to 3496, or eight times as much.

Laſtly, againſt 9 in the Tabellet, you have in the firſt place 4, ſet that down; in the next you have 1 and 5, which is 6, ſet 6 down; in the next place you have 6 and 8, which is 14, ſet down 4, and carry 1 to the next Rhom. where you find 7 and 3, that is 10, which with 1 which you carried makes 11, ſet down

down 1, and carry 1 to the next Rhom. where you find only 2, and the 1 carried makes 3, therefore set down 3, and so you have 31464 which is Noncuple to 3496, or nine times as much as the tabulated number.

Thus have I given you Examples, in shewing you how the Numbers upon the Rods are to be read and written down; and in the delivery of this Example, I have made the whole work which is to follow, so plain and easie, that the meanest capacity (I think) if he can but tell his figures, and add any two figures together, he may by this here delivered, read or write down any, number that can be tabulated; and that you may throughly understand this Chapter before you proceed further, I will give you the Products of 7009078 multiplied by all the nine Digits, which I would have your self to tabulate, and see if you find your working by your Rods to agree with those which are here written, which numbers if they do, you need not scruple at the most difficult that can be proposed to you, therefore study it, and try it.

			7009078
	2		14018156
	3		21027234
7009078 being multiplied by	4	Produceth	28036312
	5		35045390
	6		42054468
	7		49063546
	8		56072624
	9		63081702

Thus have I sufficiently described these Rods, and the manner of Numbring upon them; and now I think it time to apply them to that use for which they were intended,

namely the more difficult parts of Arithmetick, as Multiplication, Division and Extraction of Roots: But first let me give you

An Admonition concerning Addition and Substraction.

Whereas it was the difficult operations of *Arithmetick*, which by the benefit of these Rods, the Inventor chiefly aimed at (of which kind he esteemed *Multiplication, Division*, and *Extraction of the Square and Cube Roots*) he omitted to say any thing concerning *Addition* and *Substraction*, as things obvious to every *Tyro*, he therefore omitting them, begins to shew the use of his Rods in *Multiplication*, whose Method I shall here follow.

SECT. VI.

Multiplication by the Rods.

IN Multiplying by the Rods, you are to consider (as in Vulgar Arithmetick) three Terms, Things, or Numbers, *viz.*

1. The *Multiplicand*, which is the Number to be multiplied.

2. The *Multiplier*, which is the Number by which the *Multiplicand* is multiplied.

3. The *Product*, which is the sum produced by the multiplying of the two former together.

And here note, that the *Product* doth contain the *Multiplicand*, so many times as there be *Unites* in the *Multiplier*.

Thus for the definition of *Multiplication*, now for the working thereof by the Rods, for which this is

THE

THE RULE.

First, Set down upon your Paper the Multiplicand, and orderly under it the Multiplier. It matters not greatly which of the two given Numbers be made *Multiplicand* or *Multiplier*, but it is usual and best to make the greatest Number *Multiplicand*, and the lesser *Multiplier*. *Then draw a Line with your Pen under them, and having Tabulated your Multiplicand (or greater Number) look what Numbers in your Rods stand against the first Figure towards your right hand, and that Number which you shall find upon your Rods standing against that first Figure found in your Tabellet, set down under your Line which you formerly drew under your Multiplicand and Multiplier : And having so done with the first Figure of your Multiplier, do so with the rest, setting them down one under another, removing every Figure one place more towards the left hand, then that which went before it, as is done in common Multiplication, and as you see in the following Example.*

Example. 1. *Let it be required to multiply* 3496, by 489. As if it were required to know how much 489 times 3496 would amount unto.

First, Set down your given Numbers 3496, and 489, one under another, and draw your Line under them, as here you see done.

```
   3496 Multiplicand,
    489 Multiplier,
 -------
  31464
 27968
13984
 -------
1709544 Product.
```

Secondly, 3496 your Multiplicand being Tabulated, and 9 being the first Figure to the right hand in your Multiplier, look upon your Rods, what sum there stands against 9 in the side of your Tabellet, and you shall find (as by the Rules in the second Prop. of the Fifth Chap. you were directed) 31464,

31464, which is the Product of 3496 multiplied by 9, wherefore ſet down this Number 31464 under your Line, as you ſee in the Example.

Thirdly, Look what ſum upon the Rods ſtands againſt 8, which is the ſecond Figure of your Multiplier, and you ſhall find 27968; ſet this Number under the former, moving it one place forward towards the left hand.

Fourthly, Look what ſum upon the Rods ſtands againſt 4, which is the third Figure in your Multiplier, and you ſhall find 13984, which ſet down under the other, one place more to the left hand.

Laſtly, Under theſe three ſums draw a Line, and add the three ſums together, and they make 1709544, which is the Product of 3496 multiplied by 489 and this 1709544 the Product, contains 3496 the Multiplicand, 489 times.

Practiſe well this firſt Example, and compare it with the Rods as they are Tabulated in Figure 4 at the beginning of the Book, as alſo with the Rules in the Fifth Chapter, and you may perform any Multiplication. However I will give you one or two more Examples, and ſome other ways of *Multiplication*.

Example 2. *Let it be required to multiply the ſame ſum* 3496, by 261.

```
  3496
   261
------
  3496
20976
6992
------
912456
```

Set the Numbers down as here is done, then look upon the Rods for the Product of 3649 by 1, and you ſhall find it to be the ſame, wherefore ſet down 3496 under the Line—then look upon the Rods for the Product of 3496 by 6, and you ſhall find it to be 20976, which ſet down under the other Number, one place more towards the left hand.——Again, look in the Rods for the Product of

3496

3496 multiplied by 2, and you shall find it to be *6992*, which set down under the other two.

Lastly, Draw a Line under them, and add the three Numbers together in order as they stand, and the sum of them will be 912456, which is the Product of 3496 multiplied by 261.

Example 3. *Let it be required to multiply the same Number* 3496 by 520.

```
   3496
    520
 ------
    692
  17480
 ------
1817920
```

Set down your Numbers as here you see done—— Then because the first Figure of your Multiplier towards your right hand is a Cypher, wholly omit it, and multiply 3496 by 52 only, so shall you find the Product of 3496 by 2, to be *6992*, which set down: Also the Product by 5 will be 17480, which set down under the other, one place further; Then draw a Line——and add these two sums together, and they make 181792, to the which if you add a Cypher for the Cypher which you omitted in your multiplier, the sum will be 1817920, which is the Product of 3496 by 520.

Example 4. *Let it be required to multiply the same* 3496 by 7003——

```
    3496
    7003
    ----
   10488
 24472..
--------
24482488
```

Set down your numbers as before, and as you see here done; Then having Tabulated 3496, see what the Product thereof is upon the Rods, being multiplied by 3 the first Figure in your Multiplier, and you shall find it to be 10488, which set down under the Line——Then the two next places of your Multiplier being Cyphers, make two pricks under the former Number, one under 8, the other under 4, as you see in the Example; or instead of 2 pricks you may make two

two Cyphers,——Then look in the Rods for the Product of 3496 by 7, and you ſhall find it to be 24472, which ſet down under the other ſum, beginning your number at the fourth place, or beyond the two Pricks or Cyphers. Laſtly, draw a Line and add theſe two ſums together, and their ſum is 24482488, which is the Product of 3496 multiplied by 7003.

Thus have you four Examples in *Multiplication*, in which are included all the Varieties that may at any time happen in that Rule, *viz.* Two where the Multiplier conſiſted all of Figures, as in the firſt and ſecond Examples they did.—Another where the latter place of the Multiplier conſiſted of a Cypher——And this laſt Example where Cyphers were intermixed among the Figures.

And thus much for this kind of Multiplication, but before I leave, I will ſhew you

Another Form of

MULTIPLICATION.

Whereas in the foregoing Form of Multiplication, which is the beſt and moſt uſual, (only I inſert this following for variety.) You began (your Rods being Tabulated) with that Figure of your Multiplier which ſtands next your right hand, but there is no neceſſity for that, for you may begin with that Figure which ſtandeth next to your left hand, and by ſo doing, and placing your ſeveral Products one place more to the right hand, as you did before, place them to the left hand, thoſe Products added together in the Form they then ſtand, ſhall produce a Product equal to the former.

Example, For our Example we will take the firſt Example before-going at the beginning of this Section,

on, where it was required to Multiply 3496 by 489. Set the numbers down as before in that first Example, and as you see here done.

Then 3496 being Tabulated, look upon your Rods, for the Product thereof multiplied by 4, (which is the first Figure of your Multiplier towards your left hand) and you shall find the Product thereof to be 13984, which set down.— Secondly, look the Product of 3496 by 8 (your second Figure) and you shall find it to be 27968, which must not be set down as in the other first Example but as you see it in this, 8 the first Figure thereof must be set one place forwards towards the right hand, as in the other it was set a place backward towards the left. ——Lastly, seek in your Rods for the Product of 3496 by 9 your last Figure, and you shall find it to be 31464, which set under the other two Numbers, yet one place more to the right hand.——So a Line being drawn under, and these three Numbers added together produce 1709544 equal to that in the first Example: And that you may the better see the difference of the work, I have set them one by the other.

```
  3496
   489
------
13984
 27968
  31464
-------
1709544
```

As in the first Example.	As in this Example,
3496	3496
489	489
31464	13984
27968	27968
13984	31464
1709544	1709544

One

One Example more in Multiplication, which shall be for Advertisement and direction, I will give, and so conclude Multiplication.

I said in the general Rule for working of *Multiplication* (at the beginning of this Section) that it mattered not which of your Numbers were made the Multiplicand, or which the Multiplier, of which I will here give you a President where the lesser Number shall be Tabulated, and the greater Number onely set down; and I will work it here according to this last way of Multiplication, and the Example shall be as followeth.

Example, *Let it be required to multiply* 868437 by 3496, and let 3496 (the lesser Number) be Tabulated.

Let the Numbers be set as you here see, then 3496 being Tabulated, begin with the first Figure towards the left hand of your Multiplier, which here is 8, and upon your Rods find the Product of 3496 multiplied by 8, which is 27968, set that down under the Line —— then find the Product of 3496 by 6, the second Figure of your Multiplier, and you shall find that to be 20976, set this number under the former, one place more towards the right hand —— Again the third Figure of your Product is, 8 whose Product is 27968, as before, set that under the other, still one place more to the right hand. —— In this manner do with the other Figures of the Multiplier, as 4 the next Figure, whose Product is 13984, which also set down a place forward. —— So also the Product of 3 which is 10488, which set down. —— And lastly,

```
    3466
  868437
  ------
  27968
   20976
    27968
     13984
      10488
       24472
  ----------
  3036055752
```

lastly, of 7, which is 24472. —— All these Products being set down in the order as you see them in the Margent, if you add them together, the sum of them will be 3036055752, which is the Product of 3496 multiplied by 868437, the lesser number being Tabulated.

Other wayes of Multiplication I could have added, but these I esteem sufficient.

SECT. VII.

DIVISION *by the Rods.*

As in Multiplication, so in Division there are three Numbers, Terms, or things required, *viz.*

1. The *Dividend*, or number to be divided.

2. The *Divisor*, or Number by which the Dividend is divided, and

3. The *Quotient*, which is the Number issuing from the Dividend's being divided by the Divisor; And this *Quotient* doth always consist of so many *Unites* as the *Divisor* is times contained in the *Dividend*.

Thus much for the *Definition* of *Division*, now let us come to the *Practice* of it by the *Rods*, to perform which this is

THE RULE.

Tabulate the Divisor, (which is alwayes the lesser Number of the two given) and set down the Dividend, and set the Divisor on the left hand, and draw a crooked Line on the right hand for your Quotient, as in common Arithmetick.

tick. Then look upon your Tabulated Rods (always) for the Number less than the Number in the first figures of your Dividend, and what figure stands against that Number on the edge of your Tabellet must be the Figure you must put in your Quotient, and that Number you must always substract from the Figures of your Dividend, and to the remainder add another Figure, so proceeding from Figure to Figure till your Division be wholly ended.

Example, *Let it be required to divide* 1709544, *by* 3496. Having Tabulated 3496, set down your Dividend, your Divisor on the left hand thereof, and a crooked Line for the Quotient on the right hand thereof, as by the Rule preceeding you were directed, and as you see done in the Example adjoyning.

And because at your first setting down of your Divisor 3496, it would reach (if it were set under your Dividend 1709544) as far as the Figure 5, therefore under the figure 5 make a prick, to intimate how far you are gon on in your work, and under this prick draw a Line quite under your Dividend; then is your sum set down ready for work, and will appear as here you see;

3496) 1709544 (

Your Sum thus prepared, ask, how often can you have 3496 in 17095, look in your Tabulated Rods for 17095, which you cannot there find, but the nearest number thereunto amongst the Rods, which is less than 17095 (for you must always take a less number) is 13984, which number stands against the figure 4 in the Tabellet, wherefore set 4 in your Quotient, and 13984 under the Line, and substract 13984 from 17095, and there will remain 3111, so is the first part of

of your Division ended, and your work will stand thus.

3111
3496) 1709544 (4

13984

Then make another Prick under 4, the next Figure of your Dividend, so will the remaining number be 31114,—Then look among your Rods for the number 31114 (or the nearest less than it) and the nearest less you shall find to be 27968, which stands against 8 in your Tabellet; put 8 in your Quotient, and set 27968 under 31114, and substract 27968 from 31114, so will there remain 3146, which set over head, so is the second part of your Division ended, and your work will appear thus.

3146
3111
3496) 1709544 (48

13984.
27968

Lastly, Make another Prick under the next Figure of your Dividend, which is 4 also, making the remaining number to be 31464, seek among your Tabulated Rods for this number (or the nearest less) but looking you shall find the very number, against which stands on your Tabellet the Figure 9; set 9 in the Quotient, and the number 31464 under the Line, and substract it from 31464 the remainder which stands above

bove the Line, and nothing remains, and being there is never another Figure in your Dividend, your Division is ended, and your work will ſtand thus, and 3496 is contained in 1709544, 489 times.

```
             00000
              3146
              3 111
Diviſor, ) 1709544 ( Quotient
  3496   )     ... ( 489
          ---------
           13984
            27968
             31464
```

Another Example, and by another way of Diviſion.

Let it be required to divide 912456 by 3496, ſet down your Dividend and Diviſor, draw a crooked Line for your Quotient, and alſo make a Prick under the fourth Figure of your Dividend, and draw a Line under your Dividend, ſo is your Sum prepared to be divided, and will ſtand thus;

```
3496 ) 912456 (
          .
       ------
```

Then your Diviſor 3496 being Tabulated, look amongſt your Rods for the neareſt number to 9124 which is leſs, and you ſhall find it to be *6992*, againſt which, ſtands on your Tabellet the Figure 2, ſet 2 in the Quotient, and this Number under the Line, and ſubſtract it from 9124, and there will remain 2132, to which number add the next Figure of your Dividend, namely 5, and it makes 21325, under which number draw

draw a Line, then will your Sum stand thus,

```
3496 ) 912456 ( 2
        ..
       ------
        6992
        21325
       ------
```

Then among you Rods seek the nearest number to 21325 and you shall find 20976 to be the nearest number less, against which, in your Tabellet stands 6, set 6 in the Quotient, and 20976 under the Line, substracting it from 21325, which when you have done, there will remain 349, to 349 add the next Figure in your Dividend, which is 6, your last Figure, and it makes 3496, under which draw a Line, and your work will stand as here you see.

```
3496 ) 912456 ( 26
        ...
       ------
        6992
        21325
       ------
       20976
         3496
       ------
```

This done, look amongst your Rods for the nearest number to 3496, and you shall find the exact number at the top of the Rods, against which stands the Figure 1 on the Tabellet, set 1 in the Quotient, and substract 3496 from 3496, the remainder is nothing, and so is your Division ended, the work standing thus, and 3496 the Divisor is contained in 912456 the Dividend, 261 times.

3496) 912456 (361
...

6992
21325

20976
3496

3496
0000

A third Example ready wrought by the last and best way of Division. I will only set it down ready wrought, leaving the practice of it to your self.

Let it be required to divide 73020506 by 3496.

3496) 73020506 ($20886\frac{3050}{3496}$
.....

6992
31005

27968
30370

27968
24026

20976
3050

This Sum thus divided, produceth in the Quotient 20886, and 3050 remaining, so that the Quotient with Fraction and all is, 20886

$20886 \frac{3050}{3496}$ which shews

that 3496 the Divisor is contained in 73020506 the Dividend, 20886 times, and 3050 remaining.

This Example well practised, together with them before-going, are sufficient instruction for any Student whatever, and he that can perform these, need not despair of the most difficult that can be proposed. And so I conclude with Division.

Sect. IX.

Of the Extraction of ROOTS.

THe Extraction of *Roots*, which is the difficultest part of Multiplication and Division, is expeditiously and certainly performed by the Rods, for the easie and expedite performance of which, there are two Rods on purpose, one for the Square, the other for the Cube Root, of which I will speak ; first, Of their Fabrick : secondly, of thir Use.

Of the Fabrick of the Rods for Extracting of Roots.

Of the same matter, and of the same length and thickness of your other Rods, let there be made another Rod, but three times the breadth of the former, the Inscription on the one side serving to extract the Square, and that on the other side for the Cube Root, each of which are divided into three Rows or Columes, That which serveth for the Square Root, hath in

the top or uppermost Square between the Diagonal thereof, these figures 0-1, in the second 0-4, in the third 0-9, in the fourth 1-6, in the fifth 2-5, in the sixth 3-6, in the seventh 4-9, in the eighth 6-4, and in the ninth or lowermost 8-1, which are the Square Numbers belonging to the nine Digits.

In the second Column of the same Rod, in the first Square is inscribed 2, in the second 4, in the third 6, in the fourth 8, in the fifth 10, in the sixth 12, in the seventh 14, in the eighth 16, and in the ninth 18.

In the last or third Column, there are the nine Digits orderly descending, namely, 1,2,3,4,5,6,7,8,9. This Rod thus made, is fitted for the Square Root.

That which serveth for the Cube Root, hath in the top or uppermost Square of the first Column towards the left hand, between the Diagonal thereof, these Figures, 0-01, in the second 0-08, in the third 0-27, in the fourth 0-64, in the fifth 1-25, in the fixth 2-16, in the seventh 3-43, in the eighth 5-12, and in the ninth 7-29. which are Cube numbers orderly descending.——The second Column of this Rod contains these Square Numbers, 1, 4, 9, 16, 25, 36, 49, 64, 81, orderly descending. ——The third and last Column of this Rod hath in it the nine Digits, 1,2,3,4,5,6,7,8,9, orderly descending also.

This Rod thus prepared and inscribed, is fit for extracting of the Square and Cube Roots, a Figure of either side whereof you have at the beginning of the Book: That which serveth for the Square Root having the word *Square* written by the side, that for the Cube Root, hath *Cube* written by the side.

Thus having given you the Fabrick and Inscription of these Rods, I will now shew you their Use; And first,

SECT.

SECT. IX.

The Extraction of the Square Root.

TO extract the Square Root of any number, you must first prepare it, that is, set down the number on a paper, then under the first & lowest figure next the right hand, make a point with your pen, and under the third figure make another, under the fifth another point, and so forth, under every second figure of the number make a point, always leaving between each point one figure unpointed, according to the ordinary Rule by the pen; this being done, you shall see how many figures will be in the Root for so many points as you have, so many figures shall you bring into the quotient for the Square Root, of the number given; next draw a quotient-line, as in Division, and your number is ready prepared for Operation, and will stand as in the Example following, where the number given is 119025, and the Root square thereof is required, this number being set, and pointed as afore is shewed, you may perceive that the Root thereof will be of three figures, because there be three points under the number given, the two figures belonging to the highest point next the left hand are 11. the two figures belonging to the second point are 90, & the two figures belonging to the third point are 25. and the figures for the Root answerable to those several points, are to be found by the Rods, as followeth.

$$\begin{array}{l} 119025 \,(\\ \;\;\cdot\;\;\;\cdot\;\;\;\cdot \end{array}$$

Take the Frame, the Rods, and Lamina, and lay

them before you; and first place the Lamina in the Frame next to the left hand ledge, with that side upwards, whereupon are the inscriptions belonging to the Square Root, and marked at the top with the Letter S, then consider what is the greatest square number in 11. the figures belonging to the first point; the Lamina presently sheweth you that the greatest Square number in 11 is 9, and his Root is 3, for 3 times 3 is 9; therefore put 3 in the quotient for the first figure of the Root, then set 9 under the 11 and substract it therefrom, and there will remain 2, this 2 set over 11, and cancel the 11 as you use to do in Division, and so you have gained the first figure of the Root, and the work will stand as in this Example.

```
2
119025 ( 3
 .  .  .
9
```

Having the first figure of the Root; to get the second, and so all the rest in order, you must proceed in this manner; duple the Root found, which duple place or Tabulate upon the Rods between the Lamina and the ledge of the Frame; As in this example, the duple of 3, the Root found is 6, therefore place a Rod that hath in his top or upper Square 6, between the ledge and the Lamina; Then look upon your number given, what figures, or number it is that belongeth unto the second point, which you shall see will be 290 in this our Example; Then turn your eye to the Lamina and Rod now Tabulated, and search thereupon what number will (being less yet) come nearest unto 290, the number out of which the second figure of the Root is to be found, And there you may soon see it is the number 256, which of any number upon the Rods, less than 290 cometh nearest thereunto, for the next greater number upon the Rods, above 256, is 325, which is greater than 290, and therefore cannot be

be taken out of it; but 256 is the only number to work withal, against which, on the ledge and Lamina, is this figure 4, this 4 must you put in the quotient for the second figure of the Root, and then substract 256 out of that 290, and there will rest 34, this 34 set over the 290 in its due place, and cancel the 290, and the work will stand as in the Example. If you please, you may write your numbers to be substracted, under the number out of which Substraction is to be made, as I have done here in this Example, for instruction sake, or you may omit that if you please being you have them before you upon the Rods.

```
 234
119025(34
 ˙ ˙ ˙
 9
 256
```

And now for the third figure of the Root, look upon the number given, and there you shall see that the Remainder 34, with the two figures 25 still uncancelled, belonging to the third point, being all joyned together, make 3425, out of which the third figure of the Root is to be extracted; To find out what this third figure shall be, duple the Root found already, which is thus done very readily; Take forth a Rod that on his top Square hath 8 the duple of 4, the figure last found, and this Rod put into the Frame between the Lamina and the Rod that is already Tabulated; This being done, you have no more to do, but to look over the Rods and Lamina for such a number as will come nearest unto the number 3425, that belongeth to the third point, and is less than it, and the figure that you see against that number so found, put

in the quotient for the third figure of the Root; Thus looking upon the Rods you shall at the first sight finde the very number it self 3425 that belongeth to the third point, in the fifth line of Squares, against the figure 5, therefore put 5 in the quotient for the third figure of the

234	
119025	(345
· · ·	
9	
256	
3425	

Root, and if you substract 3425, the number now found from 3425, that number belonging to the third point, there will be no remainder, so is the work done, and the number given, 119025 is a perfect Square number, and the square Root thereof is 345, which was the thing required to be found.

Now if you multiply this 345 by it self, it produceth 119025, the first given number, which proveth the work to be truly wrought; for note this evermore, that for proof of the Extracting the square, Root, you must multiply the Root found, by it self; (to the product adding the remain, if any be) and the total will produce the first number given, if the work be truly wrought, otherwise not.

For a second Example, let there be given this number 117716237694, and the Root thereof required: Now to perform the work, first write down the number, and then prepare it with points and a quotient-line, as you were instructed before in the former Example: This being done, the number will appear as here you see in this Exmple; Now in 11. the figures belonging to the highest point, the greatist square number is 9, whose Root is 3, and 2 remaineth to be set over head;

221	
117719623769 4.	(34
· · · · · ·	
9	
256	

Now for the second figure of the Root, Tabulate the duple of the Root found

found, and that is no more to do, but to place a Rod that hath 6, the duple of 3, the Root found, in his top square, between the Lamina and the Ledge, and on the left hand of the Lamina, the said Lamina, being Tabulated with that face upward, that is, for the square Root; then seeing that the number belonging to the second point, out of which the second figure is to be extracted, is in this Example 277, therefore search upon your Tabulated Rod and Lamina for such a number, as will come nearest to that 277, which you shall quickly find to be 256, and right against, it, on the right hand Column of the Lamina is the figure 4, thereof put 4 in the quotient for the second figure of the Root, and substract 256 from 227 and set the remainder 21 over the head of it, and also cancel the 227, and so have you done with your second point.

And now for the third figure of the Root, observe that the 21 remaining with the other 16, the two figures uncancelled belonging to the third point, being joyned together, make 2116, out of which the same third figure is to be extracted; to perform which work, take a Rod that carrieth in his upper square the figure 8, the duple of 4, the figure last found, and put that Rod into the Frame between the Lamina and the other Rod before Tabulated, then look for the number upon those two Rods, and Lamina, that will nearest take away 2116, the number belonging to that third point, and at first sight

```
22167
117716237694 (3430
 .  .  .  :  .  .
9
256
 1049
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you shall find that the next number lesser than 2116 is 2049, and his figure for the quotient, 3, therefore put 3 in the quotient, for the third figure of the Root and substract 2049, from his respective number 2116, and

and there remaineth 67 to set over head, this being done, and the 2116 being cancelled, you have done with the third point, and third figure, and are to proceed to the fourth.

Where first you see that the 67 last remaining, with the 23, make 6723, the figures belonging to the fourth point, whereout you must Exstract the fourth figure of the Root. Therefore go on as before, taking such a Rod, as in its top Square hath 6, the duple of 3, the figure last found, and Tabulate it between the Lamina and the other Rods, and then seek what number there can be found, that will be less than 6723, but at the very first sight you shall see no number upon the Rod so small as that 6723, for the very first number against 1, is 6861, which is greater than 6723, and therefore cannot be taken out of it, so that here you can put no figure in the quotient; but must supply the place with a Cypher, therefore put a 0 in the quotient four the fourth figure of the Root, all the rest of the number standing as it did.

```
     5
  221674895
 117716237694 ( 34309
  . . . . . .
 9
  256
   2049
   617481
```

Now to the next point, which is the fifth, you have this number belonging 672376 whereout the fifth figure of the Root is to be found. Now here, inregard your last figure of the Root found is a 0, you must ever in such a case (in stead of putting a Rod, that hath the duple of the figure found in his top square,) you must take such a Rod as upon one of his faces carrieth only Cyphers, and Tabulate it between the Lamina and the other Rods already Tabulated; This being done, question the Rods, what number you must take from that

that number *672376*, or what number it is there, that being less than *672376* yet cometh nearest unto it; and also what the figure shall be that you must put in the quotient for the respective figure of the Root; The Rods will suddenly resolve you, that their greatest number less than *672376* is 617481, and its respective figure for the Root is 9, therefore put *9* in the quotient for the fifth figure of the Root, then if you will write 617481 under *672376*, and make substraction, your remainder will be 54895, which joyned to the 94, the two last figures of the number given, make 5489594, for the number out of which the sixth and last figure is to be found.

To find this sixth figure, you must Tabulate upon the Rods the duple of the figure last found between the Lamina, and the Rods already tabulated; but here, because the duple of *9* is 18, consisting of two figures, therefore this 18 cannot be Tabulated upon one Rod, as before we did use

```
    5
  22167489590
 117716237694(343098
  . . . . . .
  9
  256
   2049
    6174891
     5489504
```

to do, when the duple was contained of but one figure, now in this case, (and so of all the like (First, Tabulate a Rod bearing 8 in its top square, between the Rods formerly Tabulated and the Lamina, and next to the Lamina, then for the unite 1, being the highest figure of the 18, you must Tabulate that upon the last and lowest Rod, formerly Tabulated, which is done by encreasing the figure in its top square, one unite more than it was before; so here the last Rod before Tabulated, carrying only 0; either turn it, or else

elſe take it away, and place ſuch a face of that Rod, or of ſome other, that hath in his upper ſquare the unite 1 in room and ſtead of that face, with Cyphers, ſo is your number 18 the duple of 9 Tabulated, this done, look over the Rods, for a number that will come neareſt unto 5489594, in the Eighth line is this number 5489504, and its reſpective figure for the Root, 8: this 8 put into the quotient, and Subſtration being made as is uſed to be done, there will remain 90, and ſo is your whole work ended, and the Square Root of 117716237694 is 343098, if you multiply this Root by it ſelf, and to that product add the 90 that remaineth, you ſhall produce again the firſt number given, which argueth that the work is truly wrought.

But now when any thing remaineth, the Extraction being ended, as here it doth, make a Fraction of that remainder as you do in Diviſion, in this manner; Set the number ſo remaining after this Extraction is ended, over a line for Numerator, and for the Denominator, ſet the duple of the whole Root found, with one added thereunto, as here in this example 90 remaineth; this 90 put over a line for the Numerator of the fraction, then double 343098 the Root, and it is 686196, to which add one unite, and then it will be 686197, this ſet down under the line for Denominator to the fraction, and then the true Root ſquare of the number given will be 343098 $\frac{90}{686197}$ and will ſtand as in the Example. This is the vulgar, and ordinary way to make a fraction of the remainder.

But the beſt and moſt certain way to attain unto the true value of the fraction remaining, and that too by the Rods, very eaſie and ſpeedy, is to add two, four, or ſix Cyphers to the remainder and continue the work of Extraction, and then your fraction will be in primes, ſeconds, thirds, *&c.* that is in 10 parts, 100 parts, and 1000 parts, in the ſame manner as in Diviſion; for note,

note, that for every two Cyphers that be added or adjoyned to the number given, you shall have one fractional figure in the quotient, which will represent the fraction in Decimal parts of an unite, we will add 6 Cyphers to the remainder in this our last Fxample, and it will then be 90000000, and then we will continue the work, therefore Tabulate 16, the duple of 8, the figure last found; which to do, put a Rod that carrieth 6, the lowest figure of 16 next the Lamina, between it & the Rods afore Tabulated, & then instead of that Rod last in place next the Lamina, put another Rod that hathin his upper square one unite more than that, as here, change the Rod from 8 to 9, and the Rods are Tabulated, and you are now to look out a number that will nearest take away 9000, the number belonging to the first fraction-point, but the Rods give you none so small, therefore put a o in the quotient for the first figure of the fraction, and because there is no more to do about this first figure, you are next to Tabulate a Rod with Cyphers next the Lamina, and then see for a number that will come nearest unto 900000, the number belonging to the second point of the fraction, but yet you shall have none upon the Rods so small, therefore put another o in the quotient for the second fraction figure: again, Tabulate a Rod with o next the Lamina, and you shall yet again find no number on the Rods so small as 90000000, the number belonging to the third point, therefore put a third Cypher in the quotient for the third figure thereof. Thus have you done with your three points of Cyphers, which you first added, but because you are resolved to get at least one fraction-Figure, therefore add two Cyphers more to the remainder, making it 9000000000, and also Tabulate a Rod with o next to the Lamina, and then see if you can find a number little enough upon the Rods. And now here at last you shall find, one

one significative figure in the quotient of your Fraction, for the number now belonging to the fourth Fraction point is 9000000000, consisting of 10 places of Figures, and the number Tabulated is also now become to be of 10 places, and withal the higest figures less than the highest figures of that number which belongeth to that fourth point, therefore seek upon the Rods for a number less than that 9000000000, and that you shall have in the second line upon the Rods, and it is 6861960004, whose respective figure for the root is 2; now substraction being made, there will remain 2138039996. Thus have you gotten one figure into the quotient of your fraction, and that in the fourth place Descending, and may be thus expressed fraction-wise $\frac{0002}{10000}$, or thus, $\frac{2}{10000}$ signifying 2 parts of 10000 of an Unite; for note, that so many fractional points as you bring into the quotient to produce a new Numerator, the Denominator is always an Unite, with as many Cyphers as you have made fractional figures. This new found fraction joyned to the whole parts of the Root found, will stand as here in the Example; Or else without a Denominator, thus, which is all one with that other; Or else thus, according to *Simon Stevens*, and is thus to be read 343098, 0 primes, 0 seconds, 0 thirds, 2 fourths.

2138039996
9000000000 (0002
. . . .
6861960004

343098 $\frac{2}{10000}$

343098 0002

1 2 3 4
343098. 0. 0. 0. 2.

These Examples might be sufficient to shew the excellent use of the Rods in Exracting the square Root of any

any number; but yet to ſhew the more variety of works, take one Example more; and if in any thing I be thought too tedious, know, that it is out of a deſire to plainneſs, even of ſuch a plainneſs as is anſwerable to that of the Rods.

Let there be given this number, 97419256 and the ſquare Root thereof required, Write down the number, and prepare it with points under each ſecond figure, and a quotient Line, and then proceed as before, and firſt look upon the Lamina, what is the greateſt ſquare number there, that can be had out of 97, the two figures belonging to the firſt and higheſt point; the Lamina ſheweth that it is 81, whoſe Root is 9, put this 9 in the quotient for the firſt figure thereof, and then ſubſtract 81 from 97, and that 97 cancel, the remain is 16 for the ſecond figure, Tabulet 18, the Duple of 9, upon two Rods, between the Lamina and the Ledge, and then upon thoſe two Rods and Lamina ſeek out the number that comes neareſt to 1641, the number to the ſecond point belonging, and that you ſhall find to be the number 1504, and his reſpective figure for the Root 8, which being put into the quotient, and ſubſtraction made, according to the inſtructions afore-delivered, the number remaining to the Third point will be 13792, and to find out his proper figure for the Root, Tabulate 16 the Duple of 8 laſt found, in this manner, place a Rod that carrieth in his top ſquare the figure 6 betwixt the Lamina and the former Rods, and increaſe the former lowermoſt Rod one unite, by changing it from 8 to 9, then ſhall you ſee the number upon the Rods neareſt un-

to 13792 is 13769 in the seventh line, and after Substraction made, there will rest 23, making the number for the least point to be 2356, now to find the respective figure of that fourth point, Tabulate 14, the duple of 7 as before you were instructed, and then you shall see at the very first, that no number upon the Rods is so small as that 2356, the number belonging to the last point, therefore put 0 into the quotient for the last figure of the Root, and so have you ended the work, for the whole part of the Root sought for, which in this Example appeareth to be 9870, and the Remainder 2356. But now to make a Fraction of this Remainder, as you were before shewed, set the same 2356 over a line, at the end of the whole part of the Root found 9870, and then duple the Root found, and to that duple add one unite, and the total will be 19741, which set under the line for Denominator, and then the whole work is finished, and the true Root found answering the demand is 9870 $\frac{2356}{19741}$, and standeth as in the example it appeareth.

1
163723
97419256 (9870$\frac{2356}{19741}$
. . . .
81
1504
13769

But if you desire to be yet more exact, and would have the truest value and estimate of your fraction; then turn it into a Decimal, and proceed as before; first add to the remainder so many times two Cyphers as you desire to have figures for your Numerator of your new fraction, that is to say, two Cyphers if you would have but only one figure, four Cyphers for two figures, six for three figures, and so forth, for as many as you would have, and until you think your fraction is small enough: In this Example we add to the remainder three points of Cyphers, that is, six Cyphers,

because we would have three Decimal figures in our fraction-Root; then Tabulate the duple of that figure of the Root last found, but because that is a 0 which doth neither increase nor diminish, therefore Tabulate a 0 next the Lamina, between it and the other Rods; next see upon the Rods what number there will come nearest to 235600, the number that now belongeth to the highest point of the fraction-points, and that is only the very first, *viz.* 197401, that will do it; therefore put 1 in the quotient, for the first fraction-figure, and then make Substraction, and there will remain for the second fraction-point 3819900.

To find the second fraction-figure, Tabulate 2, the duple of 1, and seek the number that cometh nearest to that remainder 3819900, and that is again only the first, therefore put 1 in the quotient, and make Substraction, and then to your third point will belong 184587900, and to find its respective figure for the Root, Tabulate 2, the duple of 1, last found, and inquire what number upon the Rods will come nearest to that 184587900, and that you shall find in the ninth line of squares to be 177662061, therefore put 9 in the quotient, and when Substraction is made, you will have remaining 6925839, thus have you three figures in the quotient, which are enough to give the fractions value in any ordinary question; if you please to continue the work the fourth figure will be 3, and now is your fraction turned into a Decimal fracti-

1
600
1849238
381585867
235699793951
974192256.00000000 (1193
. . . .

197401
1974021
177662061
512207149

or, whose Numerator is 1193, and his Denominator 10000, and being set fraction-ways, will stand thus $\frac{1193}{10000}$, Or it may very well be expressed without the Denominator, only with a line, or point of distinction, thus, 9870, 11393, and so the value of this fraction is 1193 parts of 10000. Or according to *Simon Stevens*, 1 prime, 1 second, 9 thirds, 3 fourths, and more briefly may be thus expressed $1^{1}, 1^{2}, 9^{3}, 3^{4}$, or thus, 1^{I}, 1^{II}, 9^{III}, 3^{IIII}. If you will, you may continue the Extraction unto 9 or ten degrees, but 3 or 4 for ordinary works is enough. If you multiply this Root found 9870. 1193 by it self, the Product will be 97-419254. 99623249, from whence, by a point or line cut off 8 places of figures, according to the rules of Multiplication in Decimal Arithmetick, and the number remaining towards the left hand, will want (the fraction considered,) but a very little more than one unite of the number first given, but if you take the pains but to continue the work to one place lower, it will not want an unite, and so the lower you work the fraction, the nearer still you come to the exact truth.

SECT. X.

The Extraction of the Cube Root by the Rods.

TO Extract the Cube Root of any number, you are first to wr ite down the number given, whose Root is required, and make a point under the lowest figure next to the right hand, and another point under the fourth figure, and so under every third figure, omitting between every two pionts two figures unpointed, and then so many points as you have

have under your number, so many figures shall you have in your Root; next draw a quotient-line, as in the Extraction of the Square-Root, this being done, the number is prepared for Extraction; then to go to the work; first, seek what is the greatest Cube number in the number standing above, or belonging to the highest point next the left hand, which the Lamina will shew upon his left side of that face for the Cube Root, and in the Column, upon the right side of the same face is the Root thereof, and when either by your memory or Lamina, you have found the greatest Cube Root in that number belonging to that first point, then (as in Extraction of the square Root) substract the greatest Cube number, to that first point belonging, or that i the number, to the said first point belonging, can be found from the said number, &c. then cancell it, and set the remainder over head thereof, as before in the Extraction of the square Root, and put the respective Digit number found in the quotient, for the first figure of the Root: Now to find the second figure of the Root, (and so all the rest, how many so ever they be) you must always Triple the Root found, and that Triple multiply again by the Root found, and that last product Tabulate upon the Rods on the left hand of the Lamina, as before in the Extraction of the square Root, you did the duple of the root found, then look upon those Tabulated Rods and Lamina together, what number you can find upon them will come nearest to the number belonging to the number next following, and less than it, which number is the Divisor, and the figure on the right hand of the Lamina is the figure for the Root answerable to that point, and that figure put in the Quotient for the respective figure of the Root, belonging and answerable to that point; but note always that you take the Divisor so often, and no oftener, but that you may

yet alſo take another number from the number belonging to that point, which other number is ſquare of the Digit new found, multiplied into the former Triple, the product add to the Diviſor, with this proviſo, that you place this new product one place higher towards the left hand than is the Diviſor, that is to ſay, ſet the loweſt place of that new product under the ſecond place of the Diviſor, the total of this Addition Subſtract from the number belonging to the point in action, cancelling the ſaid number, and the remain ſet over head thereof, as you uſe to do in Diviſion and Extraction of the ſquare Root; and ſo proceed to the next point, if you have any more: But to make all plain, we will illuſtrate this by variety of Examples in all the kinds and differences of works.

Firſt, let 110592 be a number given, and the Cubick Root thereof required; this Cubick Root is thus found, firſt prepare your number, that is to ſay, write it down, and make a point under 2, the loweſt figure thereof next the right hand, and one other point under 0, the fourth figure thereof, leaving two figures between unpointed, then draw the quotient-line, and then the number will ſtand ready prepared as in the example, with two points, whereby it appeareth that the Root will conſiſt of two figures, which are to be found out according to the former directions; and firſt obſerve that 110 is the number belonging to the firſt point, and upon the Lamina you may alſo obſerve that the greateſt Cube number in that 110 is 64, and his Cube Root 4, therefore put 4

```
46
110592 (4
  .  .
64
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in the quotient, for the firſt figure of the Root, and then Subſtract 64 from 110, and there will remain 46, this 46 ſet over the 110, and cancell 110, and the work

work of the first point is done, and here you may now observe, that the 46 remaining, with the other figures 592, make 46592, which is the number belonging to the second point, and where out the second figure is to be found.

To obtain this second figure, proceed in this manner, triple 4, the Root found, and it is 12, and that triple multiply by the Root found 4, and the product is 48, this Product 48 Tabulate upon the Rods on the left hand of the Lamina, between it and the Ledge, then view over these Rods and Lamina thus Tabulated, what number there will come nearest unto 46592 the number belonging to the second point, and be less than it, you shall see the number that cometh nearest to it, is that in the ninth line, 43929, and his respective figure for the Root is 9; now square this 9, and it is 81, this 81 multiply by the former Triple 12, and it yieldeth 972, this 972 add unto 43929, the number found on the Rods (being set in Addition, one place higher than is ordinary, as was before shewed) and the total will be 53949, which if you compare with 46592, the number belonging to the point in action, you shall see it is too great to be taken out of it: whereby it appeareth that you must not take 9 for your Root, for by your general rule you must not take the Divisor no oftner, but that you may take also the Product made by the square of that new Digit number multiplied into the first triple, out of that number belonging to that second point, therefore take a less number upon the Rods, as the number 38912 in the eighth line, which will surely serve the turn, wherefore put 8 in the quotient for the second figure of the Root sought for, now write (if you will) this number 38912, just under 46592, the number belonging to the said second point; then according to the Rule, Square this Digit 8 new found, and it giveth 64, this

64 multiply by the former triple 12 produceth 768, write down under the former number 38912, seting the lowest figure 8 of this new product directly under 1, the second figure of 38912, and its second figure 6 under the third figure 9, and so of the other in Order, then add these two numbers together, and the total will be 46592, which is equal to that number above, therefore if Substraction be made, there will nothing remain, and so the work is ended, whereby you may conclude that 110592, the number given, is a right Cubick number, and that 48 is the Cube Root thereof, which was the thing required to be found.

```
      46
  110592(48
   .  .
  64
  38912
   768
  -----
   46592
```

Now if you would at any time prove your work, whether you have wrought truly or not; multiply the Root found Cubelickly, and add the remainder, when any is, to that product, and if the total be the first number given, then the work is truly wrought, or else not, as here in this Example: if you multiply the Root found 48 by 48, it is 2304, and this 2304, multiply again by 48, produceth 110592, the number first given, and therefore conclude that the work is truly wrought.

For a second Example, we will take this number, 41063625 and seek the Cube Root thereof, first, prepare the number; by writing it down, and making a point under the lowest figure 5, and another under 3, the fourth figure, and another under 1, the seventh figure, and draw the quotient-line, these three points do declare that the Root will consist of three figures: Now to fall to work, to find the first of them, consider

what

what the greatest Cube number in 41 is, which appeareth on the Lamina to be 27, and its Root 3, therefore put 3 in the quotient for the first figure of the Root, and then substract 27 from 41, and set the remainder 14 over head, and the work of the first figure is ended, and the number that belongeth to the second point is 14063, out of which the second figure is to be found, behold the Example: to get this second figure, triple 3, the Root found, makes 9, this triple 9 multiply by 3, the Root found giveth 27, this product 27 tabulate upon the left hand of the Lamina. and then the number that will come nearest unto that 14063 belonging to the second point now in action, remembring the former Caution, that it be taken no oftner, but that withal there may be taken from thence also the product, produced by multiplying the square of the respective figure into the former triple, as here the divisor may be had five times, but by reason of that other number, that must also be taken from thence, cannot be taken also, therefore it can be had but only 4 times, wherefore put 4 in the quotient for the second figure of the Root, and set the number 10864 found in that fourth rank of squares under the number 14063, the number belonging to the second point, then multiply 16 the square of 4 the digit now found by 9, the product is 144, this 144 set under the former 10864, but according to the former provisal, one place higher towards the left hand,

```
14
41063625 ( 3
 .  .  .
27
```

as set its lower figure 4 under 6, the second figure of that 10-864, the next 4 unde 8, and the uppermost being 1 under 0, as you see it stand in the example, then add these two numbers together, and they make 12304, this take from 14063 leaveth 1759, thus is the work of the second point at an end; behold the Example.

```
   1
   14759
   41063625 ( 34
    .  .  .
   27
   ------
   10864
    144
   -------
   12304
```

Now for the third figure of the Root, you are first to observe, that the number belonging to the third and last point is 1759625, from whence the third and last figure of the Root is to be extracted, which figure to find out, triple the Root found 34, & it is 102, & that multiply again by 34, the Root found yieldeth 3468, this product 3468 is the divisor, and this Tabulate upon the Rods as before, and joyn the Lamina close to them, then seek upon those Rods and Lamina what number will come nearest unto that 1759625, the number belonging to the third point now in action, (remembring the former Caution, but here is no need of that in the work of this point,) you shall find the number for the purpose, to be the number 1734125 standing in the fifth line and its respective figure for the Root 5, therefore put 5 in the quotient for the third figure of the Root, now transcribe the number 1734125, from off the Rods into the paper, just under the 1759625, then to find the other number to be hereunto added

```
   1
   14759
   41063625 ( 345
    .  .  .
   27
   10864
    144
   ------
   12304
   ------
       1734125
          2550
   -----  -----
       1759625
```

added; square 5 the figure last found, and it makes 25, this square 25 multiply by the former triple 102, and the product is 2550, this set under the other number 1734125, according to the former proviso, as you see it stand in the Example, and add these two numbers together, and their total will be 1759625, and is equal to that above, belonging to that third point, so that if substraction should be made, there would nothing remain which declareth the number given to be a perfect Cubick number, and the Cubick Root thereof to be 345 which was the thing required to be done; if you will multiply this Root 345 Cubickly, it produceth the number first given 41063625, which proveth the work to be truly wrought, the like is to be observed in all other works of this nature whatsoever.

For a third Example, we will take at an adventure this great number 859271650667, and seek the Cube Root thereof. This number being prepared with points, and a quotient-line; sheweth by his four points, that his Cube Root will consist of four figures, and by the former directions, the first figure will appear to be 9, for the greatest Cube number in 859, the number belonging to the first point, is 729, which taken from that 859, leaveth 130, which with the 271 between that 130 and the next point, make 130271, out of which the second figure of the Root is to be extracted; this second figure by the former rules, will be found to be 5, after the work of this second point is ended, there will be remaining to the third point 1896650, and the Root

```
 1
130896
859271650667 (95
 .  .  .  .
729
------
 121625
  675
---------
 128375
```

found

found is 95, this tripled, and also multiplied again by the Root, produceth 27075 for the divisor, this divisor tabulated, there is no number to be found upon the Rods so small as is 1896650, the number belonging to that third point, therefore put a o in the quotient for the third figure of the Root, and so have you done with that third point,

1
100896 3842
859271650667(9507
729
———
121625
975
———
128375
———
1895250343
139650
———
1896646843

Now to find the fourth figure answerable to the fourth point; you need here do no more, but Tabulate two o between the Lamina, and the other Rods, without any altering of the Rods already Tabulated, and then seek the fourth figure as before, the reason is, because o doth neither multiply nor divide but only raiseth the places of figures higher towards the left hand, for here if you triple 950, the Root found, it yieldeth 2850, and this multiplied again by 950, the said Root found, produceth 2707500, which is the same with that former tabulated number, saving only the two Cyphers, and therefore it is, that when there is a o in the quotient, there needs no more to be done, but to Tabulate two Cyphers between the Lamina and those Rods before Tabulated, when you sought for that last figure, which happeneth to be a o. Now upon those Rods thus Tabulated, seek what number upon those Rods and Lamina will come nearest unto that number, which belongeth to the fourth point, which is here 1896650667. By viewing the Rods, you shall find that number in the seventh rank

of

of squares to be the number that will serve the turn, *viz.* 1895250343: and its respective figure to be put into the quotient 7. To this number add 139650, the number made by multiplying 49 the square of 7, the Digit now found by 2850, the Triple of the Root afore found, and the total is 1896646843. This substracted from that number to the last point belonging, leaveth remaining over head 8824.

Thus have you finished all your points, and have found the Cube Root of your number given to be 9507. And being there is a remainder left, it appeareth that the number first given is not a perfect Cube number; but the greatest Cube number therein is 859271646-843, and his Cubick Root is 5907. The truth of this work you may examine by multiplying the Root found Cubickly, which if you do, and add the remainder to the product, you shall produce the first number given.

Now for the remainder, to make a fraction thereof in such sort, that it may aptly express the nearest Cube Root, I could shew several ways delivered by several Authors, how to bring it nearest to the truth. But for a brief and easie Rule, and which in my conceit is the most exact, is, that which that good Artist, my old acquaintance Mr. *John Tap* delivereth in his Book of Arithmetick, and which he saith, is the Method used by *Gosselin* upon *Tartaglia*. And the Rule in his own words is this. The Root in whole numbers being extracted, set the remainder over a line for numerator, and then for the denominator, triple the Root found, that triple multiply again by the Root found, and to the product add the former triple, the total set under the line for denominator. According to this Rule, if you triple the Root found 9507, it giveth 28-521, and that multiplied again by 9507, the Root found

found 9507, it giveth 28521, and that mutiplied again by 9507, the Root found produceth 271149147, to this product add the former triple 28521, and it giveth this total ſum 271177668 to be ſet under the line for a Denominator to 3824 the remainder; And ſo the neareſt Cube Root of the number given, is found to be, as here it ſtandeth in this Example. This Rule, of a brief Rule, is the beſt I know.

$$9507\frac{28521}{271177668}$$

But the moſt abſolute and beſt rule to get the Cube Root of any number not Cubick, is this: Add to the remainder ſo many ſeveral points, or Ternaries of Cyphers, as you deſire to come nearer to the true Root, (in the ſame manner as you did in the extraction of the ſquare Root) and then continue on the work for extraction, as you do in whole numbers, and the fraction will be turned into a Decimal fraction, the numerator whereof will not be the remainder as before it was, but other figures new found, and then ſo many as you add Ternaries of Cyphers, and the Denominator will have an Unite, with as many Cyphers, as you added Ternaries, or points of Cyphers to the remainder; a ſhort Example will ſerve to explain this Rule.

Let 2112 be a number given, whoſe Cube Root is required, This number I can perceive is no Cubick number, therefore I ſet it down, and to it add three Ternaries of Cyphers, and then prepare it with a quotient-line, and points, as you uſe to do, where by the points you may ſee that you ſhall have two figures for the whole part of the Root, and three that you would have in the fraction. Now to go to work, the greateſt Cube number in 2 is 1, and 1 is the firſt figure of the Root, and by the former Rules, the ſecond figure of the Root is found to be 2, ſo that the whole part of the Root of that number given 2112 is 15, and the remainder

mainder is 348, whereof a fraction is to be made. Thus having done with the whole part, go on now to the fraction, as you use to do with whole numbers, for indeed it is no other but a whole number.

Therefore triple 12, the Root found, and it makes 36, that multiply again by the Root found 432, this product Tabulate, and joyn the Lamina to the Rods, and you shall find that you may have this Divisor 8 times out of that number 384000, the number belong-

```
 1 340
 12 315000
 1384 848513856
 2112.000000000 ( 12.134
 .  .   .  .  .

 608
  12
 ----
 728
 ----
 346112
   2304
 ------
 369152
 ------
 12501 627
     31 86
 ---------
 12533487
 ---------
 1975306864
      20528
 ----------
 1975512144
```

ing to the first fraction-point, put 8 therefore in the quotient

quotient for the first fraction-figure; when you have finished this work, as you have been before instructed, you will find remaining to the next point 14848000, whose respective figure for the Root will be found to be 3, and to the third point remaineth 2315513000, whose respective figure for the Root is found to be 4, thus have you gained three figures into the quotient, for the Numerator of your new fraction, whose Denominator will be one Unite, with three Cyphers, because three Ternaries of Cyphers were added, and so the Root found is $12\frac{834}{1000}$ representing 12, and 834 such parts whereof 1000 make one Unite, or 8' 3 4', that is to say, 12 Unites, 8 Primes, 3 Seconds, 4 Thirds.

If you seek a Denominator for this remainder 384 by that other Rule, you shall find the Root to be $12\frac{384}{474}$, but this last Rule by addition of Cyphers, though not so easie as that other, yet is the most exact, as by tryal you shall find.

Thus much of the Extraction of Roots.

Example

Example in the Rule of Three Direct.

IF 37 *Ells and a half of Linnen Cloth cost* 24 *l.* 7*s.* 8 *d. what shall* 283 *Ells and an half cost ?*

First, set down your 37 Ells, then if you look in your Scale for your half Ell, you shall find it to stand against 500 in the Scale of 1000, which 500 may be called 5 only, for the two Cyphers may be omitted. Then set down your 24*l.* and for your 7 set down 3 decads, which is 6 *s.* and look in your Scale for the Decimal of 1 *s.* 9 *d.* which you shall find to be 875 ; Lastly, set down your 188 Ells, and for your half Ell set down 5 as before, so will your numbers stand thus,

Ells		*li.*		*Ells*
If 37.5	cost	24.3875	what	185.3

```
        183.5
      ---------
      1219475
     731625
    1951000
   243875
   ----------
   4475.10625
```

37.5 (4475.10625 (119.3361
or
(119 *li.* 6 *s.* 8 *d* 2 *q.* $\frac{250}{1000}$

375
725

375
501

3375
1260

1125
1356

1125
2312

2250
625

375
250

Your numbers thus taken out of your Scale, and placed as here you see, if you multiply the second and third together, you shall find the product of that multiplication to be 4475. 10625, which divided by the first number 37. 5, giveth in the quotient 119.3361, which is 119 pounds, 3 decades, or 6 shillings, and 361, which reduced by the Scale, giveth 8 pence, 2 farthings, and something more.

Example

Example in the Rule of Three Reverse.

IF *when the price of a Quarter of Wheat is* 1 li. 5 *s.* 6d. *the peny white loaf shall weigh* 12 *ounces* 16 *peny weight; I demand what the peny white loaf shall weigh, when the price of the Quarter of Wheat is* 7 *s.* 9 d.?

If you place your numbers according to the tenor of the Question, they will stand as followeth,

li.	*s.*	*d.*	*ou.*	*pw.*	*s.*	*d.*
1—	5—	6——	12—	16——	7—	6

But being taken out of the Scales of Money and Troy weight, they will stand thus,

```
  li.            ounces          s.
 1.275 ———— 12.8 ———— .375
  12.8
 ————
 10200      .375 ) 16.3200 ( 435
 2550                ...          or
1275               ————
                    1500     ounces .pw
————                1320     43—10 75/[illegible]
16.3200          ————
                   1125
                    1950
                  ————
                    1875
                      75
```

Here if you multiply 1.275, which is the Decimal of 1 *li.* 5 *s.* 6 *d.* by 12.8. which is the Decimal of 12 ounces 16 peny weight, you shall find the product of that multiplication to be 16.3200, which being divided by .375, which is the Decimal of 7 *s.* 6 *d.* the quotient will be 43.5, which is the Decimal of 43 ounces, 10 peny weight 3 grains; and so much ought the peny white loaf to weigh, when the quarter of wheat is sold for 7 *s*, 6 *d*.

Example in the Double Rule of Three.

If 24 yards of stuff of three quarters broad cost 4 l. 14 s. what shall 328 yards of the same stuff cost being 5 quarters broad.

If you place your numbers according to the directions of this Rule, they will stand thus,

	yards		*quarters*		*li.*	*s.*			*yards*		*q.*
If	24	of	3	cost	4	14,	what	shall	328	cost of	5

But if you take your fraction-numbers out of their proper Scales, they will stand thus,

yards	*quarters*	*li.*	*yards*	*quarters*
24	3	4.7	328	5
3			47	
72			2296	
			1312	
			15416	
			5	
			770800	

72)

72) 7708.000 (105055

72 — or
508 — li. s. d.
— 107 — 1 — 1
504
400

360
480

360
40

First, multiply the two first numbers, as 24 and 3 together, they make 72 for Divisor, then multiply 4.7, which is the Decimal 4 *l.* 14 *s.* by 328, and the Product is 15416, which again multiplied by 5, the last number giveth 77080; unto this Product, (that there may be a competent number of figures in the quotient,) I add two Cyphers, making it 7708000, which I divide by 72, and the quotint is 107.055, which is 107 *l.* 1 *s.* 1 *d.* and so much is 328 yards of stuff worth, being 5 quarters broad.

Example in Barter.

TWo *Merchants having two several Commodities, are willing to Barter, or Exchange the one with the other. The one hath Indigo, which he will sell at* 4*s. the pound for ready mony, but in Barter he will have* 4 s. 9 *d. the pound, the other Merchant hath Kersies, which for ready mony he will sell for* 3 *s.* 6 *d. the yard. Now the question is, at what price he must rate his Kersies in Barter, to equalize the* 9 d. *advanced upon the pound of Indigo?*

The tenor of the Question is this.

If 4 s. *in Barter require* 9 d. *what shall* 3 s. 6 d. *require?*

Your numbers placed will stand thus,

s.	d.	*s.*	*d.*
4	9	3	6

But being taken out of your Scale, they will stand thus,

Decades

Decades	*d.*	*s.*
2	.375	1.75

```
   .375
   1.75
  ------
   1875
  2625
  375
  ------
2 ) .656|25 (.328
  ------
   65
    4
  ------
   16
```

Say then by the Rule of Three Direct, If 2 Decades or 4 *s.* in Barter require .375, which is the Decimal of 9 *d.* what shall 1 75 require? which is the Decimal of 3 *s.* 6 *d.*

First, multiply .375 by 1.75, the product is .65625, but being it is a fraction, I cut off the two last figures, because we require only three figures in the quotient, which divided by 2, giveth in the quotient .328, which is the Decimal of 7 *d.* 3 *q.* this 7 *d.* 3 *q.* added to this 3 *s.* 6 *d.* maketh 4 shillings 1 peny 3 farthings, and so much ought he to rate his Kersies at by the yard in Barter, to save himself harmless.

Example in Fellowship.

THree Persons A, B and C. bought 4000 Sheep, which cost 483 li. 6 s. 8 d. of which money A. paid 203 li. B. paid 165 li. 6 s. 8 d. and C. paid 114 li. 11 s.

First, say by the Rule of Three Direct.

1. If 483 li. 6 s, 8 d. buy 4000 sheep, how many sheep shall 203 l. (which is A's share) buy? Answer 1680.

2. Say, If 483 li. 6 s. 6 d. buy 4000 sheep, how many sheep shall 165 li. 15 s. 8 d. (which is B's share) buy? Answer 1372.

3. Say again, if 483 li. 6 s. 8 d. buy 4000 sheep, how many sheep shall 114 li. 11 s. (which is C's share) buy? Answer 918.

Your numbers being taken out of your Scale, proceed as followeth.

First for A.

		sheep		*li.*	
If 483.3333	buy	4000,	what	103	
				4000	
				812000	

483.3333)8120000000(1680
....

4833333
32866670

28999998
38666720

38666664
00000560

Secondly for B.

li.		*sheep*		*li.*
483.3333	——	4000	——	165.7833
				4000
				663133000

483.3333)6531332000(1372
....

4833333
17979990

14499999
34800010

33833331
9666790

9666666
124

Thirdly for C.

li.	*sheep*	*li.*
483.3333 —	4000 —	114.55
		4000
		458200.00

483.3333) 4582000000 (948

43499997
23200030

19333332
38666980

38666664
316

The manner of Work.

For *A*. multiply 203 *l.* (which is *A*'s share) by 4000 (which is the number of sheep bought) and the product is 812000, which number should be divided by 483.3333, but being it is greater than 81200, I therefore add four Cyphers thereto, that I may have four figures in the quotient, and it makes 812000000, which divided by 483.3333, giveth in the quotient 1680, and so many sheep belong to A.

2. For *B*, multiply 165.7833 (which is the Decimal of *B*'s share) by 4000, (the number of sheep bought) and it produceth 6631332000, which divided by 4833333, giveth in the quotient 1372, and so many sheep belong to *B*.

3. For

3. For *C*. multiply 114.55, (which is the Decimal of *C*'s share (by 4000, (the number sheep bought) it produceth 45820000, which number should be divided by 483.3333, but being it is not large enough to give figures enough in the quotient, I therefore add two Cyphers, makeing it 458200 0000, which divided by 483.3333, giveth in the quotient 948, and so many sheep ought *C*. to have.

Now for proof, if you add the number of sheep that *A*, *B* and *C*. should severally have, you shall find them in all to make 4000, which demonstrates the Work to be true.

A	1680
B	1372
C	948
	4008

Examples in Loss and Gain.

I*F one Yard of Stuff cost* 6 s. 8d. *and I sell the same again for* 8 s. 6 d. *what shall I gain in laying out* 100 li. *upon such a Commodity?*

Take the difference between the price that your Commodity cost, and the price for which you sell it, that is, in this Example, the difference between 6 *s.* 8 *d.* and 8 *s.* 6 *d.* which is 1 *s.* 10 *d.* then say by the Rule of Three Direct,

If 6 *s.* 8 *d.* gain 1 *s.* 10*d.* what will 100 *li.* gain?

If you place your numbers according to the Rule of Three Direct, as they are here given, they will stand thus,

6 *s.* 8 *d.*

s. d.		s. d.		li.
6 8	———	1 10	———	100

But being taken out of your Scale and placed, they will stand as followeth.

```
 s.          s.        li
.3333———— .425 ———— 100
            100
       ——————————
.3333)42500|00(127.5
         .....
       ——————————
       3333
        9170
       ————————
        6666
        25040
        ————————
         23331
          17090
         ————————
          16665
            425
```

Your numbers being placed, multiply .425, which is the Decimal of 1 *s*, 10 *d*. by 100 *li*. and the Product is 42500, to which I add two Cyphers (that I may have a competent number of figures in the quotient) and it makes 42500'00, which divided by .3333, the decimal of 6 *s*. 8 *d*. giveth in the quotient 127.5, which is 127 *li*. 5 Livers or 10 *s*. so there is 17 *li*. 10 *s*. gained in laying out of 100 *li*.

I will here prove this question by the converse.

If by one yard of Stuff which is sold for 8 s. 6 d. *there was gained* 27 li. 10 s. *in laying out of* 100 li. *I demand what the said stuff cost a yard at the first hand?*

Add 100 *l.* and 27 *l.* 10 *s.* together, and they make 127 *li.* 10 *s.* Then say by the Rule of Three Direct,

If 127 *li.* 10 *s.* give 100 *l.* what shall 8 *s.* 6 *d.* give?

Take your numbers out of your Scale, and place them as here you see.

li. *li.* *s.*
If 127.5 give 100, what .425
127.5) 42500|0(3333

3825
4250

3825
4250

Here if you multiply 425, which is the Decimal of 8 *s.* 6 d. by 100, you shall have 42500, to which if you add a Cypher, you make it 42500.0, this number being divided by 127,5, which is the Decimal of 127 *li.* 10 *s.* giveth in the quotient 3333, and if you had added more Cyphers to the Dividend, you should have had more threes in the quotient, and no other figures, but these four threes are enough, and are a Decimal fraction representing 6 *s.* 8 *d.* and so much did the yard of Stuff cost at the first hand.

Examples

Examples in Loss and Gain upon Time, wrought by the Double Rule of Three.

IF *one Ell of Lockeram cost me* 2 s. 8 d. *ready mony, and I sell the same again for* 2 s. 10 d. *the Ell, to be paid at the expiration of three months; I demand what I shall gain in* 12 *months, laying out* 100 li. *upon that Commodity?*

This and such like questions, although they may be wrought by the Rule of Three Direct, at two Operations, yet they are best performed by the Double Rule of Three compounded of five numbers, wherefore the question may be thus stated.

If 2 *s.* 8 *d.* in three months, gain 2 *d.* what shall 100 *li.* gain in 11 months?

If you take your numbers out of your Scale, and place them according as was directed in the first part of this Book, you shall find them to stand thus,

	s.		*mo.*		*d.*		*li.*		*mon.*
If	1.333	*in*	3	*gain*	83,	*what shall*	100	*gain in*	12

```
      3            100
  ---------     ---------
    3.999          8300
                     12
                ---------
                  16600
                  8300
                ---------
       3.999 ) 99600 ( 25 fere
                 ..
                ---------
                  7998
                  19620
                ---------
                  19995
                    375
```

Your numbers being placed according to the tenor of the question, if you multiply 1.333, which is the Decimal of 2 *s.* 8 *d.* by 3 months, the product will be 3.999, which must be your Divisor, then multiply. 83, which is the Decimal of 2 *d.* by. 100 *li.* and it makes .8300, that again multiplied by 12 months, giveth for the product 99600 for your Dividend, wherefore if you divide 99600 by 3999, it will give you in the quotient 25 almost, which is 25 *li.* for the Decimal fraction remaining is so small, that it wanteth not near a farthing of 25 *li.* and therefore we call it 25 *li.* and so it is exactly, as you may try, if you reduce all the numbers to their least denominations, and work as is before taught in *Vulgar Arithmetick.*

I will prove this question by the converse.

If one Ell of Lockeram cost me 2 s. 8 d. *ready mony, for what price shall I sell the same again to be paid at the end of three months. So that I may gain* 25 li. *in* 100 li. *for* 12 *months ?*

Say by the Rule of Three Direct

If 100 li. *in* 12 *months gain* 25 li. *what shall* 2 s. 8 d. *gain in* 3 *months ?*

If you take your numbers out of your Scale, and place them according to the Double Rule of Three, they will stand as followeth,

```
li.     m.      li.     s.          mon.
100 --- 12 ---- 25 ---- 1.333 ----- 3
 12                        25
-----                   ---------
 200                     6665
100                     2666
-----                   ---------
 1200                   33325
                            3
                        ---------
                1200 )99975 (.83

                          ---------
                          9600
                           3975
                          ---------
                           3600
                            375.
```

Your numbers being thus placed, if you multiply 100 *li.* by 12 months, you shall find the product to be 1200, which is your Divisor. Then multiply 25 *li.* by

by 1.333, which is the Decimal of 2 *s.* 8 *d.* and the product thereof will be 33325, which multiply again by 3, & the product will be 99975 for your Dividend, this 99975 divided by 1200, giveth in the quotient .83, which is the Decimall of 2 *d.* which 2 *d.* added to 2 *s.* 8 *d.* the price which the Ell of Lockeram cost, giveth 2 *s.* 10 *d.* and at that price must you sell the same at 3 months time, so that you may gain 25 *li.* in the 100 *li.* in 12 months.

I might further proceed to shew you Examples in divers other Rules, As in *Alligation*, *Position*, &c. but those Rules being already handled in the First Part of this Book, it will be easie to apply them to the Scales. And (as I intimated at the beginning of this Third Part,) that although I have only made choice of the eight *Scales* there mention'd and described, yet it was easie to contrive Scales for the *Coins*, *Weights* and *Measures* of other *Countries*, and not only so, but when the value of the *Mony*, *Weight* or *Measure* of one *Country*, and the *Mony*, *Weight* or *Measure* of another Country is known, it is easie to contrive two *Scales*, which facing one another, shall immediately tell you how many *Pounds* in one place shall be equal to so many *Crowns*, or other *Coin* in another place; but this I do only intimate, that such as are desirous, may fit themselves with Scales answerable to their most necessary occasions. And thus shall I conclude this Third Part, referring some things necessary to such a Work, to the Appendix following.

The End of the Third Part.

AN ABRIDGEMENT OF THE PRECEPTS OF ALGEBRA:

The Fourth Part.

VVritten in French
BY
JAMES de BILLY.

And now Translated into English.
With divers Questions added, which were not in the Original.

Published by Will. Leybourn.

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MDCLXXXIV.

ALGEBRA BREVIS.

The Fourth Part.

I Have been always of opinion, that the practice of *Algebra* should not be entangled with great number of precepts: This Science is of it self dark enough, without adding unto it new obscurity, by the confusion of many different operations. You have here an Abridgement which hath pleased many of good Judgments, and I hope, such as will with attention read it, shall from thence receive both satisfaction and profit.

I shall in the first place set forth a Table of three Ranks: In the first of which there is a Progression natural, of which terms are disposed in that order, that immediately under them you have the Cossick Characters, of which they are Exponents, and in the lowest Cell, a Progression Geometrical, which beginning with an Unite, may be doubled, trebled, quadrupled, *&c.* We have for the greater ease only doubled them. Observe then that R is a Cossick Character signifying a Root, and that the Exponent thereof is marked above in the uppermost Rank, and C is the Cossick Character of a Cube, whose Exponent is

is 3, and ſo of the reſt. I call thoſe terms, which are in the upper Cell, Exponents, becauſe they expound the Coſſick Characters, and the Numbers of the Geometrical Progreſſion which are below. Mark well this manner of ſpeaking, conſider diligently this Table, and for the preſent content your ſelf with this.

Expo-nents	0	1	2	3	4	5	6	7	8	&c.
Chara-cters. Coſſick	N Numbers abſolute	R Root	Q Square	C Cube	QQ Biquadrate.	S Surſolide.	QC Square Cube.	S 2 Solide ſecond	QQQ Square ſquare ſquare	&c.
Prog. Geo. metr.	1	2	4	8	16	32	64	128	256	&c.

You may continue this Table, if you pleaſe, infinitely in this manner. Take two numbers, which if you multiply, they will produce ſome Exponent, you ſhall preſently ſee what a Character is to be put under that Exponent. For Example, If you would know the Character of the Exponent 6, take the numbers 2 and 3 (becauſe theſe multiplied together make 6) after that add their Characters which are Q and C, and you ſhall have QC for the Character of the Exponent 6. In like manner the Character of the Exponent

nent 8 is QQQ, because under the Exponents 2 and 4, which multiplied together produce 8, are contained the Characters Q and QQ. Also the Characters of the Exponent 12 will be QQC, because 12 is a number produced by the Multiplication of 2 and 6, or of 3 and 4.

But if the Exponent be a first number (that is to say, a number not produced by the Multiplication of any two other) mark in what order it is after the Exponent 5, and call it Sursolide Second, Third, or Fourth, *&c.* according to its Rank. The Character of 5 is solid, of 7 solid second, of 11 solid 3; and so consequently under such Exponents as are first numbers, under which only are found such solids.

CHAP. I.

The Alegorithm of Cossick Numbers, *simple, compounded, or diminished.*

BY the word *Alegorithm*, I mean all the Operations comprehended under these four kinds, *Addition, Substraction, Multiplication*, and *Division.* By the word *Cossick Simple*, I understand such as have not this + (which signifies *Plus*, northis— signifying *Minus*, expressed before them. On the other side, by *Numbers Composed*, are meant such as have the sign +, and by diminshed, such as have the sign— Note such Numbers as have no sign expressed, are supposed to have this of +.

Sect. 1. Addition *of Simple Cossick Numbers.*

All Simple Cossick Numbers of the same Denomination (that is to say, have the same Character) or

of different: If they be of the ſame Denomination, the Addition is as in Common Arithmetick. *Example*, 5 Q added with 3 Q, makes 8 Q.

If they be of different Denominations, they muſt be added by the interpoſition of the Figure +, as 6 R added to 4 Q, makes 6 R + 4 Q in like manner 3 added to 4 R makes 3 + 4 R.

Sect. 2. Subſtraction of *Coſſick Numbers.*

Either they are of the ſame, or different Denominations: if of the ſame, you muſt ſubſtract as in ordinary Arithmetick; for Example, 3 Q ſubſtracted from 8 Q, there reſts 5 Q.

If of different, you muſt ſubſtract by the interpoſition of the ſign — as 6 R ſubſtracted from 4 Q, there reſts 4 Q — 6 R, ſo 3 C ſubſtracted from 66 there remains 66 — 3 C.

Sect. 3. Multiplication *of Simple Coſſick Numbers.*

You muſt here have regard both to the abſolute Number, and to the Coſſick Characters: If therefore a Coſſick Number be to be multiplied by an abſolute, you muſt multiply the abſolute Numbers, and unto the Product give the ſame Character, as 5 R, multiplied by 12 produce 60 R.

But if you multiply Coſſick Numbers by Coſſick, you muſt multiply the abſolute Numbers together, and to the Product give the Character of that Exponent, which is made by the Addition of the Exponents belonging unto the aforeſaid Coſſick Characters. For Example, 2 R multiplied by 3 Q make 6 C, becauſe the Exponent of Q is 2, added to 1, the Exponent R, makes 3 the Exponent of C, which in this reſpect ought to be given to the Product. In like manner 5 R mul-

multiplied by 4 C it makes 20 QQ for the reaſon above given.

Sect. 4. Diviſion *of ſimple Coſſick Numbers.*

The ſpeculation of this is marvellous. But the practice of it is by putting the Diviſor under the Number to be divided, drawing between them a little ſma'l line in manner of common Fractions. For example, 13 Q divided by 7 R, the quotient is $\frac{13q}{7R}$ and 6 QQ divided by 5 C, the quotient is $\frac{6qq}{5C}$.

Sect. 5. Addition *of Numbers compoſed and diminiſhed.*

Some order is to be obſerved in this, in which the Numbers are to be diſpoſed in ſuch manner, that thoſe that are of the ſame denomination muſt be put right under one another. After you have done this, if they have the ſame ſign, they are added as in common Arithmetick, and to the product give the ſame ſign. As for example, 7 Q—4 C added to 3 Q—2 C, the ſum is 10 Q—6 C.

But if the Numbers be of different ſigns, the leſſer muſt be ſubſtracted from the greater, and to the reſidue you muſt give the ſign of the greater number as 6Q+7 R added with 7Q—12 R, give for the ſum total 13 Q—5 R.

$$\begin{array}{r} 6\,Q + 07\,R \\ 7\,Q - 12\,R \\ \hline 13Q - 05R \end{array} \qquad \begin{array}{r} 6Q - 07\,R \\ 7Q + 12\,R \\ \hline 13Q + 05\,R \end{array}$$

Sect. 6. Subſtraction *of numbers compoſed and diminiſhed.*

There is nothing more intricate to beginners than

the precepts commonly given for Substraction. You have here an order plain, sure, and very easie to practise.

Change the sign of the particulars of that Number you desire to substract, and after this change add them with the Number from which the Substraction is to be made, and you shall have the residue: as if from 6 Q — 10 R you would substract 18 Q — 15 R by the 5 Sect. the residue will be 5 R — 12 Q So likewise if you would substract — 8 R = 9 Q from 16 R + 6 Q the residue will be 24 R + 15 Q

$$\left.\begin{array}{r} 6\,Q - 10\,R \\ -18\,Q + 15\,R \end{array}\right\}\text{Add} \quad \left.\begin{array}{r} 16\,R + 6\,Q \\ 8\,R + 9\,Q \end{array}\right\}\text{Add}$$

—-12 Q + 5 R Resid. 24 R + 15 Q Residue.

Sect. 7. Multiplication *of Numbers composed and diminished.*

Mark what I said of Cossick simple in the 3 Sect. and remember that the same signs have always the sign + in the product and different — and there is no difficulty in Multiplication, so as you multiply every particular of the Multiplicand by every particular of the Multiplicator, as in common Arithmetick, as if you multiply 3 Q — 2 R by 8 R, 2 R by 8 R, it makes 18 Q and because the Multiplicator and Multiplicand have diffirent signs, the product must have the sign of — : and therefore that shall be 16 Q, farther 3 Q by 8 R make 24 C, to which you must give the sign +, Because the Multiplicator and the Multiplicand have the same sign, so that the product of this Multiplication will be 24 C —-16 Q. So 2 R + 4 Q multiplied by 3 Q —-5 the product is 6 C + 12 QQ — 10 R.— 20 Q.

3 Q—

3 Q — 2 R Multiplicand	2 R + 4 Q multipl.
8 R Multiplicator	3 Q — 5 multipl.
24 C — 16 Q Product.	— 10 R — 20 Q
	6 C + 12 QQ
	6 C + 12 QQ — 10 R — 20 Q.

Sect. 8. Division *of Numbers composed and diminished.*

There is no great difficulty in this, only put a line between the Dividend and the Divisor, and you have the Quotient, as 4 C — 3 Q + 2 R divided by 5 R — 4 C make for this quotient.

$$\frac{4\,C - 3\,Q + 2\,R}{5\,R - 4\,C}$$

Sect. 9. Algorithm *of Fractions.*

I shall not give any particular precepts, because if a man understand the Fractions of common Arithmetick, and practise according to what is before said, there will be no need of them.

CHAP. II.

The Rule of Algebra, with the explication thereof.

IT was meerly necessary by the precedent rules, to trace out insensibly the way to *Algebra*, which cannot be practiced without *Addition*, *Substraction*, *Multiplication*,

plication, and *Division*. Having therefore made plain these difficulties, we will proceed in the *proposition* of the Rule of *Algebra*, and the explication of every part of it briefly and plainly.

Sect. 1. *The Rule of* Algebra.

You must first for the Number unknown put 1 R, and after examine this root according to the tenour of the question, until you come to an Equation. Secondly, this Equation is to be reduced, if need require. Thirdly, you must divide every part of the Equation by the Number of the greatest Cossick Character. After which, either the quotient, or some root of the quotient, will give the root unknown. This is the Rule of *Algebra*, let us now explain it.

Sect. 2. *How the* Equation *must be found.*

The Rule saith, this is done by examining the question propounded, according to the tenour of the same. That is to say, you must well observe all the conditions of the question propunded, to the end you may fully accomplish it. For after you have gone thorow it, you shall find an Equation between two Numbers. As if I search a Number, which added with its square shall make 20, I suppose this Number unknown to be 1 R, the square thereof is 1 Q, (because every Number multiplied by its self, makes its square) then 1 Q + 1 R is equal to 20. See thus an Equation found between 1 Q + 1 R and 20.

Sect. 3. *How your* Equation *must be reduced.*

Your Equation must be found, it is reduced by adding the same Number to both the terms of the Equation,

quation, or by ſubſtracting from them the ſame number. So is it performed by multiplying, or dividing both the terms by the ſame number. For by this means your Equation ſhall remain the ſame after theſe things done. As for example, $1\,R + 1\,Q = 20$, adding throughout $2\,C$, you ſhall have alſo an Equation between $1\,R + 1\,Q + 2\,C$ and $20 + 2\,C$. So ſubſtracting $1\,R$ from the terms, you have $1\,Q = 20 - 1\,R$. Likewiſe multiplying, or dividing both your terms by 3, you ſhall have by the Multiplication $3\,R + 3\,Q$ equal to 60, and by the diviſion the Equation will be between on $\frac{1}{3}\,R + \frac{1}{3}\,Q$ and $\frac{20}{3}$.

Now to make your reduction judiciouſly and profitably, You muſt take care always that your greateſt character remain alone on one ſide of your Equation. As of all the reductions before made, there is none uſeful but the ſecond, becauſe in that only you find on the one ſide alone $1\,Q = 20 - 1\,R$ which is the only end of your reduction.

I ſaid in the rule of *Algebra*, that your Equation muſt be reduced, if it be neceſſary; becauſe it ſometimes happens, that there is no need of it: as when your Equation falls out between two ſimple collateral numbers, I call thoſe numbers collateral, whoſe exponents do not ſurpaſs one another by more than an unite.

Sect. 4. *When you muſt* extract *the root.*

When your coſſick numbers are ſimple and collateral, you muſt not extract any root; but if you divide by the number of the greateſt coſſick character, the quotient ſhall ſhew you the value of the root, which is all you ſeek for in *Algebra*. For example, If you find an Equation between $2\,R$ and 28, dividing ſimply 28 by 2, the quotient ſhall be the value of $1\,R$.

But

But when the terms of your equation are not collateral, you must extract some root; either square cube, squared square, &c. according to the Cossick character which remains after your Hypobibasm.

Now Hypobibasm is nothing else but an abatement, or depression of the character, and is done by substraction of the lesser exponent from the greater. As if you find an equation between 10 QC and 90 QQ, take notice of the exponent of QC in the Table inserted at the beginning, the which exponent is 6; afterwards look to the exponent of QQ which is 4, substract 4 out of 6 there rests 2, of which the Cossick character is Q. From hence I conclude, that 10 Q are equal to 90, after dividing 90 by 10, and finding 9 in the quotient, I conclude that the square root of 9 must be extracted by reason of the character Q.

Sect. 5. *How to* extract *the* square root *of numbers, compound and diminished.*

No man hath yet perfectly found out the way to extract the root of numbers compound and diminished, unless the exponents of three terms of the equation keep between them in some situation or other, an Arithmetical proportion, that is to say, the same distance. As if the equation be between 1Q and 20—1 R, you may now extract the root of 20—1 R, because the exponents of the three numbers which make the equation, are 2, 0, 1, which thus place, 0, 1, 2, keep the same distance.

The greatest Cossick character left after the Hypobibasm, shews the root to be extracted; as in this example, before the square root is to be extracted, because the greatest characters is Q, the method to be followed in the extraction, see here propounded in general terms.

Take

Take first the half of the number of roots. Secondly, to the square of this half, add or from it subduct your absolute number according to the sign + or—. Thirdly, extract the square root of this sum, or of the residue. Fourthly, to this root add or from it substract half the number of roots, and this last sum or residue shews you the value of the root unknown. For example, I would find a number, the double whereof added to its square, should be equal to 24, I shall find an equation between 2 R + 1 Q and 24 by the second Section. Moreover, I shall reduce this equation after this manner 1 Q equal to 24—2 R by the third Section. Then if I divide 24 — 2 R by 1, the number of the greatest Cossick character, there still remains 24—2R, because an unite doth neither multiply nor divide. Then in as much as the three terms of the equation do keep an Arithmetical proportion, I extract the square root of 24—2 R in this manner. I take first half the number of roots, which is 1. Secondly, the square of 1 is 1, to which I add the absolute number, which is 24, because of the sign + before it, that makes 25. Thirdly, I extract the square root of 25, which is 5. Fourthly, from this root I extract the moiety of the number of roots which is 1, because of the sign— the residue will be 4; whence I conclude that to be the value of one root, and that the number sought is 4, whose double is 8, added to the square (of 4,) 16, makes 24.

Here note that numbers diminished, where the absolute number hath the sign—, have two roots. The greater is extracted, as before we have taught; the lesser is found out by substracting the square root of the residue from the half sum of the roots. As if I seek a number whose octuple diminished by 12, shall be equal to its square, you will find an equation between 1 Q and 8 R—12. The greatest root is 6, the lesser

lesser is 2, here both the roots answer the question. But this happens not often.

But if you be to extract the biquadrate root: First, extract the square root, as is taught, and again extract the square root of this, and this shall be your biquadrate root. As if the equation be between 1 QQ and 2 Q + 8, you may find the square root to be 4 by the method taught, first, taking half the number of the squares, *&c.* and afterward you must exstract the square root of 4, which is 2, this shall be the value of the root. In like manner, if your equation be between 1 QC and 2 C + 48, first, I extract the square root of 2 C + 28, which is 8, of which extract the square root 2, because the root to be extracted is the square cube, as the character QC, which is one of the terms of the equation, denoteth.

Sect. 6. *How to know if the* question *be impossible, vain, or ill propounded.*

You may know the question to be impossible, if you come to an equation impossible: As if following the conditions of the Probleme, you meet with an equation between 6 R and 24 R, or between 3 Q + 5 and 4 + 2 Q.

Secondly, the question is vain, when the equation is between two equal numbers of the same denomination, as between 6 Q and 6 Q.

Thirdly, the question is ill propounded, when without any difficulty many numbers will answer the probleme propounded.

CHAP. III.

Algorithm *of Second Roots, with their Use.*

ALgebraists sometimes use more than one root to find out divers numbers propounded, and then to the end they may proceed with less confusion, they usually help themselves with second roots which they express thus, 1 A, 1 B, *&c.*

Sect. 1. Addition *of Second Roots.*

If your Second roots be of the same denomination add the numbers, and to the sum give the same denomination, as 5 A added to 4 A make 9 A, if they be of different denomination, add them with the sign of + as 5 A added to 6 B make 5 A + 6 B.

Sect. 2. Substraction *of Second Roots.*

If they be of the same Denomination, substract one Number from the other, and to the residue give the same Denomination, as 5 A taken from 9 A, there remains 4 A, if different, they are substracted with the sign — as 6 B taken from 8 A, there rests 8 A — 6 B.

Sect. 3. Multiplication of *Second Roots.*

If they be of the same Denomination, do as you do with the first Roots, as 4 A multiplied by 7 A make 28 A Q, if of different, both Denominations are retained in the Product, as 3 R multiplied by 5 A make 15 R A.

Sect. 4. Division *of Second Roots.*

Division is only performed by the interposition of a little line, as is before taught, notwithstanding it is to be observed, that if for Example 3 A R be to be divided by the divisor 1 R, the quotient shall be A, because in such case there needs nothing else but to take from the Dividend the Character of the Divisor.

Sect. 5. The Extraction and Use of Second Roots.

After you have found and reduced your Opinion, according to the manner of Working in second Roots, you must extract the Root after the manner taught in the precedent Chapter, As if 1 A be equal to 25, I say that 5 is the value of the second Root, and if 1 A Q be equal to 4A+12, you must take the moiety of the number of Roots, *&c.* As is said in the 5 Sect. of the precedent Chapter, and you shall find 6 to be the value of 1 A.

Now since the end of the second Roots is to be reduced to first, you must not forget after you have found the value to begin again your work, and to put in first Roots that which you have found to be the value of the second, as I shall shew you in some Examples in a Chapter following.

CHAP.

CHAP. IV.

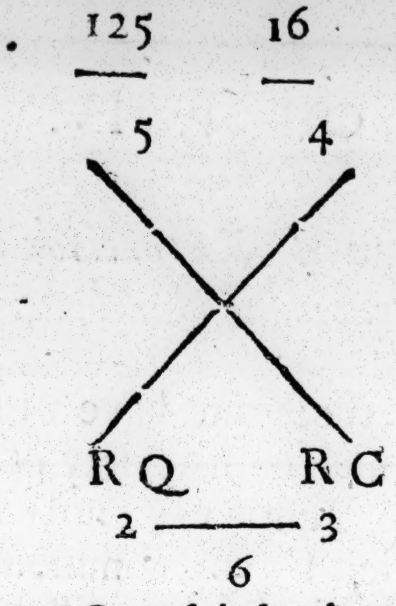
The Algorithm and Extraction of the Roots of Surd and Irrational Numbers.

Surd Roots are those that have a radical sign before them, and which in propriety of speech ought to be called absolute numbers, notwithstanding they cannot be expressed by any common Number, neither whole, nor broken; we will hereafter express the Radical Sign by this Character R.

There are many Sorts of Surd Roots, some are simple, as R Q 5, that is to say, the root square of 5, others are compound, as R Q 5 + R C 6, that is to say, the root square of 5, *Plus* the root cube of 6; some are universal, whose radical character extends to all the particulars following it, and for that end are enclosed in a parenthesis in this manner, R Q (14 + R Q 4) the root universal of 14, joyned with the root square of 4, all which number is 4 for 14 + the root square of 4 which is 2, maketh 16, whose root is 4.

Sect. 1. Reduction of Surd Roots Simple to the same Denomination.

First, you must put the radical signs under the numbers to which they belong. Secondly, you must multiply the numbers by the signs across, for to get new ones. Thirdly, you must add the signs together, which is done by multiplying their Exponents, and give the Character of the product common to the two new products; as if you would reduce to the same denomination R Q 5 and R C 4, you must first place them as followeth.



Secondly, you muſt multiply the the numbers 4 and 5 by their ſigns a croſs, that is to ſay, you muſt take the ſquare of 4, and the cube of 5, which are 16, and 125. Thirdly, the exponents of the ſigns R Q and R C, which are 2 and 3, ought to be multiplied together, the product is 6. I ſook then in the Table what coſſick character is under the exponent 6, and finding Q C, I take that for my common denominator, and inſtead of my two firſt ſurd roots, which were of different denomination, that is to ſay R Q 5, and R C 4. I have two new ones of the ſame denomination, that is to ſay, R Q C of 125, and RQC of 16.

Sect. 2. Multiplication *and* Diviſion *of Surd Simple Roots.*

If the roots be of the ſame denomination, you muſt onlymultiply and divide by the numbers by themſelves, and to the product and quotient give the ſame radical ſign; as R Q 7 by RQ 2, give for the product R Q 14. In likemanner R Q 36 divided by RQ 12, gives for the quotient R Q 3.

But if the Roots be of different denomination, you muſt reduce them to the ſame denomination by the

precedent

precedent Paragraph, and after multiply and divide as ſhall be ſhewed; for example, R Q 3 multiplied by 2, the product is R Q 12, and R Q 12 divided by 2, the quotient is R Q 3.

Sect. 3. *How to know whether two Surd Roots be commenſurable or not.*

You muſt divide the greateſt root by the leſſer, if the quotient be rational, the two roots are commenſurable: if otherwiſe, they are not. As becauſe R Q 24 divided by R Q 6, gives for the quotient R Q 4, which is 2 a rational number; I conclude theſe two roots R Q 24 and R Q 6, to be commenſurable. In like manner, ſince root ſquare 24 divided by R Q 8, the quotient will be 3, a ſurd number and irrational, you may conclude thoſe two roots, R Q 24, and R Q 8, to be incommenſurable.

Sect. 4. Addition *of Simple Irrational Roots.*

If the roots be incommenſurable, you muſt add them only by the ſign + as R Q 24 added unto R Q 8 makes R Q 24 + R Q 8. But if they be commenſurable, you muſt add a unite to their quotient rational, and you ſhall have a ſum, which being multiplied by the leſſer of the two roots to be added, will give a product which ſhall be the ſum ſought. As R Q 24 added with R Q 6 makes R Q 54, becauſe R Q 24 divided by R Q 6 gives 2 for the quotient rational, to which I add a unite, and it is 3, by which (always reducing them to the ſame denomination) I multiply R Q 6, which is the leſſer of my two roots, and I find for my ſum R Q 54.

Sect. 5 Subſtraction *of Simple Irrational roots.*

If they be incommenſurable, you muſt ſubſtract

them by prefixing the ſign—as R Q 8 ſubſtracted out of R Q 24, the reſidue ſhall be R Q 24—R Q 8.

But if they be commenſurable, you muſt take away a unite out of the quotient rational, and you ſhall have the reſidue, the which being multiplied by the leſſer of the roots given, ſhall give a product which ſhall be the reſidue ſought; as if I be to ſubſtract R Q 6 from R Q 24, dividing the greater by the leſſer, the quotient is 2, from which if you take 1, there remains 1, by which (reducing them firſt to one denomination) if you multiply the leſſer root, that is to ſay, R Q 6, the reſidue will be R Q 6.

Sect. 6. Addition *and* Subſtraction *of Surd Numbers, compoſed and diminiſhed.*

I have here no new Precepts, only advertiſe you, that you may remember what I have ſaid before of Coſſick Numbers, touching the ſigns of + and — in the fifth and ſixth Paragraph of the firſt Chapter, and what is delivered in the fourth and fifth of this Chapter, touching the Addition and Subſtraction of Simple Surd Numbers; and theſe will be no difficulty, as if you be to add 5 + R q. 24 with 3 + R q 6, you will find 8 + R q. 54. In like manner, if you ſubduct 3 — R q 6, from 5 + R q. 24, there reſts R q. 54. — 42.

Sect. 7. Multiplication *of Numbers ſurd, compoſed, and diminiſhed.*

This Multiplication hath no great difficulty, nor needs new precepts. Remember only that the ſame ſigns have in the Product + and different — with this, that your Multiplication is not good, if the particulars to be multiplied be not firſt reduced to the ſame denomination. For example, 5 + Rq. 24 by 3 — R q. 6,

q. *6*, 'tis to be done after this manner: + R q. 24 by — R q. 6, maketh — R q. 144, or — 12 after + 5 by — R q. *6*, maketh — R q. 150. Further + R q. 24 by 3 is + R q. 216. Lastly, + 5 by 43 is + 15, then the whole Product will be 15 + R q. 216, — Rq 150. — R q. 144, or 3 + R q. 316 — Rq. 150, because R q. 144 is a rational number, to wit 12, which being subducted out of 15, because of the sign — leaveth 3.

Example:

5 + R q. 24
3 — R q. 06

— R q. 150 — R q. 144
15 + R q. 216.

15 + R q. 216. — R 150 — R q. 144
3 + R q. 216 — R q. 150.

25 | | 9
6 5 | | 24 3
X | | X
R q. 0 | | R q 0
R q. 150 | | R q. 216

Sect. 8. Division *of Surd Numbers, compounded or diminished.*

If the Divisor be simple, the Division is made by the interposition of a line between the Divisor and the compound number to be divided; as if R q. 2 + R q. 5 be divided by 8, the quotient will be $\frac{\text{Rq. } 2 + \text{Rq. } 5,}{8}$ and so of others.

But because it sometimes may fall out (though

very ſeldome) if the Diviſor alſo will be a Binome, or compound number, that is to ſay, a ſurd number compounded of two particulars with the ſign +, or a Trinome that is compounded of three particulars, &c. See here the manner to divide in ſuch a caſe.

If the Diviſor be a Binome, you muſt multiply by his Apotome, as well the number to be divided, as the Diviſor (and if the Diviſor be an Apotome, you muſt divide by the Binome, as well the Dividend, as the Diviſor) by means of this Multiplication you ſhall have a new Dividend, and a new Diviſor. Now this new diviſor will be alwayes rational, and therefore needs only to be ſet under the Dividend with a line between. As for example, R q. 6 — 2 by R q. 5 + R q. 3, I take the Apotome of my Diviſor (that is) R q. 5 — R. q. 3, by which I multiply both my Dividend and my Diviſor, of one by the Multiplications is produced R q. 30 — R q. 20 — R q. 18 + R q. 12, for my new Dividend, and by the other is produced 2 for my new Diviſor. So that the quotient of my Diviſion will be

$$\frac{\text{R q. 30} - \text{Rq 20} - \text{R q. 18} + \text{R. q. 12.}}{4}$$

becauſe 2 muſt be firſt ſquared, and then 4 put underneath the Dividend.

If your Diviſor be a Trinome, you muſt obſerve the ſame method, multiplying the Dividend and the Diviſor by the Apotome of the Diviſor, that is to ſay, by the ſame Diviſor onely, changing the ſign of the laſt particular. After this done, you ſhall have a new Dividend and a new Diviſor, which ſhall be a Binome. Then you muſt again ſeek a new Dividend and Diviſor, which now will be ſimple and rational.

Laſt of all, if you will not take this pains, the Diviſion is good, if under the Dividend you ſubſcribe the Diviſor with a line between.

Sect. 9. Multiplication *of roots univerſal.*

You muſt reduce the root to be multiplied, and the Multiplicator to their ſquares or cubes, according to the radical ſign prefixed, and afterward perform your multiplication, as is taught in Sect. 7. of this Chapter. Afterward you muſt affix the ſign radical, and incloſe all in a Parentheſis.

This is better underſtood by an example, as if you multiply ℞ q. (7 ℞ q. 3) by 2, the ſquares of the one and other numbers are 7 + ℞ q. 3 and 4, then the firſt being multiplied by the laſt, maketh 28 + R q. 48, and therefore if you cloſe this number within a Parentheſis, and put before it the ſame radical ſign, the product will be ℞ q. (28 + ℞ q. 48.)

In like manner, if you would multiply this number $\frac{7}{2}$ — $\frac{1}{2}$R + R q. ($\frac{49}{4}$ — $\frac{3}{4}$ q. — $\frac{7}{2}$ R) by it ſelf, to get the ſquare of it, you muſt call to mind the fourth propoſition of the ſecond Book of *Euclide*, which ſheweth that a line being divided into two parts. the ſquare to the whole is equal to the ſquare of the parts, and to double their Rect-angles, you muſt therefore conceive this number, as divided into two parts, of which the firſt is $\frac{1}{2}$ — $\frac{1}{3}$ R, and the laſt ℞ q, ($\frac{29}{4}$ — $\frac{3}{4}$ q. — $\frac{7}{2}$ R) take then the ſquares of the parts, which are $\frac{42}{4}$ + $\frac{3}{4}$ q. — $\frac{7}{2}$ R, the double of the Rect-angle of the parts is ℞ q. ($\frac{9604}{16}$ + $\frac{229}{16}$ q. — $\frac{2744}{8}$ R + 7 C — $\frac{32}{16}$ q.q.) then the ſquare of the number propoſed is the ſum of theſe three numbers, that is to ſay, $\frac{49}{2}$ — $\frac{1}{2}$ q. — 7 R + ℞ q. ($\frac{9604}{16}$ + $\frac{220}{16}$ q. — $\frac{2744}{8}$ R + 7 C $\frac{11}{16}$ q. q.) The ſquare of the Apotome is the ſame number, putting only the ſign — before the univerſal root and the ſum of the two ſquares is 49 — 1 q. — 14 R.

Sect. 10. Diviſion *of Roots univerſal.*

You muſt reduce the roots to be divided, and the

Divisor to their Squares, Cubes, *&c.* And after divide them as is taught in the 8 Sect. and when this is done, enclose all in a Parenthesis, with the same radical sign which was before. As if you divide R q (13 + R q. 17) by the root square of 5, their squares are 13 + R q. 17 and 5, then the first being divided by the last, the product will be 2 $\frac{3}{5}$ + R q. $\frac{17}{25}$. then the quotient of the division proposed will be R q. (2 $\frac{3}{5}$ + R q. $\frac{17}{25}$.)

Sect, 11. Addition *and* Subduction *of roots universal.*

Many trouble themselves to give precepts intricate enough, the short and most certain is to add them with the sign +, and subduct them with the sign —. As for example, R q. (3 + R q. 2) added with R q (Rq. 5 + 6) will be Rq. (3 + R q. 2) + Rq. (Rq 5 + 6) and the same first root subducted from the last, the residue is Rq. (Æ q. 5 + 6) — Rq. (3 + Rq. 2,)

Sect. 12. Extraction *of the roots of* Binomes *and* Apotomes.

1. Take the difference of the squares of the one and the other part of the Binome. 2. Add and subduct the square root of this difference from the greatest part of the Binome. 3. Conjoyn the square root of the moiety of the sum, with the square root of the moiety of the residue by the sign +, if it be a Binome, and by the sign —, if it be an Apotome; and thus the extraction is finished. As if you would extract the square root of this Binome $\frac{3}{2}$ + Rq. $\frac{5}{4}$, you shall first take the square of the first part, which is, $\frac{9}{4}$, and the square of the second, which is $\frac{5}{4}$, the difference of these two is $\frac{4}{4}$, that is to say, 1. Secondly, you must extract the square root of this difference, which is 1, you shall add and take it away from the first part of the Binome, by the addition you shall have for your sum, and by subduction $\frac{1}{2}$ for your residue.

Thirdly,

Thirdly, joyn the root ſquare of the ſum, with the root ſquare of the reſidue by the ſign + and you ſhall have Rq.$\frac{5}{2}$ + Rq. $\frac{1}{2}$ for the ſquate root of your Binome propounded, and conſequently Rq. $\frac{5}{2}$ —Rq. $\frac{1}{2}$ ſhall be the root ſquare of the Apotome $\frac{3}{2}$ — Rq.$\frac{5}{4}$.

CHAP. V.

The Uſe of Algebra.

'TIS much to have taken the pains to learn all that we have hitherto ſhewn or taught: but I dare boldly ſay, that thoſe that ſhall reſt here, do as yet know nothing to the purpoſe, although they may know all the precepts; it behoveth us then to make a ſtep further, to apply and bring thoſe precepts into uſe and exerciſe. 'Tis that which I deſire to demonſtrate in this Chapter, by ſome *Queſtions*, the ſolutions of which will give great light to the attaining of perfection in this Art. Wherefore I intreat thee (Reader) not to omit this Chapter, in which I pretend to yield thee ſome pleaſure and delight, as alſo an illuſtration of what hath been before treated of.

Sect. 1. *Queſtions reſolved by one ſimple* Equation.

Queſtion. I.

ALexander *one day told* Epheſtion, *That he was elder than him by two years; thereupon* Clitus *tells them, That he was as old as both of them (their ages added together) and four years over and above. The Philoſopher* Calliſthenes *being preſent at this diſcourſe (ſaith he) I well remember that my father, who was 96 years old, had the age of you three. It is demanded here, how old* Alexander *was when he held this diſcourſe,*

course, as also how old Clitus *and* Epheſtion *were?*

I put for the age of *Epheſtion* 1 R of years, whence it follows, that *Alexander* had 1 R + 2; therefore *Clitus* had 2 R + 6, and thoſe three together, according to the condition of the queſtion, ought to be equal to 96: therefore there is an equality between 4R + 8 (which is the ſum of the three ages) and 96, take away 8 from both parts of the equation, ſo there will remain on one ſide 4 R equal to 88, which divide by the number of the greateſt Coſſick Character, that is to ſay by 4, the quotient gives 22 for the value of one root, which was ſuppoſed for the age of *Epheſtion.* Therefore *Epheſtion* was at that time aged 22 years, *Alexander* 24, and *Clitus* 50, which altogether make 96 years.

Queſtion II.

A Hare is a 100 *Geometrical paces diſtant from a Dog that ſwiftly purſues her, and the Dog runneth two times and half faſter than the Hare. It is demanded how many Geometrical paces the Hare will have run when the Dog overtaketh her?*

I put for theſe Geometrical paces 1 R; therefore the Dog which runs 100 paces more than the Hare, will have run 100 + 1 R: and for that the Dog runs twice and an half ſwifter than the Hare, I take two numbers in like proportion to one another, that is to ſay 5 and 2, and conclude that there is the ſame proportion between 100 + 1 R to 1 R, as between 5 and 2; therefore the product of the firſt number 100 + 1 R multiplied by the laſt number 2 (which is 200 + 2 R) is eqnal to the product of the two means 1 R and 5 (which will be 5 R) therefore if you take from both parts 2 R, there will remain 200 by 3, which is the number of the greateſt Character, and find

find in the quotient $66\frac{2}{3}$, which will be the value of the root. I say therefore that the Hare will have run 66 Geometrical paces and $\frac{2}{3}$, when the Dog shall have overtaken her, and the Dog will have run $166\frac{2}{3}$ paces, which make twice and an half more than $66\frac{2}{3}$.

Question III.

The Architect Vitruvius *in his* Ninth Book Chap. 3. *tells us, That* Archimedes *found the quantity of silver that a Goldsmith had mixed in a Golden Crown which he had made for the King* Hiero (*who was obliged by Vow to present it to the Gods*) *weighing* 100 *pounds. It is demanded by what means* Archimedes *could arrive to the knowledge of that secret?*

The common Opinion is, that he took two masses, the one of gold, the other of silver, which weighed as much as the Crown; afterward he filled a Vessel up to the brim with water, which Vessel was placed in some great Bason, that the water that should be forced out of the first Vessel, might be preserved and not lost. Thirdly, he gently put in the two masses and the Crown, one after the other, into the prepared Vessel, taking exact notice of the quantity of water that issued out of the Vessel at each time, and concluding from thence, that the Goldsmith had mingled 6 pounds and $\frac{2}{3}$ parts of silver. We will suppose then, that the masse of gold weighing 100 pounds, did cast out of the Vessel 60 pounds of water, and that the masse of silver also weighing 100 pounds, cast forth of the Vessel 90 pounds of water, and that the Crown cast forth 65 pounds. I put afterward for the silver mixed in the Crown 1 R, and constitute twice the Rule of Three after this manner.

If 100 pounds of gold give me 60 pounds of water, how much will 100 — 1 R? and I find $\frac{6000 - 60R}{100}$ for my fourth number.

Secondly

Secondly, if 100 pounds of silver give me 90 pounds of water, how much shall 1 R? and I find $\frac{90R}{100}$. Now these pounds of water $\frac{6000-60R}{100}$ and $\frac{90R}{100}$ added together do make $\frac{6000+30R}{100}$ pounds of water cast out, which ought to be equal to 65 pounds of water, cast forth by the Crown, and therefore if we reduce them, we shall find 6000 + 30 R equal to 6500 (this reduction is made by multiplying the denominator 100 by 65, for seeing that this fraction $\frac{6000+30R}{100}$ is equal to 65, it shall be also equal to $\frac{65}{1}$, and therefore there will be the same proportion of the numerator 6000 + 30 R to the denominator 100, as of the second numerator 65 to 1. Therefore the product under the extreams 6000 + 30 R (is equal to the product of the means 6500) take away therefore from both parts 6000, and there will remain an equation between 500 and 30 roots, and therefore divide 500 by 30, the number of the greatest character, you shall have the value of the root 16 $\frac{2}{3}$ for the pounds of silver mingled by the Goldsmith in the Crown.

Mark well this kind of reduction once for all.

Sect. II. *Questions resolved by an* Equation *compounded.*

Question. I.

TO *divide* 8 *into two such numbers as their squares being added together, may make* 34?

I put for the first 1 R, therefore the second shall be 8 — 1 R, their squares are 1 Q, and 64 + 1 Q — 16 R; which added together, do give for their sum 64 + 2 Q — 16 R, the question imports that the sum of the squares is 34. Therefore there is an Equation between 64 + 2 Q — 16 R, and 34, which being reduced by addition and substraction, there will remain also

alſo an Equation between 2 Q and 16 R — 30, and the whole divided by 2, which is the number of the greateſt Coſſick Character, there will yet remain an Equation between 1 Q and 8 R—15, from which I extract the root, as hath been ſhewn in the fifth Section of the ſecond Chapter. The half of the root is 4, his ſquare is 16, from which take the abſolute number 15, reſt 1, whoſe ſquare root 1 added to the half of the number of roots, gives for its ſum 5, which is the value of the root; therefore the two numbers ſought ſhall be 5 and 3.

Queſtion II.

To find two numbers whoſe product may be 12, *and the difference of their ſquares* 32?

I put for the one of them 1 R; therefore ſeeing that the product is 12, the other number ſhall be $\frac{12}{1R}$ (for if the product of two numbers be divided by one of thoſe two numbers, the quotient ſhall be the other number) their ſquares are 1 Q and $\frac{144}{1Q}$, whoſe difference is $\frac{144}{1Q}$—1 Q equal to 32, as appears by the queſtion; therefore there will be an equality between $\frac{144}{1Q}$ and 32+1Q, and therefore if we make the reduction, as in the third Queſtion of the firſt Section, we ſhall alſo find an Equation between 144 and 32 Q + 1 QQ, alſo between 144—3 Q and 1 QQ; it behoveth then to extract the ſquare root of the number 144—32 Q. The half of 32 is 16, whoſe ſquare is 256, to which add 144, makes 400, whoſe ſquare root is 20, from which take the half of 32, to wit 16, there remains 4. See here the ſquare root, but ſeeing that the ſquared ſquare root ought to be taken, I take again the root of 4, and I find 2 for the value of the root. Therefore ſeeing that the ſecond number hath been put $\frac{12}{1R}$ the ſame ſecond number ſhall be $\frac{12}{2}$, that is to ſay 6.

Queſt.

Question III.

Two Merchants joyn in Company, and together bring the sum of 165 *Crowns; but the first mans money hath been exposed twelve months intire, and the second mans money only eight months: it happens that they gain but* 28 *Crowns, which added to* 165 *make* 193, *which they distribute to one another in such sort, as the first takes* 67 *Crowns, as well for his principal money as for his profit, and the second takes* 126 *Crowns, the question is what each of those Merchants brought into the Stock?*

I put for the money of the first man 1 R, therefore seeing that the sum of both was 165 Crowns, the second money is 165 — 1 R. Now if you take away 1 R, which is the sum the first man brought in, from the sum he received, which was compounded of the principal and profit; you will find that the first mans profit will be 67 — 1 R, and by the same Argument you shall find, that the second mans profit will be 1 R — 39. Now you must find what one root gaineth in eight months, which will be done by the Rule of Three, thus, If in 12 months there be gained 67 — 1 R, how much will there be gained in eight months, and the fourth number shall be $\frac{134}{3}$ — $\frac{2R}{3}$ for the first mans profit in eight months; after that I seek what the second man hath gained by another operation of the Rule of Three, saying, if 1 R gain $\frac{135}{3}$ — $\frac{3R}{2}$ what will 165 — 1 R gain? and I find for my fourth number $\dfrac{7370 + \frac{2}{3}\,Q - \frac{464}{3}\,R}{1\,R}$ which is equal to the second mans profit, which we have already found to be 1 R — 39, and therefore by Reduction there will be an equation between 1 Q — 39 R and

7370

$7370 + \frac{2}{3}Q - \frac{464}{3}R$, all which 39 R being added and $\frac{2}{3}Q$ taken away, there will be an equation between 7370 and $\frac{1}{3}Q + \frac{347}{3}R$, and consequently between $7370 - \frac{347}{3}R$ and $\frac{1}{3}Q$, and therefore multiplying all by $\frac{1}{3}$, which is the number of the greatest character, there will be yet an equation between 1 Q and 22110–347 R, out of which the square root must be extracted. The half of the number of roots is $\frac{347}{2}$, whose square is $\frac{120409}{4}$, which added to 22110, make $\frac{208849}{4}$, whose square root is $\frac{457}{2}$, from which if you take half the number of roots, there will remain $\frac{110}{2}$, that is to say, 55 for the value of one root, and was the mony the first man put in bank; and for that we have found in the persuit of these operations, that the first mans profit was 67—1 R, that same profit will be 67 — 55, that is to say 12, by the same reason the second mans stock shall be 110, and his profit 16.

Sect. III. *Questions resolved by* Surd Numbers.

Question I.

To divide any given number (as for example 4) *according to mean and extream reason*, that is to say, *to divide* 4 *in two numbers, in such manner as that the whole* 4 *may bear the same proportion to its greater part, as the greatest part bears to the least?*

I put for the greatest part 1 R, therefore the least shall be 4—R. Therefore there is the same proportion of 4 to 1 R, as of 1 R to 4— 1 R, and therefore the square of the middle part 1 Q is equal to the product of the extreams 16 —4 R, from which I extract the root according to the rule before prescribed. The half of the number of roots is 3, whose square 4, add to 16, makes 20; out of which the square

root

root ought to be extracted according to the precept; but seeing it is no square number, I must content my self by putting the radical sign before it thus, R Q 20, from which I take the half of the number of roots, and I have for residue R Q 20 — 2, which is the value of the root; by which I shall find with facility, that the other part will be 6 — R Q 20. For the proof of this operation, it behoveth that these two parts added together make 4, and that the lesser 6 — R Q 20, being multiplied by 4, make the product equal to the square of the greater part R Q 20 — 2.

Question II.

To divide 8 *into two parts, between which* 2 *may be mean proportional.*

I put for the first part 1 R, therefore he lesser shall be 8 — 1 R, and seeing that 1 R and 2, and 8 — 1 R, ought to be proportional, it behoveth that the square of 2, which is 4, be equal to the product of the extreams, which is 8 R — 1 Q, and therefore after the reduction, it will be found that 1 Q is equal to 8 R — 4, whose square root is R Q 12 + 4. For the one part of 8, and for the other part 4 — R Q 12, both the one and the other root do resolve the question, as you will find, if you will take the pains to examine it.

From this practice may be framed an universal Canon, which may serve for the resolution of an infinite number of Algebraical Problems, which may be conceived after this manner. The sum given, which containeth the two extreams, ought to be distributed into two equal parts, that the square of the half may be taken, from which the square of the mean proportional given, must be taken, and the square root of the residue added, and taken from the half of the given sum, will shew the two parts sought.

As for example, I take the half of 8, which is 4, whose square is 16, from which I take 4, the square of 2, which is the given mean, and there remains 12, whose square root added to the same half, makes, 4 + R Q 12, and taken from the same half, makes 4 − R Q 12.

Question III.

To divide any given number (as for example 4) *into three numbers continually proportional, in such sort as that the squares of the extreams joyned together, may be triple the squares of the mean.*

I put 1 R for the number of the middle part, then seeing all three ought to make the same sum of 4, we shall have for the sum of the extrems 4 — 1 R. Now seeing that of three numbers continually proportional, the square of the sum of the extreams is equal to the squares of the extreams, and to the double of the square of the middle part; I take the square of this sum 4 — 1 R, which is 16 + 1 Q — 8 R, from which I take 2 Q, which is the double of the square of the middle part, and there will remain 16 — 1 Q — 8 R for the sum of the squares of the extreams: Therefore seeing that the condition of the question requireth a triple proportion, there will be an equation between 16 — 1 Q — 8 R and 3 Q; add therefore 1 Q on both sides of the equation, and you shall have 4 Q equal to 16 — 8 R, and dividing the whole by 4, which is the number of the greatest character, the equation will be 1 Q equal to 4 — 2 R, whose square root is R Q 5 — 1 for the middle number sought, and the sum of the extreams shall be 5 — R Q 5, which being divided into two parts, by the Canon of the precedent question, in such sort as R Q 5 --- 1, be the middle proportional, you will find that the extreams are 2 and 3 --- R Q 5, therefore the three numbers are

2 and R Q 5 1 —, and 3 — R Q 5, which altogether make 4, and are in continual proportion, and the ſquares of the extreams are triple the ſquare of the mean.

Sect. IV. *Geometrical queſtions reſolved by* Algebra.

Queſtion I.

There is a piece of ground of a greater length than breadth, whoſe angles are right angles, and in a triple proportion, and their ſquares taken together, are quintuple their ſum. The ſides, the diameter, and the capacity or ſuperfices of that piece of ground is demanded.

Before you attempt the reſolution of ſuch queſtions, you muſt draw their figures.

I put for the leaſt ſide 1 R, therefore ſeeing they are in a triple proportion, the other ſide ſhall be 3 R, their ſquares ſhall be 1 Q and 9 Q, which added together make 10 Q, which ought to be quintuple, the ſum of the numbers. Now the ſum of the numbers is 4 R, and its quintuple 2 R, and by conſequence, ſee here an equation between 10 Q and 20 R, which are two collateral Characters, and therefore dividing 20 by 10, which is the number of the greateſt character, you ſhall find 2 for the leaſt ſide, therefore the greateſt ſide ſhall be 6. Therefore the ſuphurficies ſhall be 12, and the diameter R Q 40.

Queſtion. II.

There is an equilateral triangle, whoſe ſuperficies is R Q 243. *The ſide and perpendicular is demanded.*

Suppoſing that the perpendicular of an equilateral triangle doth always divide the ſide into two equal parts, I put for the half of the ſide divided, 1 R; therefore the ſide ſhall be 2 R. Now ſeeing that in every

every equilateral triangle; the square of the side is equal to the square of the perpendicular joyned to the square of the half of the side, which is 1 Q of the square of the whole side, which is 4, I shall have 3 Q for the square of the perpendicular, and so R Q 3 Q shall be perpendicular, which is multiplied by the half of the side, which is 1 R (reducing it first to its square because of the radical sign, which is in the number multiplied) you shall have R Q 3 QQ, for the surperficies of the triangle: therefore there will be an equation between R Q 243 and R Q 3 QQ, and therefore there will be also an equation between their squares, which are 243 and 3 QQ, and the whole being divided by 3, the number of the greatest character, there will be yet an equation between 81 and 1 QQ. I extract therefore the squared square root of 81, and have 3 for the half of the side, 6 for the side, R Q 27 for the perpendicular, and R Q 243 for the superficies of the triangle.

Question III.

There is a Semicircle, whose Diameter is devided according to mean and extream reason; on which there is raised a perpendicular produced to the circumfernce, and the lesser line which is drawn from the extremity of the diameter, to this point of the circumference, is R Q 20—2, the quantity of the Diameter, and of its parts, and of this perpendicular, is demanded.

To resolve this question, it is presupposed that the greater part of the diameter shall be equal to the given line, as may with facility be Geometrically demonstrated. That being done, I put for the lesser part of the diameter 1 R; therefore seeing that the other part is given R Q 20—2, the whole diameter shall be R Q 20—2+1 R, which multiplied by 1 R, giveth

for the product R Q 28 — 2 R + 1 Q, equal to the ſquare of the given quantity, which is 24—R Q 320, and by due tranſpoſition you ſhall find 1 Q equal to 24—R Q 320 + 2 R — R Q 20, from which the ſquare root ought to be extracted, taking exact notice that the particles which have the Coſſick Characters may hold place, with the number of roots. I conſider therefore in this term of the equation, the number of roots which is 2—R Q 20, whereof I take the half, which is 1— R Q 5, to whoſe ſquare 6 R Q 20, the abſolute number 24—R Q 320, ought to be added, and the ſum will be 30—R Q 500, whoſe ſquare root ought to be extracted as from Apotomes, as hath been ſhewn in the laſt Section of the forth Chapter, that root is 5—R Q 5, which added to the half of the number of the roots 1—R Q 5 gives for the ſum 6—R Q 20, which is the value of 1 root, that is to ſay, of the leſſer part of the diameter; and therefore if you add it to the greater part, you ſhall have 4 for the quantity of the whole diameter, from whence the perpendicular is eaſily known, provided the rules of Geometry be in any reaſonable manner underſtood.

Sect. 5. *Queſtions reſolved by the* Second Roots.

Queſtion I.

Three men have amongſt them a ſum of mony: The firſt ſaith to the ſecond, If you deliver me the half of your money, I ſhall have 100 *Crowns: The ſecond ſaith to the third, If you deliver me* $\frac{1}{3}$ *of your mony, I ſhall have* 100 *Crowns, The third ſaith to the firſt, If you deliver me* $\frac{1}{4}$ *of your money, I ſhall have* 100 *Crowns. I demand how much money each one hath.*

I put for the firſt mans money 1 R of Crowns, and for

for the second mans money 1 A, and for the third mans money 1 B: therefore the first which hath 1 R with $\frac{1}{2}$ of the second mans mony shall have 1 R + $\frac{1}{2}$ A equal to 100, and by consequence $\frac{1}{2}$ A shall be equal to 100 — 1 R, and multiplying the whole by 2 1 A shall be equal to 200 — 2 R. I begin again therefore the operation, and instead of 1 A, I put for my second number 200 — 2 R. Now the question requires that the second man (with $\frac{1}{3}$ of the third mans) shall have 100: therefore there will be an equation between 200 — 2 R + $\frac{1}{3}$ B, and btween 100, add to both parts of the equation 2 R, and take away 200, there will yet remain an equation between $\frac{1}{3}$ B and 2 R — 100, and multiplying the whole by 3, you will have 1 B equal to 6 R — 300: That being found, I begin again the work, and instead of 1 B I put for the third number 6 R — 300, which added to $\frac{1}{4}$ of the first mans, make $\frac{25}{4}$ R — 300, which ought to be equal to 100, therefore if you add on both parts 300, there will be an equation between $\frac{25}{4}$ R and 400, therefore if you divide 400 by $\frac{25}{4}$, you shall find 64 for one root, therefore the second, which had 200 — 2 R, shall have 200 — 128, that is to say 72, and the third shall have 84. Those three numbers do perfectly satisfie all the conditions of the question.

Question II.

Two men divide between themselves three hundred Crowns, in such sort, as that the second mans money divided by that of the first mans, makes $\frac{3}{2}$. It is demanded how much each of them hath.

I put for the first mans mony 1 R, and that of the second 1 A, there is therefore an equation between 1 R + 1 A, and 300, and therefore 1 A is to equal to 300 = 1 R, therefore $\frac{300-1R}{1R}$ is equal to $\frac{3}{2}$, and by con-

consequence in cross multiplying these two Fractions, I shall have 600—2 R equal to 5 R, and dividing 600 shall be also equal to 5 R, and dividing 600 by 5, I shall find 120 for the first mans mony, so shall the other have 180.

Question III.

To find two numbers whose product may be 10, *and the sum of their squares* 29.

I put for the first 1 R, and for the other 2 A, the product is 1 R A, equal to 10. therefore in dividing the whole by 1 R, there will be an equation between 1 A and $\frac{10}{1R}$, and therefore I begin again the oprati-on, and put for the first 1 R, and for the second $\frac{10}{1R}$, their spares are 1 Q $\frac{100}{1q}$ equal to 29, and after the reductions and extractions of the roots, I find 5 and 2 for my sought numbers.

Sect. VI. *Questions* resolved indefinitely.

THat queston is said to be resolved indefinitely, in which the numbers are demonstrated in such Algebraical terms, as do satisfie all the conditions of the question proposed.

Question I.

To divide 12 *into four numbers Arithmetically and continual.*

I presuppose that when there is four numbers in Arithmetical proportion, the sum of the extreams is alwayes equal to the sum of the means: whence it follows, that in our question the sum of the exstreams shall be 9, and the sum of the means also 6, I put for the second 1 R, therefore the third shall be 6 = 1 R,

their

their difference is 2 R — 6, presupposing 1 R to be the greater number of the two; if therefore I add this difference to 1 R, I shall have 3 R — 6, and if I take it from 6 — 1 R, I shall have 12 — 3 R, and therefore the four numbers in continued Arithmetical proportion, shall be 3 R — 6 | 1 R | 6 — 1 R 12 — 3 R | and the question is indefinitely resolved, in pursuit of which you may take such a number as you please for the value of 1 R, provided nevertheless, that you do admit of fained numbers less than nothing: However, if you will have no other numbers than what are real, you ought to take the value of 1 R, beneath 4 and above 2, which will be easily understood by a little experience.

Take for example $\frac{5}{2}$, for the value of 1 R, therefore the first number, which is 3 R — 6, shall be $\frac{15}{2}$ — 6, that is to say $\frac{3}{2}$, the second shall be $\frac{5}{2}$, the third $\frac{7}{2}$, and the fourth $\frac{9}{2}$, which added together make 12, and are in continued Arithmetical proportion. And so you may take an infinite number of others.

Question II.

A Vintner hath three sorts of Wine. the first is worth 4, *the second* 6, *and the third* 10 *pence the pint; of these three sorts of Wine he desires to fill a Vessel which contains* 80 *pints, which may be worth* 8 *pence the pint; I demand how many pints he ought to take of each sort.*

You ought here to consider, that the number 80 must be divided into three such numbers, as the first multiplied by 4, the second by 6, and the third by 10, the sums of the three products added together, may make 640 (because that all the Wine which shall be in the Vessel to be filled will cost 640 pence, seeing that if one pint be worth 8 pence, 80 pints will be worth 640 pence) I put therefore for the third number 1 R,

which multiplied by 10, makes 10 R, which being taken from 640, doth leave for the residue 640—10R, which is a number containing the first 4 times, and the second 6 times. On the contrary, seeing that the third hath been put 1 R, therefore 80 — 1 R, shall make the sum of the first and the second, which multiplied by 4, will give 320—4 R, which being substracted from 640 — 10 R, will leave 320 — 6 R double to the second number, and therefore the second shall be 160 — 3 R. In like manner the same sum 80 — 1 R multiplied by 6, will produce 480—6 R; therefore if you take way 640 — 16 R, there will remain 4 R—160, double to the first, and therefore the first shall be 2 R — 80. See here the Question resolved indefinitely, the first number is 2 R— 80, the second 160 — 3R, and the third 1 R. The terms between which you ought to take the value of 1 R are $53\frac{1}{3}$, and 40; if therefore you take 46 for the value of 1 R, you shall have 46 pints of Wine of 10 *d.* the pint, 22 of 6, and 12 of that of 4 pence the pint.

A special Note. Here I intreat you to consider, That it is impossible perfectly to understand the Rule of Allegation, without the knowledge of *Algebra*: For if you propose this question to one skilful only in Arithmetick, he will give you for the 3 sought numbers, 40, 20, and 20. And if you tell him, that of the sort of Wine of 4 pence the pint, you have but 16 pints, he will remain astonished, whereas by your question resolved indefinitely by *Algebra*, you will be able to give satisfaction to this condition infinite kinds and manners.

Question. I.

Two numbers are sought, which have 56 for the difference of their Cubes, and which added together make 6.

I

I put for the difference of thoſe numbers 1 R, and if from the difference of thoſe two Cubes you take the Cube of the difference of the ſides, dividing this reſidue by the triple of the difference of the ſides, you have for the quotient the product of the ſides, it follows, that if from 56 you take 1 C, and that the reſidual 56 — 1 C be divided by the difference of the ſides, which is R, the quotient $^{56}\frac{10}{3R}$- ſhall be triple the the product of the ſides; and therefore if you divide this quotient by 3, you will have for the product of the ſides $\frac{56}{3}-\frac{1q}{R}$, which is equavalent to $\frac{56}{3R} — \frac{1}{3}q$ therefore if you divide 6 (which is the ſum of your two ſought numbers) into two parts, whoſe product may be $3\frac{36}{R} — \frac{5}{3}q$, you will have the queſtion reſolved indefinitely. Now to attain this, I have given you a Canon in the ſecond Queſtion of the third Section, the half of the ſum is 3, whoſe ſquare is 9, from which if you take the product found, there will remain $9+\frac{1}{3}q -- \frac{56}{3R}$ whoſe ſquare root added, and taken from the half of the ſum, gives for the reſolution $3 + R\ (9+\frac{1q}{3R} --- \frac{36}{3R})$ which ſhall be the greateſt ſought number, and $3 — R\,Q\,(9 + \frac{1}{3}q --- \frac{56}{3R}$ which ſhall be the leaſt. Now theſe two numbers reſolves the Queſtion indefinitely in ſuch ſort, that if you take two for the value of the root, you will find that your two ſought numbers ſhall be 4 and 2, and every other number (taken for the value of 1 R) above 2, will reſolve the Queſtion.

Appendix.

Appendix.

Questions in *Algebra*, most of which require the Rule of Three in their Operation.

Question I.

A Merchant receives in exchange for 568 Crowns, four kinds of Mony, of the first 7 make one Crown, of the second 18, of the third 21, and of the forth 28 make one Crown. Moreover, he recived of each sort of Mony alike number. I demand how much he received of each kind of Money?

Put 1 R for the quantity of each kind of Money, and then constitute the Rule of Three after this manner:

$$\text{If}\left\{\begin{array}{c}7\\18\\21\\28\end{array}\right\}\begin{array}{l}\text{Pieces of}\\\text{money be}\\\text{worth}\end{array}\left\{\begin{array}{l}\text{Crown}\\1\ \text{what}\\\text{is}\end{array}\right\}\begin{array}{c}\text{money}\\1\text{R}\end{array}\left\}\begin{array}{l}\text{I an-}\\\text{swer}\end{array}\right\}\begin{array}{c}\text{Crowns.}\\\frac{1}{9}\text{R}\\\frac{1}{18}\text{R}\\\frac{1}{21}\text{R}\\\frac{1}{28}\text{R}\end{array}$$

For if 7 pieces of mony make 1 Crown, 1 R of mony will make $\frac{1}{7}$ R of Crowns, and so of the rest. Now their Fractions described in the fourth place, do make together $\frac{71}{252}$ R of Crowns, equal to the number of 568 Crowns, Divide therefore 568 by $\frac{71}{252}$ and you have for the value of 1 R 2106, and so many pieces of each kind of money he received: which thus I prove:

							Crowns.
If	7 18 21 28	Pieces are worth	1 Crown	what	Pieces 2016 Answer		288 12 96 72

For 2016 pieces of the first kind of mony do make 288 Crowns, and as many of the second kind make 112, as many of the third kind make *96*, and of the fourth kind 72, which added together, do make 568 Crowns.

Question II.

A certain man hath two measures of Wine, the one worth 12 *Crowns, the other* 15. *Now he desires of both these Wines to fill another equal measure, whose worth may be* 13 *Crowns. I demand what part of each of those Wines he must take to fill the other to be worth that price?*

Put 1 R for the part of the measure of the worst Wine, and for the part of the measure of the best Wine 1 — 1 R, then work by the Rule of Three thus:

	Meas.	Cro.		Crowns
Worst Wine	1	12	1 R?	make 12 R
Best Wine	1	15	1—1R?	make 15 — 15 R

For if one measure of the worst Wine be worth 12 Crowns, 1 R of one measure of the same Wine will be worth 12 R of Crowns, and if one measure of the best Wine be worth 15 Crowns 1—1 R of a measure of the same Wine will be worth 15—15 R. Therefore 1 R measure of the worst Wine, and 1—1R measure of the Best will be worth 15 —3 R Crowns, which ought to be equal to 13 Crowns. Add therefore 3 R to each part of the equation, and the equation will be between 2 R + 13 and 15; take away therefore 13 from both sides, and the equation will be between

tween 3 R and 2. Divide therefore 2 by 3, and you shall have $\frac{2}{3}$ for the value of 1 R, and so much ought to be taken of the measure of the worst Wine, and $\frac{1}{3}$ part of the measure of the best Wine, which thus I prove by the Rule of Three:

	Meas.	Cro.			Crowns	
Worst Wine	1	12	$\frac{2}{3}$?	worth	8	13 Cr.
	1	15	$\frac{1}{3}$s		8	

For $\frac{2}{3}$, of a measure of the worst Wine is worth 8 Crowns, and $\frac{1}{3}$ of the measure of the best Wine is worth 5 Crowns, which added together make three.

Question III.

I have a measure of Wine worth ten Crowns; How much Water must I mix with one measure, that a mixed (like) measure may be worth seven Crowns?

Put for the measure of Water 1 R, then frame the question by the Rule of Three, thus:

Meas. Wine,	Meas. Water,	Crowns	Meas. mixt		Crowns
1 + 1 R		10	1?	worth	$\frac{10}{1 + 1\,R}$

For if a measure of wine, together with 1 R of a measure of Water, be worth 10 Crowns, one measure of Wine and Water mingled together will be worth $\frac{10}{1 + 1\,R}$ and so the equation will be between $\frac{10}{1 + 1\,R}$ and 7, which by cross multiplication is reduced to 10 and 7 + R, take away 7 from each part of the equation, and it will be between 3 and

7 R;

7R; divide 3 by 7, and you have for 1 R, $\frac{3}{7}$, and so much Water ought to be mingled with a measure of wine, that a measure of the mixture may be worth seven Crowns, and is thus proved:

Measure of Wine and Water	Crowns	Measure	Crowns
$1\frac{3}{7}$	10	1? worth	7

For if 1 of Wine and $\frac{3}{7}$ of Water be worth 10 Crowns, 1 of that mixture is worth 7 Crowns.

Question IV.

There are in a certain Vessel 20 measures of Wine, of which each of them are worth 12 Crowns the measure: Now this Vessel is filled up with Water, and then one measure of this mixture is worth 12 Crowns, I demand the content of the Vessel.

Put for the measure $20 + 1\,R$, then work by the Rule of Three thus:

Measures of Wine	Measures of Water	Cro.	Meas. Mixt	Crowns
$20 + 1\,R$		240	1? worth	$\frac{240}{20 + 1\,R}$

For if one measure of Wine be worth twelve Crowns, a measure of Wine 20 + 1 R measure of Water together will be worth 240 Crowns: therefore one measure mixt will be worth

$\frac{240}{20 + 1\,R}$ Crowns, so the Equation is found be-

tween $\frac{240}{20 + 1\,R}$ and 10, which by cross multipli-
cation

caution is reduced to 240 and 200+10 R; take away 200 from both parts, and there will remain an equation between 40 and 10 R: Divide 40 by 10, and you have 4 for the price of the root, and so many measures of Water were put into the Vessel, and therefore the whole Vessel contains 24 measures, thus proved: 24 measures Wine and Water worth 240 Crowns, 1 measure worth 10 Crowns.

Question V.

Two Letter-Carriers belonging to two Cities distant each from other 140 leagues set forth towards one another, at one and the same time; the one travels eight leagues a day, the other six. I demand on what day they shall meet together.

Put 1 R for the day, then work by the Rule of Three thus;

Dayes	Leagues	Dayes	Leagues
1	8	1 R?	8 R.
1	6	1 R?	6 R.

The first therefore in 1 R of days shall travel 8 R of leagues, and the later 6 R. and both of them together will have measured 14 R, of leagues, that is 140 leagues. There is therefore an equation between 14 R and 140. Divide then 140 by 14, and you shall have ten for the value of 1 ℞. So the tenth day finished, they met together, which thus I prove.

Dayes	Leagues	Dayes	Leagues
1	8	10?	80
1	6	10?	60

For the first in 10 dayes went 80 leagues, and the later went 60 leagues, which both together make 140, the distance of the two Cities from one another.

Question

Question VI.

A certain Merchant bought a quantity of Wool, and another quantity of wax, which cost him together 124 Crowns. Now 100 pound weight of Wooll cost him 7 Crowns, and 100 pound of wax cost him 14 Crowns; but the quantity of Wooll that he bought was double to the quantity of Wax. I demand how many pounds of each sort the Merchant bought?

Put for the wax 1 R, and for the wooll 2 R of pounds, then work by the Rule of Three thus:

Pounds	Crowns	Pounds	Crowns
100 Wooll	7	2 R Wooll?	$\frac{14}{100}$ R
100 Wax	14	1 R Wax?	$\frac{28}{100}$ R

Therefore there will be an equation between $\frac{14}{100}$ R of Crowns, and 124 Crowns. Divide therefore 124 by $\frac{28}{100}$, and you shall have for the value of 1 R $442\frac{6}{7}$, and so many pounds of Wax he bought, and of Wooll $885\frac{5}{7}$, which is thus proved:

100 Pounds,	Wooll 7 Crowns	885 Pounds $\frac{5}{7}$	*62* Cr.
100 Pounds,	Wax 14 Crow.	442 Pounds $\frac{6}{7}$	62 Cr.

And so for Wooll and Wax, together he expended 124 Crowns.

Question VII.

One buys a number of Ells of Velvet, which he selleth agan; he buys 5 Ells for 7 Crowns, and sells 7 Ells for 11 Crowns, and gained on the Bargain 100 Crowns. I demand how many Ells of Velvet he bought and sold in all?

Put

Put for the quantity of Ells 1 R, then work by the Rule of Three thus:

Ells	Crowns	Ells	Crowns
5	7	1 R?	$\frac{7}{5}$ R
7	11	1 R?	$\frac{11}{7}$ R

You see if $\frac{7}{5}$ R of Crowns which he laid out be substracted from $\frac{11}{7}$ R of Crowns which he received, there will remain $\frac{6}{35}$ R of Crowns for the profit. Therefore the equation will be between $\frac{6}{35}$ R and 100 R, divide 100 by $\frac{6}{35}$, and you have 583 $\frac{1}{3}$ for the value of 1 R, and so many Ells he bought and sold. Thus proved:

Ells	Crowns	Ells	Crowns
5	1	583 $\frac{1}{3}$?	816 $\frac{2}{3}$
7	11	583 $\frac{1}{3}$?	916 $\frac{2}{3}$

By which you may perceive there is 100 Crowns gotten.

Question VIII.

A certain man buys a number of Ells of Velvet, paying 11 *Crowns for* 7 *Ells: Now he sells the whole again after the rate of* 5 *Ells for* 7 *Crowns, and lost* 100 *Crowns by the bargain. I demand how many Ells he bought and sold in all.*

Put for the number of Ells 1 R, and then work by the rule of Three thus:

Ells	Crowns	Ells	Crowns
	11	1 R?	$\frac{11}{7}$ R.
5	7	1 R?	$\frac{7}{5}$ R.

Then

Then if $\frac{7}{5}$ R of crowns which he received, be substracted from $\frac{11}{7}$ R of crowns which he expended, the loss will happen to be $\frac{6}{35}$ R of crowns. So the equation will be between $\frac{6}{35}$, R of crowns, and 100 crowns. Divide therefore 100 by $\frac{6}{35}$, and you shall have for the value of 1 R, 583 $\frac{1}{3}$ Ells, and so many Ells were bought and sold, which thus I prove:

Ells	Crowns		
5	11	583 $\frac{1}{3}$?	916 $\frac{2}{3}$
7	7	583 $\frac{1}{3}$?	816 $\frac{2}{3}$

Where you see he lost 100 crowns by the bargain.

Question IX.

A man buys 100 *pounds of Wax for* 17 *crowns. I demand how many pounds he must sell for one crown, that so on* 102 *crowns he may gain* 18 *crowns?*

Put 1 R for the number of pounds, then work by the Rule of Three thus framed:

Crowns	Pounds	Crowns	Pounds
17	100	102	600
5	1 R	132 + 18 ?	128 R

For 102 crowns do give 600 pounds, and if for one crown there be given 1 R of pounds, there will be 120 R of pounds given for 102 + 18, which said 120 are equal to 600 pounds, the quantity of the Wax sold. Therefore the equation shall be between 120 R of pounds and 600 pounds. Divide therefore 600 by 120, and so you have for the value of 1 R, 5 pounds, and so many pounds are sold for one crown; so as that in 600 pounds 18 crowns may be gained on 102, thus proved:

Pounds	Crowns	Pounds	Crowns
200	17	600 ?	102
5	1	600 ?	120

Where in the first example 600 pound made 102, in this it makes 120, that is 102 + 18.

Question. X.

A Man buys 100 pounds of Wax for 17 crowns, disposing of which he loseth 18 crowns, on 102 Crowns. I demand how many pounds he sold for one crown?

Put for the number of pounds 1 R, then work by the Rule of Three, thus constituted.

Crowns	Pounds	Crowns	Pounds
17	100	102 ?	600
1	1 R	102 — 18 ?	48 R.

So there will be equation between 84 R of pounds and 600 pounds. Divide therefore 600 by 84, and the value of 1 R will be 7 pounds and $\frac{1}{7}$, and so many pounds he sold for one crown, and lost 18 crowns on 102 by the bargain, which thus I prove:

Pounds	Crowns	Pounds	Crowns
100	17	600 ?	102
7 $\frac{1}{7}$	1	600 ?	84

Where you see he laid out 102 crowns for 600 pounds, and instead thereof received but 84 crowns, that is 102 — 18 crowns.

Question XI.

A certain man agrees with a servant for 12 months service, for ten crowns and a coat; but at the end of seven months, he gives him the coat and two crowns

crowns. I demand then at what rate he esteemed the coat.

Put for the price of the coat 1 R of crowns, and say by the rule of Three. If 12 months require 1 R + 10 crowns, how much will one month require? and you shall find that it will require $\frac{1\,R + 10}{12}$ of crowns: Again say, If 7 months require 1 R + 2 crowns, how much will one month require, and you shall find it to be $\frac{1\,R + 2}{7}$ as here under appears.

Month	Crowns	Month	Crowns
12	1 R + 10	1?	$\frac{1\,R + 10}{12}$
7	1 R + 2	1?	$\frac{1\,R + 2}{7}$

Therefore there will be an equation between $\frac{1\,R + 10}{12}$ and $\frac{1\,R + 2}{7}$ seeing both are the reward of one month, which equation by cross multiplication is reduced to 7 R + 70, and 12 R + 24 take away 24 from both parts, and the equation will be between 7 R + 46 and 12 R. Again, take away 7 from both parts, then it will be 46, equal to 5 R; divide 46 by 5, and the price of a root, and so of the coat is $9\frac{1}{5}$ crowns, as appears here under. The reward of 12 months is $19\frac{1}{5}$, and of 7, $11\frac{1}{5}$.

Months	Crowns	Months	Crowns
12	$19\frac{1}{5}$	7?	$11\frac{1}{5}$

Question XII.

A certain Citizen agrees with a slothful servant for 30 days, that every day he wrought he would give him 7 Groats; but for every day that he idled, and wrought not, he was to allow his Master five Groats: when the 30 days were past, it happens, that the servant was to receive nothing from his Master, nor the Master from the Servant, I demand then, how many days he laboured, and how many he idled?

Put 1 R for the days of labour, and 30 — 1 R for the days of idleness, and then frame the Rule of Three thus:

	Day	Groats	Days	Groats
Labour	1	7	1 R?	7 R
Idleness	1	5	30 — 1 R?	150 — 5 R

Now seeing that his work and play came to one reckoning, there will be an equation between 7 R and 150 — 5 R, add 5 R to each part, and the equation will be between 12 R and 150. Divide therefore 150 by 12, and you have for the value of 1 R, $12\frac{1}{2}$, and so many days he laboured, and 17 days $\frac{1}{2}$ he idled, which is thus proved:

	Day	Groats	Days	Groats
Labour	1	5	$12\frac{1}{2}$	$87\frac{1}{2}$
Idleness	1	1	$17\frac{1}{2}$	$87\frac{1}{2}$

Where you see the reward is the same with the mulct.

Question XIII.

One sells 20 pound weight, part Saffron, and part Ginger, for 45 crowns; but he sold 1 pound of Saffron for three crowns, and 1 pound of Ginger for $\frac{1}{2}$ a crown.

The

The question is, how many pounds of each sort he sold?

Put for the Saffron 1 R of pounds, and for the Ginger 20—1 R of pounds, then by the Rule of Three work thus:

	Pound	Crowns	Pounds	Crowns
Saffron	1	3	1 R ?	3 R
Ginger	1	$\frac{1}{2}$	20 — 1 R ?	$\frac{20 - 1R}{2}$

Therefore the equation is between the sum of 3 R of crowns, and $\frac{20 - 1R}{2}$ of crowns added together, and 45 crowns, now that sum is $10 + \frac{5}{2}$ R of crowns (for $\frac{20 - 1R}{2}$ is equal to 10 1 $\frac{1}{2}$ R, to which if you add 3 R of crowns, it makes the sum $10 + \frac{5}{2}$ R) therefore the equation shall be between $10 + \frac{5}{2}$ R of crowns, and 45 crowns. Take away 10 from both parts, and it will be between $\frac{5}{2}$ R and 35. Divide therefore 25 by $\frac{5}{2}$, and you shall have 14 for the value of 1 R. And so many pounds of Saffron were sold, and 6 pounds of Ginger, which thus I prove:

	Pound	Crowns	Pounds	Crown
Saffron	1	3	14 ?	42
Ginger	1	$\frac{1}{2}$	6 ?	3

Where you see that the price of 14 pound Saffron, and 6 pound Ginger added make 45 crowns.

Question XIV.

A certain Tradesman hath 2 sorts of coyn, in number

560 pieces, worth 160 crowns; a certain part thereof is worth each piece $\frac{1}{3}$ of a crown, and each piece of the rest $\frac{1}{4}$ of a crown. I demand the number of the first and later sort of money?

Put 1 R for the first, and 560— 1 R for the later, and then constitute the Rule of Three after this manner:

Money	Crown	Mony	
1	$\frac{1}{3}$	1 R?	$\frac{1}{3}$ R
1	$\frac{1}{4}$	560 — 1 R?	$\frac{560-1\,R}{4}$

The fourth number found is equal to 160 crowns, and the sum of their numbers makes 140 + $\frac{1}{12}$ R (for $\frac{560-1\,R}{4}$ is equal to 140 + $\frac{1}{4}$ R, to which if you add $\frac{1}{3}$ R, makes the sum 140 + $\frac{1}{12}$ R.) There is therefore an equation between 140 + $\frac{1}{12}$ R, and 160 crowns. Take 140 from both parts, and then the equation is between $\frac{1}{12}$ and 20, Divide therefore 20 by $\frac{1}{12}$, and you have 240 for the value of 1 R, and so much money there was of the first sort, of which each piece was worth $\frac{1}{3}$ of a crown of the latter kind 320, each worth $\frac{1}{4}$ of a crown, Thus proved;

Money		Money	Crowns
1	$\frac{1}{3}$	240?	80
1	$\frac{1}{4}$	320?	80

Where you see the numbers in the fourth place make 160 crowns.

Question XV.

In the Army of the Emperour, the number of the

Infantry

Infantry were octuple to the number of the Cavalry, among them there is distributed 392000 crowns, ſo as that every Foot Soldier had 5 crowns, and every Horſeman 16. The queſtion is, Of how many Horſemen the Army conſiſted, and of how many Footmen?

Put 1 R for the Horſemen, and 8 R for the Foot, according to the condition of the queſtion, and then conſtitute the Rule of Three thus:

	Crowns	Horſemen	Crowns	
Horſe 1	16	1 R	16 R	56 R
Foot 1	5	Foot 8 R	40 R	

Therefore 56 R of crowns ſhall be equal to 392000 crowns: wherefore divide 392000 by 56, and you ſhall have 1 R 7000 for the number of Horſemen, therefore the Foot ſhall be 56000, eight times as many, and ſo there will be diſtributed to the Horſemen 112000 crowns; and to the Foot 280000, which together make 392000 crowns.

Queſtion XVI.

A man hath a certain ſum of Money in a purſe, which a ſtander by judgeth to be 600 crowns, whoſe errour he thus corrects. If to what I have in this purſe, there be added $\frac{1}{2}$, $\frac{1}{3}$ and $\frac{1}{4}$, and from the ſum there be ſubſtracted $\frac{1}{12}$ part of my money, then I ſhould have 600 crowns. The queſtion is, How many crowns he had in the purſe?

Put 1 R for the number of crowns. If the parts $\frac{1}{2}$ R, $\frac{1}{3}$ and $\frac{1}{4}$ R, which together make 1 $\frac{1}{12}$ R, be added to 1 R, the whole makes 2 $\frac{1}{12}$ R; take away $\frac{1}{12}$ R, and there will remain 2 R equal to 600. Divide therefore 600 by 2, and you have 300 for the value of 1 R, and ſo much money was in the purſe. For if you add the $\frac{1}{2}$, the $\frac{1}{3}$, and $\frac{1}{4}$, to wit 150, 100, and 75, it will make the ſum of 625, and taking away $\frac{1}{12}$, to

wit 25, the rest will be the number 600, which resolves the question.

Question XVII.

A certain Traveller goes 9 miles a day, another Traveller after the tenth day past, begins his journey from the same place, and goes every day 14 miles. I demand in how many days the latter will overtake the first.

Put 1 R for the number of days, therefore the first over and above 90 miles, which he hath gone in ten days, goeth in 1 R of days, besides 9 R of miles, seeing that every day he goeth 9 miles. But the latter going 14 miles each day, goeth 1 R of days, 14 R of miles, and because the first of necessity in 1 R of days goes as many miles together with 90, which he went in ten days as the latter went in 1 R of days; sith that then they are to meet together, the equation shall be between 9 R+90 and 14 R. Take away 90 from both parts, and it will be between 90 and 5R. Divide therefore 90 by 5, and 1 R makes 18. Therefore in 18 days they shall come together. For the first in 18 days went 162 miles, which added to 90, he made the first 10 days, makes 252 miles, which the latter went in 18 days.

Question XVIII.

A Traveller goes nine miles a day, another Traveller after the end of ten days begins the same journey. I demand, how many miles a day the latter ought to travel, that so in 18 days he may overtake the first.

Put 1 R for the miles. Therefore in 18 days he will have travelled 18 R of miles. Seeing therefore that the first travelling every day 9 miles, went in 18 days 162 miles, and adding thereto 90, which he

went

went the firſt ten days before the ſecond ſet forth, it is manifeſt, that he travelled 252 miles. Therefore the equation is between 18 R and 252. Divide 252 by 18, and you have 14 for the value of 1 R, and ſo many miles the latter ought to travel, to overtake the firſt in 18 days.

Queſtion XIX.

A certain man dying, made his Will and Teſtament; leaving 3000 Crowns to be diſtributed between his Wife, Son, and two Daughters, on this condition, that the Portion of the Son might be double to that of the Mother, and the Portion of the Mother double alſo to the Portion of each of the Daughters. The queſtion how much each ones Portion was?

Put for the Portion of one of the Daughters 1 R, for the Mothers Portion 2 R, and for the Sons Portion 4 R. So there will be an equation between 8 R and 3000 Crowns.

The Portion of	one Daughter	1R	that is	375	Cro.
	the other Daugh.	1R		375	
	the Mother	2R		750	
	the Son	4R		1500	

Divide 3000 by 8, ſo the value of 1 R will be 375, the Portion of one of the Daughters, and therefore the Mothers Portion will be 750, and the Sons 1000.

Queſtion XX.

A certain man receiveth of a Merchant, a quantity of Saffron for 10 Crowns; and again he receives of the ſame man 24 pounds more of Saffron, at length he returns to him 30 pounds thereof again, and the Merchant computing the

the price of the Saffron restoreth to him 14 Crowns. I demand the price of a pound of that Saffron.

Here you see 10 crowns, + 24 pounds to be the whole debt which the Buyer owed to the Merchant, and in like manner 36 pounds − 14 crowns. Therefore there will be an equation between 10 crowns + 24 pounds and 30 pounds − 14 crowns. Add 14 crowns on both parts, and the equation will be between 24 crowns + 24 pounds and 30 pounds, take away 24 pounds from both parts, and it will be between 24 and 6 pounds. Divide 24 by 6, and you have 4 for the root, and so many crowns one pound of Saffron is worth, which I prove thus, $2\frac{1}{2}$ pounds are bought for 10 crowns, and so the Buyer received of the Merchant $26\frac{1}{2}$, which were worth 106 crowns. If therefore to the Merchant there be restored 30 pounds, the Merchant oweth to the Buyer $3\frac{1}{2}$, seeing he received only $26\frac{1}{2}$ but $3\frac{1}{2}$ pound are worth 14 crowns, which the Merchant rendered to the Buyer.

Question XXI.

Two men enter into Fellowship in Trade, now the second brings with him double the money that the first brings, and 5 Crowns over and above. They gain by their Traffick 960 Crowns, of which the first takes to himself 300 Crowns, and the second 660 I demand how much each put in bank?

Put for the first 1 R, and for the second 2 R + 5, the sum of both together is 3 ℞ + 5, which have gained 960 Crowns. Then work by the Rule of Three thus:

$$3\,℞ + 5 \qquad 960 \qquad 1\,℞\,? \qquad \frac{960\,℞}{3\,℞ + 5}$$

You shall find that the first which brought in 1 ℞, hath gained $\dfrac{960\,℞}{3\,℞ + 5}$ which number is equal

qual to 300 Crowns which he received. This equation by croſs multiplying will be reduced to 960 R equal to 900 R + 1500. Take away therefore 900 R from both parts, and there will remain 60 R equal to 1500, divide 1500 by 60; and you have 25 for one R, and ſo much the firſt put in bank, therefore the ſecond put in 55 Crowns, and is thus proved: For both put in 80 Crowns. 980. 260 25 ? 300 | 80. 960 55 ? 660.

Queſtion XXII.

Three Merchants gain together 700 Crowns, which thus they diſtribute amongſt themſelves (having regard to the ſum each one brought into Bank) ſo as that the portion of the ſecond ſurmounted the portion of the firſt by 12 Crowns, and the portion of the third ſurpaſſed that of the ſecond by 16 Crowns. I demand how much each mans portion was?

Put for the portion of the firſt man 1 R, and then ſhall the portion of the ſecond be 1 R + 12, and the portion of the third be 1 R + 28. Theſe 3 portions together make 3 R + 40, equal to 700. Take away 40 from both parts of the equation, and the equation will be between 3 R and 660. Divide therefore 660 by 3, and you ſhall have 220 for the value of 1 R, and ſo much was the firſt mans portion, ſo the portion of the ſecond ſhall be 232, and of the third 248, which all together make 700.

Queſtion XXIII.

A Caterer buys a number of Fowls, ſo as that if he had bought the $\frac{1}{3}$, $\frac{1}{4}$, and $\frac{1}{5}$ of the number, and over and above 6, he would then have 100 juſt. I demand the number of Fowls he bought?

Put 1 R for the number, whoſe $\frac{1}{3}$, $\frac{1}{4}$ and $\frac{1}{5}$ make $\frac{47}{60}$ R, which

which with 6, make $\frac{47}{60}$ R + 6 equal to 100. Take away 6 from both parts of the equation, and the remaining equation will be between $\frac{47}{60}$ R and 94. Divide therefore 94 by $\frac{47}{60}$, and the value of 1 R will be 120, to wit, the number of Fowls that were bought.

For its $\frac{1}{3}$ is 40, and its $\frac{1}{4}$ is 30, and its $\frac{1}{5}$ 24; which all together with 6 make the sum 100.

Some Examples in *ALGEBRA* concerning *Squares*.

To find two numbers in a given Excess, so as their Squares may have also a given Excess.

LEt there be sought two numbers, whose Excess is 4 and the Excess of their Squares 144.

Put for the lesser number 1 R, and therefore the greater number must be 1 R + 4, whose squares are 1 Q and 1 Q + 8 R + 16, their excess is 8 R + 16, which ought to be equal to 144. Take away 16 from both parts of the equation, and the equation will be between 8 R and 128. Divide therefore 128 by 8, and you shall have for the value of 1 R, 16 for the lesser number, the greater therefore will be 20, that the excess may be 4. The squares of those 2 numbers are 256, and 400, whose difference or excess is 144.

Two numbers being given, to find another, with whc h multiplying both the numbers, makes the greater number a square, and its lesser the side of that square.

LEt the two given numbers be 200 and 5, and let the sought number be put 1 R. Now 1 R multiplied in 200, produceth 200 R, and 1 R multiplied in 5, makes 5 R, which ought to be the side, and so multiplied

multiplied in it self, ought to make a number equal to 200. But 5 R multiplied in it self, makes 25 Q, the equation therefore is between 200 R and 25 Q. Divide therefore 200 by 25, and the value of 1 R is 8, which multiplied in 200, make the square 1600, whose side is the number 40, which is also produced by the multiplication of 8 in 5.

Some Examples relating to *Cubes*.

To find a number, which multiplied in it self, and the product multiplied by some given number, may produce a number in a given proportion to the Cube *of the found number.*

Let the given number be 20, find another number, which being first multiplied in it self, and then the product multiplied by the given number 20, may produce a number in a quintuple proportion to the Cube made of the found number.

Put for the sought number 1 R, which multiplied in it self make 1 Q, which also multiplied by 20, makes 20 Q, the Cube of 1 ℞ is 1 C, to which 20 Q ought to have a quintuple proportion; so the equation is between 20 Q and 5 C: Divide 20 by 5, and you have for the value of 1 R, 4, the number sought. This 4 multiplied in it self makes 16, and 16 multiplied by 20, makes 320, which is quintuple to 64, the Cube of 4.

To divide a given number in two parts, so as that their Cubes may make a given sum, which shall be greater than the quarter part of the Cube described of the given number.

Let the given number be 10, to be divided into two parts, whose Cubes may make 370, which number is greater than the $\frac{1}{4}$ part of the Cube of 10. Put for the first number compounded of 1 R, and half of the given number, 1 R + 5, and for the second 5 — 1 R. So these two numbers do make the given number. Their Cubes are C + 15 Q + 75 R + 125, and 125 — 75 — R + 15 Q — 1 C; their sum is 30 Q + 250: For + 1 C, and — 1 C, as also + 75 R, and — 75 R, do mutually destroy one another; and of 15 Q and 15 Q, are made 30 Q: Also 125, and 125, make 250. Therefore the equation is be tween 30 Q + 250, and 370. Take away 250 from both parts, and the equation will be between 30 Q and 120. Divide 120 by 30, and you have 4 for the value of 1 Q, and the value of 1 R, 2. The first part put 1 R + 5 shall be 7; and the second put 5 — 1 R shall be 3, The Cubes of those parts are 343 and 27; which together make 370.

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